SPACETIME CONSTRAINTS FOR BIOMECHANICAL MOVEMENTS

David C. Brogan

Department of Computer Science
University of Virginia
dbrogan@virginia.edu

Kevin P. Granata

Motion Analysis and Motor Performance Laboratory
Departments of Orthopaedic Surgery and Biomedical Engineering
University of Virginia
kpg8n@virginia.edu

Pradip N. Sheth

Department of Mechanical and Aerospace Engineering
University of Virginia
pns7q@virginia.edu

Abstract

To better understand human movements, biomechanical models must be developed that accurately describe human physiology and control strategies. Typically, biomechanical models must be adapted or hand tuned to study the countless unique users and tasks caused by human diversity and pathologic movement dysfunctions. The convergence of optimization theories, biomechanical models, and computational systems promises to alleviate these manual processes. We describe a computational technique called spacetime constraints that can be used to automatically solve for both optimal movement trajectories and joint activation torques as befitting the subject, environmental constraints, and objectives. We demonstrate the success of this technique on bipedal downhill walking by comparing our results to optimal movements and joint torques published in the literature. With this contribution to computational biomechanics, we outline a modeling framework that uses easily configurable physical models, constraints, and objective functions to determine movements and control actions.

Key Words: Biomechanics, robotics, and optimization

1. Introduction

Biomechanists face an ever-expanding family of locomotion and lifting tasks that demands analysis and synthesis of control strategies. The mechanical principles and theories of human motion that contribute to this analysis are not sufficiently general to prescribe movements for a wide variety of users and tasks while preserving efficiency, safety, and effectiveness goals. Classic examples include the inability to explain why antagonistic muscle co-contraction is recruited despite the fact it causes reduced efficiency in human movement or how compensation patterns are developed following a neuromuscular disturbance like stroke. Furthermore, there exists a rhetorical hypothesis within the biomechanics community that both movement trajectory and joint torque are modulated or adapted to accomplish desired tasks. Not only are these two control variables intricately related, but many competing objectives (speed, endurance, accuracy, etc.) inevitably contribute to the complex movements performed by humans. As a consequence, any single biomechanical model or objective function used to describe a motion is necessarily limited.

We are particularly interested in motion control problems where limb trajectories, joint torque controllers, and physiological parameters are unknown. The complexity of biomechanical analyses and the variety of user/task conditions overwhelms modeling technologies that either apply only to particular system conditions or require hand tuning. As such, we are unable to study pathologic gait, movement dysfunctions, and commonplace tasks like lifting a box where both the desired movements and torques are unknown. Computational approaches to search and optimization exist, but multiple challenges prevent their straightforward application to biomechanical applications. Human control systems are frequently modeled as layered architectures containing continuous and discrete parameters and objective functions that may not be well defined and conducive to local search strategies. For example, the existence of multiple minima in the objective functions of cyclic motions presents unique challenges to optimization techniques that typically assume initial and terminal conditions are known. We must search for state space representations and objective functions that integrate with modern optimization methods.

We are motivated by the progress of computer animation researchers who have used an optimization technique called spacetime constraints [1] to automatically generate the movements of physically simulated characters that must react to a dynamic environment. Although spacetime constraints has produced very good results for finding locally optimal solutions to articulated character animation problems, these methods have not been thoroughly tested or validated in biomechanical applications with many constraints and degrees of freedom. We use mechanical theories and biomechanical models to enhance the computational foundation of spacetime constraints. Our spacetime constraints framework supports additional flexibility by solving for cyclic (periodic) motion trajectories in addition to two-point boundary value problems. We also alter the spacetime constraints computational algorithm so the size of integration timesteps is automatically adjusted and a motion's duration is included in the state space.

By using this spacetime constraints framework, it is possible to overcome limitations in computational biomechanics and pursue application areas that were previously out of reach. We test the framework by attempting to simultaneously determine the optimal movement trajectories and joint activation torques of a simulated walking human without relying on any a priori assumptions regarding one or the other. Both the joint torques and movement trajectories can be modified as befitting the environmental constraints and objectives. The periodicity and stability required for bipedal walking present unique challenges and a non-trivial objective function to spacetime constraints implementations. Our changes to spacetime constraints permit solutions to walking problems where gait length is indeterminate and no a priori specifications constrain leg positions at the beginning and end of a cycle, so long as they form a loop. Downhill walking serves as the particular walking task because the optimal movements and joint torques are known [2] and thus serve as a basis for evaluating the optimization results. Our results indicate the spacetime constraints solution to the downhill walking problem matches the predicted theoretical behaviors.

2. Background

Spacetime constraints is an optimization method for two-point boundary value problems subject to constraints. It is commonly used to determine a set of joint torque trajectories that cause an articulated character to transition from one pre-specified configuration to another *T* seconds later while minimizing a user-defined objective function. A typical objective function minimizes the sum of joint torques during time interval *T*. This method is particularly valuable because it can "solve for a character's motion and time-varying muscle forces over the entire time interval of interest, rather than progressing sequentially through time." [1] In this section, we describe the optimization foundations of spacetime constraints and review its recent applications. We then explain how spacetime constraints can be integrated with biomechanical analyses and control.

2.1 Spacetime Constraints

The state space of the optimization problem consists of all joint trajectories and character configuration trajectories possible during the *T*-second interval. To reduce the state space, the joint and character configuration trajectories are sampled at *n* regular intervals and the resulting state vector consists of the *n* values of each joint torque and character configuration (location of the root and hierarchical joint angles). The *n* sampled joint torques and character configurations are used to compute *n* integration steps. To further reduce the state space, only those state vectors that satisfy positional and dynamic constraints are included. The first and last configurations of the character must match those that were specified by the user and Newton's Laws must be enforced at each of the *n* sampled moments during the animation. Among all state vectors that satisfy the constraints, the one with a minimal objective function value is selected. Cast in this way, the animation problem becomes a constrained optimization problem where all constraint functions must be driven to zero and both joint torques and system states are free to vary during the search process. Because the objective function evaluates the entire time interval at once, a joint torque that occurs early in the sequence is appropriately evaluated by its immediate effects and its subsequent influence on the final state. The algorithm produces a result that is spatially optimal due to its maintenance of position and dynamic constraints while providing a temporally optimal joint torque trajectory.

The authors of the seminal spacetime constraints paper [1] use a variant of sequential quadratic programming to perform an iterative, gradient-based optimization. The user provides the algorithm with an initial configuration, a final configuration, and initial guesses to populate the remaining *n-2* character configurations and *n* joint torques. We can represent the algorithm's elements symbolically by using $\vec{S}$ to represent the state vector, $\vec{C}$ to represent the vector of spatial and dynamical constraint violations, and $R(\vec{S})$ to represent the objective function. We must solve for $\vec{S}$ such that $\vec{C}=0$ and $R(\vec{S})$ is minimized. Provided an initial $\vec{S}$, three elements are computed: the constraint violations, $\vec{C}$, the Jacobian of the constraint functions,

$$J = \frac{\partial \vec{C}}{\partial \vec{S}},$$

and the Hessian of the objective function,

$$H = \frac{\partial^2 R}{\partial \vec{S}^2}.$$

A two-step process first solves for a local change in the state vector, $\vec{S}_\alpha$, that minimizes the objective function without consideration of constraint violations. To minimize the objective function, we set the deriviate $R'(\vec{S}) = 0$. We use a second-order Taylor series expansion to approximate R' at $\vec{S}$:

$$R'(\vec{S}) = 0 = \frac{\partial R}{\partial S} + \frac{\partial^2 R}{\partial S^2}(\vec{X} - \vec{S}) \cdot$$

Letting $\vec{S}_\alpha = \vec{X} - \vec{S}$, we can solve:

$$-\frac{\partial R}{\partial \vec{S}} = H\vec{S}_\alpha \cdot$$

Although we know $R(\vec{S})$ is minimized at $\vec{S} + \vec{S}_\alpha$, spatial and dynamical constraints may be violated. We must find a second change in the state vector, $\vec{S}_\beta$, that preserves the minimization of R while eliminating any constraint violations at $\vec{S} + \vec{S}_\alpha$. We pursue a second step that projects $\vec{S}_\alpha$ to the null space of the constraint Jacobian:

$$J(\vec{S}_\alpha + \vec{S}_\beta) + \vec{C} = 0 \cdot$$

Note both the constraint vector, $\vec{C}$, and the Jacobian, J, are evaluated at $\vec{S} + \vec{S}_\alpha$. Solving for $\vec{S}_\beta$ in:

$$-\vec{C} = J(\vec{S}_\alpha + \vec{S}_\beta)$$

drives $\vec{C}$ to zero.

The new value of $\vec{S}$ is incremented by $(\vec{S}_\alpha + \vec{S}_\beta)$. The algorithm computes new state vectors until the reduction in the objective requires violating constraints. The earliest results of this method demonstrated a planar, three-link system accomplishing leaping tasks.

More recent applications of spacetime constraints have demonstrated the generalizability of the technique to other animation problems. Gleicher [3] has used the technique to adapt motion capture clips (an actor's kinematic state sampled 60 frames per second) to animate characters of different sizes while preserving important elements (footplants, joint angles, distance traveled) of the original motion. Rose et al. [4] use spacetime constraints to interpolate joint angles when blending from one motion capture clip to another. Winzell [5] finds that the search for *n* discretized joint torque and state vectors can be replaced by a search for a smaller number of vectors, each of which serves as one control point of a multidimensional B-spline surface. In exchange for this reduction in state space, the trajectories of system state and joint torques must be smooth. As the animated character becomes more complex, hand-crafted hierarchical clustering of degrees of freedom [6] reduces the size of the search space, but heuristics are required to artificially constrain the simplified system.

2.2 Biomechanics

Although first developed for simulated characters in a graphical world, there is little question spacetime constraints is an important technology for studying the biomechanics of movements. We are developing extensions of spacetime constraints for its application to biomechanical movements because we foresee a convergence between optimization theories, biomechanical models, and computational systems. Spacetime constraints provides the opportunity to use guided, automated search algorithms to solve control problems for which analytical solutions cannot be found. It permits the simultaneous solution of system state and joint torque trajectories without relying on a priori assumptions regarding one or the other. The absence of such assumptions permits the study of such pathologic movement dysfunctions as leg-length discrepancy, range-of-motion limitations, and velocity constraints caused by spastic hypertonia.

The standardized definition of the spacetime constraints state vector, constraints vector, and objective function facilitates the reconfiguration and reuse of biomechanical models (see figure 1). Constraint functions can easily be changed to model different pathologies and predefined gaits, or the gaits themselves can be unspecified and solved by the optimizer. Objective functions may be rigidly defined or more loosely specified as a linear combination of goals with weightings that change throughout optimization. The scope of the models themselves may be easily changed to include additional control layers, feedback loops, and actuators. The freedom to quickly adjust models, constraints, and objectives while optimizing only a few or many system parameters is very valuable.

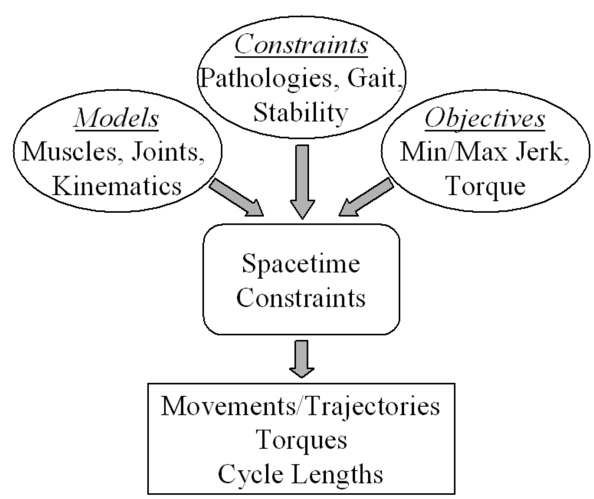

Figure 1: Spacetime constraints uses models, constraints, and objective functions to determine movements and control actions.

In order to demonstrate the applicability of spacetime constraints to biomechanical systems, we used the spacetime constraints framework to compute the optimal joint torques and limb movements required to produce stable biped walking. Previous simulations have pre-specified movement trajectory and require the actuation torques to control and maintain that movement pattern [7]. Others have pre-specified the actuation torques then solved for the resulting movement trajectory [8, 9, 10, 11]. Some advanced models have derived input joint torques from measured EMG data [12]. To our knowledge, none have simultaneously solved for trajectories and torques.

3. Methods

The goal of the current study is to implement a simulation that determines the movement trajectory simultaneously with the optimum activation torques. In this experiment, the simulated walker is placed on a downward-sloped plane such that if it were passively simulated (with no internal torque sources) it would settle into a steady gate. The movement trajectories of the passive-dynamic walker require zero-torque activation and are known to be the optimal trajectories for actively powered walkers as well [2, 7]. To investigate the validity of the spacetime constraints framework, we permitted the algorithm to explore the state space containing non-zero joint torques. However, the resulting spacetime constraint solutions are zero torque and movement trajectories compare favorably to the known theoretical optima generated by simulated passive-dynamic walkers.

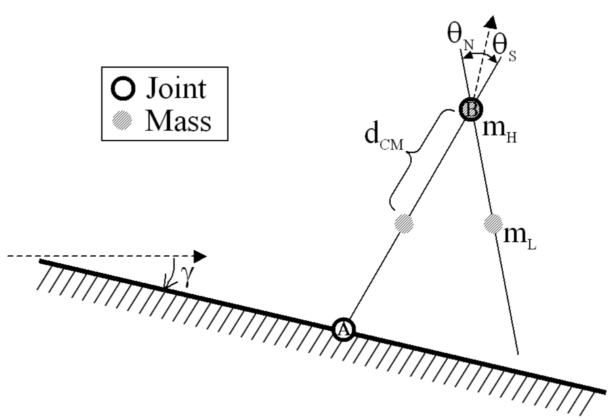

Figure 2: A bipedal walker. Lumped masses are positioned at the hip (m_H) and on each leg (m_L). The spacetime-constraints walker applies torques at the stance foot (A) and at the hip (B).

3.1 Passive-Dynamic Walker

Our simulation of a passive-dynamic walker (see figure 2) models a planar knee-less walker including two legs of mass m_L, joined by a revolute joint located at the hip with point mass, m_H, based upon successful walking models published elsewhere [2, 13]. Leg masses, m_L, are located at a distance d_{CM} from the hip along a line joining the hip to the point-foot. The walker moves along a plane of slope γ with respect to horizontal. A time-dependent vector $\boldsymbol{\theta} = [\theta_S, \theta_N]^T$ represents the walker configuration where θ_S and θ_N are the angles of the stance-leg and non-stance-leg versus ground normal. During walking only one foot is in contact with the ground at any time, i.e. single-stance. Ground clearance of the swing-leg is ignored in this treatment because simple mechanisms such as prismatic joints [14] are readily established that do not influence walker dynamics. The governing equations of motion include the differential equations of movement that model swing phase dynamics and the conservation of angular momentum that models foot-strike transitions. These models are implemented using classical homogeneous forward-integration techniques. To confirm steady state behavior, the forward-integration model is simulated for 100 consecutive steps. The model is initialized with leg angles of ±15º, stance leg velocity of 60 deg/sec and swing leg velocity of 0 deg/sec. Because this initial state is within the limit-cycle basin of attraction the behavior undergoes transient state changes in the first few steps but quickly converges within four decimal places to the steady state behavior describing the natural dynamics of the system.

3.2 Spacetime-Constraints Walker

The masses and limb lengths of the spacetime-constraints walker are modeled exactly as the passive-dynamic walker. However, the spacetime constraints algorithms permit non-zero torques about the hip and stance leg contact point and the classical homogeneous forward-integration techniques are used to compute constraint violations, not to compute movement explicitly. The state vector, $\vec{S}$, contains the entire movement trajectory and is composed of a scalar, *dt*, and angle vector, $\boldsymbol{\theta}_t = [\theta_{S\,t}, \theta_{N\,t}]^T$ for every time increment t = 1…n. By including the time-increment, *dt*, as a variable the swing period is permitted to approach an optimum. Note that the full angle vector includes two legs at *n* time increments represented in a 1-by-2n vector. The velocity and acceleration vectors are also 1-by-2n column vectors determined by multiplying the position vector $\boldsymbol{\theta}_t$ by numeric differentiation matrices, $\dot{\boldsymbol{\theta}} = V\,\boldsymbol{\theta}_t$ and $\ddot{\boldsymbol{\theta}} = A\,\boldsymbol{\theta}_t$:

$$V = \frac{1}{dt}\begin{bmatrix} 1 & 0 & 0 & 0 & \ldots & 0 & -1 \\ -1 & 1 & 0 & 0 & \ldots & 0 & 0 \\ 0 & -1 & 1 & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \ldots & -1 & 1 \end{bmatrix}$$

$$A = \frac{1}{dt^2}\begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 1 & -2 \\ -2 & 1 & 0 & \dots & 0 & 0 & 1 \\ 1 & -2 & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & -2 & 1 \end{bmatrix}$$

where *dt* is the time increment. The non-linear, second-order differential equations of motion are computed for each time step using these position, velocity, and acceleration vectors.

An arbitrary motion trajectory θ_t requires a set of actuation torques, $\tau_t = [\tau_{A,t}, \tau_{H,t}]^T$ and value of *dt* that satisfies the equations of motion and produces a zero-constraint vector, $\vec{C}$. Here $\tau_{A,t}$ represents the ankle torque of the stance leg and $\tau_{H,t}$ represents the hip torque at time t. The constraint vector, $\vec{C}$, limits feasible joint angles ±90 degrees to prevent solutions wherein the walker performs flips and whirling gait behaviors. An upper bound on the time increment, *dt*, is also established to limit the total swing period less than 2π, i.e. the swing leg is not permitted to swing back-and-forth multiple times within a single step. Finally, the constraint vector requires the system state at the initial and final time-points to align in order to assure periodicity and conservation of momentum. Using constrained optimization routines in MATLAB it is possible to solve for the movement trajectory θ_t that minimizes the objective function, $R(\vec{S})$ = sum of squares of actuation torques throughout the stride cycle ($min\ \Sigma\tau_t^T * \tau_t$), where a full stride cycle is the time between consecutive foot strikes

3.3 Results

The spacetime constraints framework successfully generates the movement trajectories for passive walking. To demonstrate the walker converges on a stable trajectory, we select initial conditions for the spacetime-constraints walker that are well outside the basin of attraction for the natural dynamics. In two independent analyses the configuration is initialized at leg angles of ±1° (or ±30°) with initial and final stance and swing leg velocities of ±3 deg/sec (or ±90 deg/sec). In both cases, the initial state vector required by the spacetime constraints algorithm is arbitrarily initialized with values that vary linearly between the positive and negative extremes. In both experiments, the simulation converges on movement trajectories that are similar to the simulated passive-dynamic walker. The leg angles and velocities are identical to the passive-dynamic walker's forward-integration results. The trajectory successfully identifies the passive walking behavior illustrated by the fact that the actuation torques approach zero, indicating a homogeneous solution.

4. Discussion

In our experiments with bipedal walking, we have extended spacetime constraints in multiple ways. The duration of a walk cycle can vary and resides under the control of the optimization algorithm. Such an extension was unnecessary for computer animators who prefer to specify when events start and end, but human movement certainly capitalizes on efficiencies obtained by changing a movement's pace. Unlike the traditional formulation of spacetime constraints, cyclical movements like walking need not have a final state explicitly defined, rather they need only ensure that the final state precede the initial state as the cyclical motion wraps around and begins again. The user no longer specifies a final state to be used as a constraint; rather the system creates a formulaic constraint that requires the final state to transition to the initial state upon integration.

For some tasks, the spacetime constraints user can define an initial guess that provides a starting point for the local optimization step. For more complicated applications, an optimization method that searches more broadly is required. We aim to develop evaluation criteria that preempt solutions that are bound to fail, and thereby reduce the search space. Can we, for example, identify during the first second of a walking maneuver that the resulting gait will be either energy inefficient or impossible to maintain? Stability analysis is a theoretical tool that provides such an opportunity to expedite the local, gradient search policy used by spacetime constraints. In the context of bipedal walking, a gradient-based search algorithm is vulnerable to falling into a local minima where additional joint torques are required to compensate for a poor, greedy decision that was made many iterations earlier. Augmented with a stability analysis algorithm, the search algorithm can examine the cyclical stability of a system state trajectory and improve the local search characteristics with good predictive evaluations.

Although our research has validated the application of spacetime constraints to biomechanical movements, we observe many opportunities to further develop its foundation and to expand its impact. Much as laser scanning devices and computer assisted design tools permit the mass production of customized prostheses, we envision biomechanical treatments that record a disabled person's movements and design assistive devices catered to their pathologies. We must explore algorithms that exchange computational effort and automated search for theoretical purity. These computational tools must support the creative and intuitive abilities of scientists, engineers, and physicians who quickly conjure experimental conditions and potential solutions, which then undergo batteries of automated testing and analysis.

5. Conclusion

Spacetime constraints permits the development of a robust motion optimization system that adapts to a variety of complex biomechanical limitations. Understanding how movement dysfunctions are related to pathologic constraints in neuromuscular dynamics is a significant challenge in clinical rehabilitation. For example, spasticity imposes a constraint on muscle lengthening velocity [15] but it is unclear how it affects movement in complex tasks such as walking. Although spasticity influences both joint torques and movement trajectory, the neurocontroller clearly adapts to the imposed constraints. The feasibility of using spacetime analyses to optimize both movement and control may permit future assessments to investigate the change in movement and joint torques following onset or treatment of neuromuscular spasticity. Ongoing studies are in the process of validating model predictions of multi-segment movement tasks with measured human movement data [16].

6. References

[1] A. Witkin and M. Kass, Spacetime Constraints. *Proceedings of SIGGRAPH*, 1988, 159-168.

[2] T. McGeer, Passive bipedal running, *Proc.R.Soc.Lond.B*. 1990a; 240: 107-34

[3] M. Gleicher, Retargetting Motion to New Characters, *Proceedings of SIGGRAPH*, 1998, 33-42.

[4] C. Rose, B. Guenter, B. Bodenheimer, and M. Cohen, Efficient Generation of Motion Transitions using Spacetime Constraints, *Proceedings of SIGGRAPH*, 1996, 147-154.

[5] P. Winzell, An Implementation of Spacetime Constraints Approach to the Synthesis of Realistic Motion. Master's Thesis, 1998, Liköping Institute of Technology, Sweden.

[6] C. Liu and Z. Popović, Synthesis of Complex Dynamic Character Motion from Simple Animations, *Proceedings of SIGGRAPH*, 2002, 408-416.

[7] J. Apkarian, S. Naumann, and B. Cairns, A three-dimensional kinematic and dynamic model of the lower limb, *J.Biomechanics*, 22, 1989, 143-55.

[8] C. Chang, D. Brown, D. Bloswick, and S. Hsiang, Biomechanical Simulation of Manual Lifting Using Spacetime Optimization, *Journal of Biomechanics*, 34, 2001, 527-532.

[9] H. Hemami amd B.F. Wyman, Modeling and control of constrained dynamic systems with application to biped locomotion in the frontal plane, *IEEE Transactions on Automatic Control*, 24, 1979, 526-35.

[10] S. Onyshko and D.A. Winter, A mathematical model for the dynamics of human locomotion, *Journal of Biomechanics*, 13, 1980, 361-8.

[11] M.G. Pandy and N. Berme, Quantitative assessment of gait determinants during single stance via a three-dimensional model, Part I. Pathologic gait. *Journal of Biomechanics*, 22, 1989, 725-733.

[12] S.C. White and D.A. Winter, Predicting muscle forces in gait from EMG signals and musculotendon kinematics, *Journal of Electrophysiological Kinesiology*, 2, 1993, 217-230.

[13] M. Garcia M., A. Chatterjee, A. Riuna, and M. Coleman, The simplest walking model: Stability, complexity and scaling, *Journal of Biomechanical Engineering*, 120, 1998, 281-288.

[14] A. Goswami, B. Thuilot, and B. Espiau, Compass-like biped robot. Part I: Stability and bifurcation of passive gaits, 1996; 2996, INRIA.

[15] A.E. Tuzson, K.P. Granata, and M.F. Abel, Spastic Velocity Threshold Constrains Functional Performance in Cerebral Palsy *Arch. PhysMed. Rehabilitation* (in press).

[16] C. Huang, Optimized Dynamic Lifting Biomechanics. *M.S.Thesis, Univiversity of Virginia*, 2002.