Feb 10 Slides

1. Power Sets
2. Disjoint Sets
3. Set-builder Notation
4. Sequences
5. Cartesian Products

Reminder: Subset Definition

Set A is a ***subset*** of set B

$A \subseteq B$

If & only if **all elements of A are also in B**

Power sets

4.1.3 Power Set

The set of all the subsets of a set, A, is called the *power set*, $\text{pow}(A)$, of A. So

$$B \in \text{pow}(A) \quad \text{IFF} \quad B \subseteq A.$$

For example, the elements of $\text{pow}(\{1, 2\})$ are $\emptyset$, $\{1\}$, $\{2\}$ and $\{1, 2\}$.

Power sets -- Break-outs

1.) What is the power-set of {}?

2.) What is the power set of {a, b, c}

3.) What is the power set of { W, X, Y, Z }

Power sets -- Break-outs

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2.) What is the power set of {a, b, c}

{ {}, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}

3.) What is the power set of { W, X, Y, Z }

Can we see a rule/pattern to determine the cardinality of a powerset?

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$$|\mathcal{P}(X)| = 2^{|X|}$$

Other Notations

$$\mathcal{P}(X)$$

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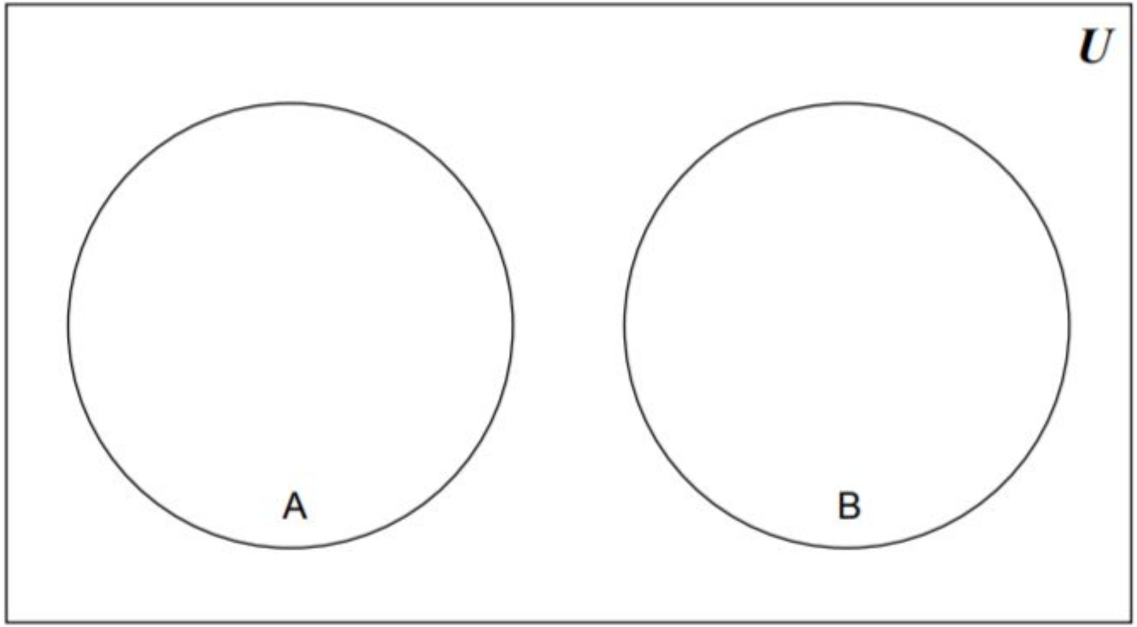
$$\mathscr{P}(X)$$

$$\wp(X)$$

Disjoint Sets



Disjoint Sets

- Formal definition for disjoint sets: *two sets are disjoint if their intersection is the empty set*
- Further examples:
  - {1, 2, 3} and {3, 4, 5} are not disjoint

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  - {New York, Washington} and {3, 4} are disjoint
  - {1, 2} and ∅ are disjoint
    - Their intersection is the empty set
  - ∅ and ∅ are disjoint!
    - Their intersection is the empty set

Ways to describe Sets

- Listing out the elements of a set works well for sets that are small and finite. What about larger sets?
- Set Builder Notation!

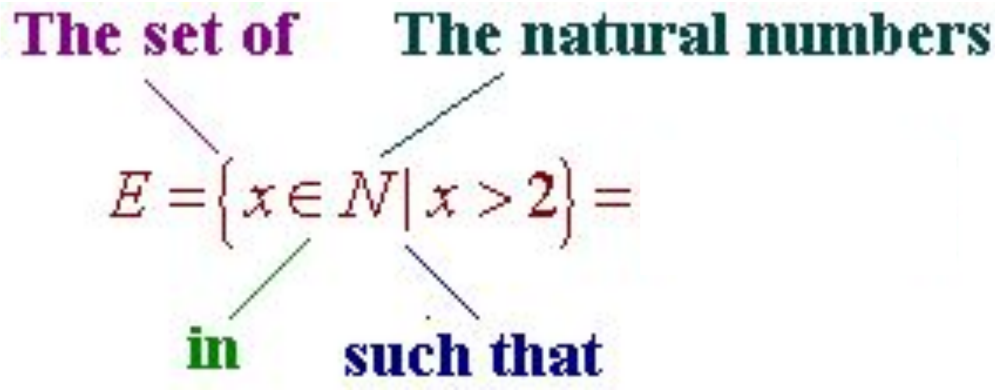
$$S = \{x \in A | x \text{ is blue}\}$$

The set of all x in A

Vertical Bar is read “such that”

Property (or properties) of x that must be met in order to be an element of S

Set-Builder Notation



https://ltcconline.net/greenl/courses/152a/definitions/SETS.HTM

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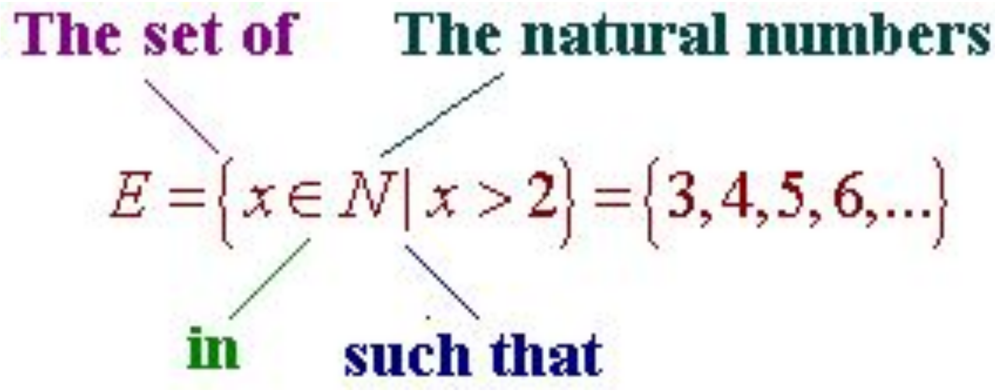

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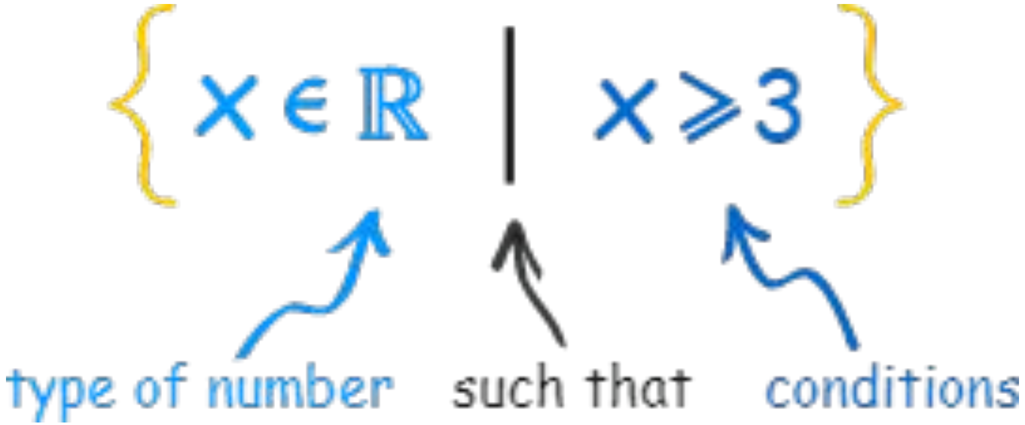

https://www.mathsisfun.com/sets/set-builder-notation.html

Set-Builder Notation

Let’s ***formalize*** our set operators in “set-builder notation”

Quick Side-Note:

-We will need to link together multiple “conditions” with “and’s”, “not’s” and “or’s”

Special symbols:

$\lor$ is “or” (notice similarity to $\cup$)

$\land$ is “and” (notice similarity to $\cap$)

$\neg$ is “not”

Set-Builder Notation -- My turn!

Intersection $S \cap T$: the elements that belong both to S and to T.

$$S \cap T =$$

For Reference:

$\vee$ is "or"

$\wedge$ is "and"

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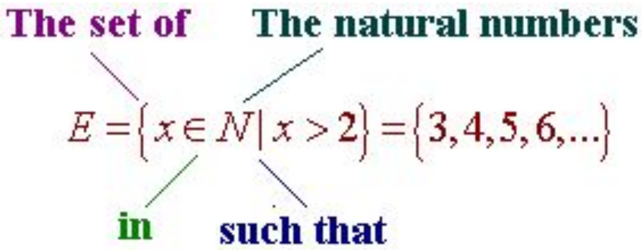

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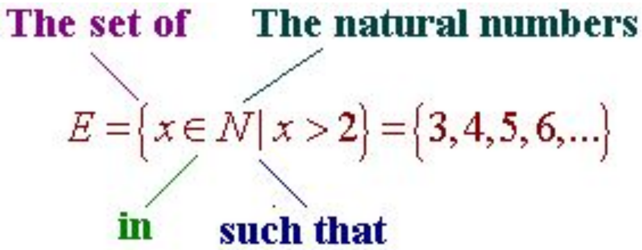

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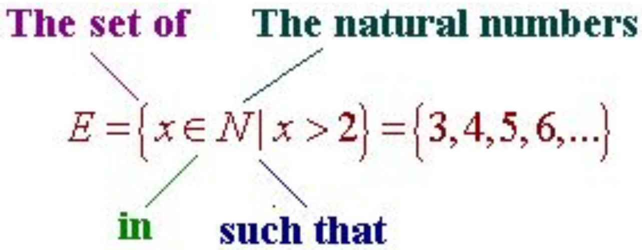

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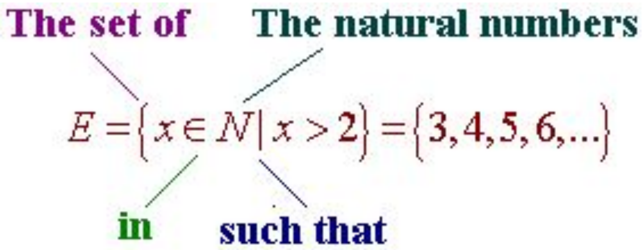

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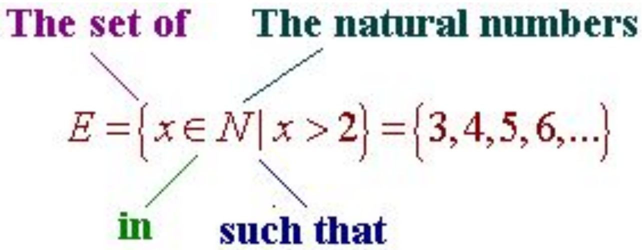

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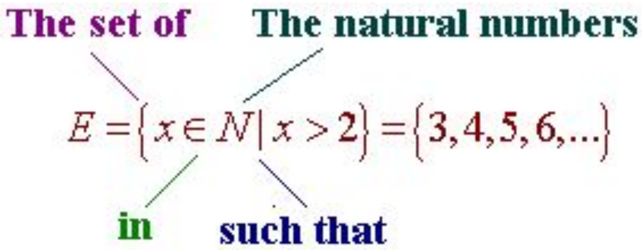

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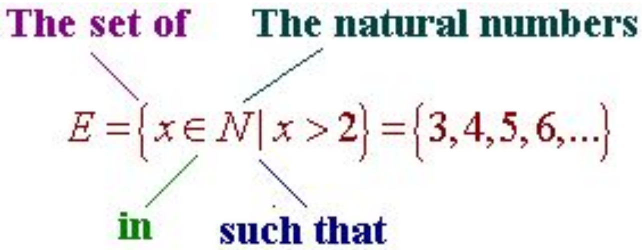

Set-Builder Notation -- Your turn!

Union $S \cup T$: the elements that belong either to S or to T (or both).

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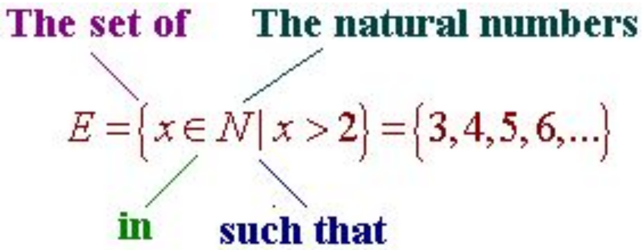

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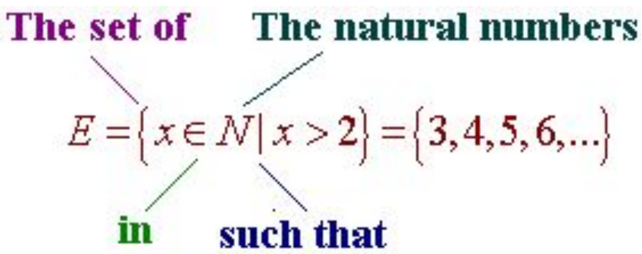

Set-Builder Notation -- Your turn!

Difference $S \setminus T$: the elements that belong to S but not to T.

$$S \setminus T =$$

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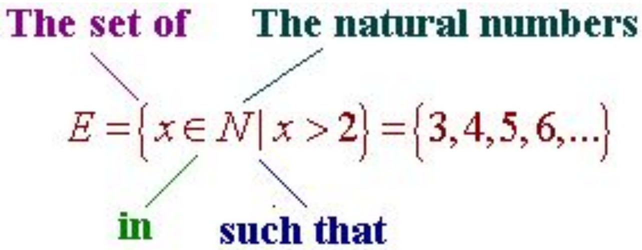

Set-Builder Notation -- Your turn!

Difference $S \setminus T$: the elements that belong to S but not to T.

$$S \setminus T = \{x \in U \mid x \in S \wedge x \notin T\}$$

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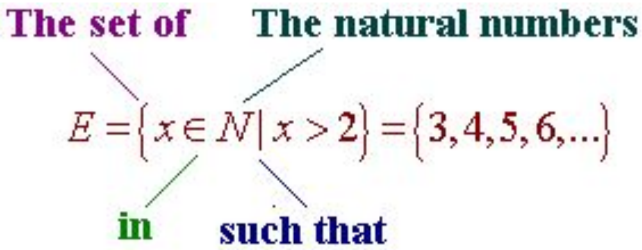

High Level: Sets vs Sequences

Both can:
-Contain anything
-Can have a sequence of sequences, set of sets, sequence of sets, etc
-Cannot be modified

Sets:
-no duplicates
-no order
-has cardinality

Sequences:
-can have duplicates
-has order
-has length

Lists, Arrays, Ordered pairs, Tuples, etc!

Cartesian Product of Sets

Ordered Pair: An ordered pair is a **sequence with 2 elements**. It is a pair of objects where one element is designated first and the other element is designated second, denoted (a, b).

Cartesian Product: The Cartesian product of two sets A and B, denoted $A \times B$, is the set of all possible ordered pairs where the elements of A are first and the elements of B are second.

In set-builder notation, $A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$.

Cartesian Product of Sets

$$\{1, 2\} \times \{3, 4, 5\}$$

$$= \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}$$

Cartesian Product of Sets

$$|\{1, 2\} \times \{3, 4, 5\}|$$

$$= |\{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}|$$

$$= 6$$

Cartesian Product of Sets

Your Turn: What is $\{1, 2\} \times \{2, 3\}$?