High Level: Sets vs Sequences

Both can:
-Contain anything
-Can have a sequence of sequences, set of sets, sequence of sets, etc
-Cannot be modified

Sets:
-no duplicates
-no order
-has cardinality

Sequences:
-can have duplicates
-has order
-has length

Lists, Arrays, Ordered pairs, Tuples, etc!

Cartesian Product of Sets

Ordered Pair: An ordered pair is a **sequence with 2 elements**. It is a pair of objects where one element is designated first and the other element is designated second, denoted (a, b).

Cartesian Product: The Cartesian product of two sets A and B, denoted $A \times B$, is the set of all possible ordered pairs where the elements of A are first and the elements of B are second.

In set-builder notation, $A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$.

Cartesian Product of Sets

$= \{(1, 3), (3\ (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}$

Cartesian Product of Sets

$$|\{1, 2\} \times \{3, 4, 5\}|$$

$$= |\{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}|$$

$$= 6$$

Cartesian Product of Sets

Your Turn: What is $\{1, 2\} \times \{2, 3\}$?

Cartesian Product of Sets

Your Turn: What is $\{1, 2, 3\} \times \{2, 3, 4\}$?

Answer: $\{ (1, 2), (1, 3), (2, 2), (2, 3) \}$

Cartesian Product of Sets

We can write **$\{1, 2, 3\}x\{1, 2, 3\} = \{1, 2, 3\}^2$** to take the Cartesian Product of a set with itself.

Your Turn: What is $\{1, 2, 3\}^2$?

Cartesian Product of Sets

We can write $\{1, 2, 3\}^2$ to take the Cartesian Product of a set with itself.

Your Turn: What is $\{1, 2, 3\}^2$?

Answer: $\{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

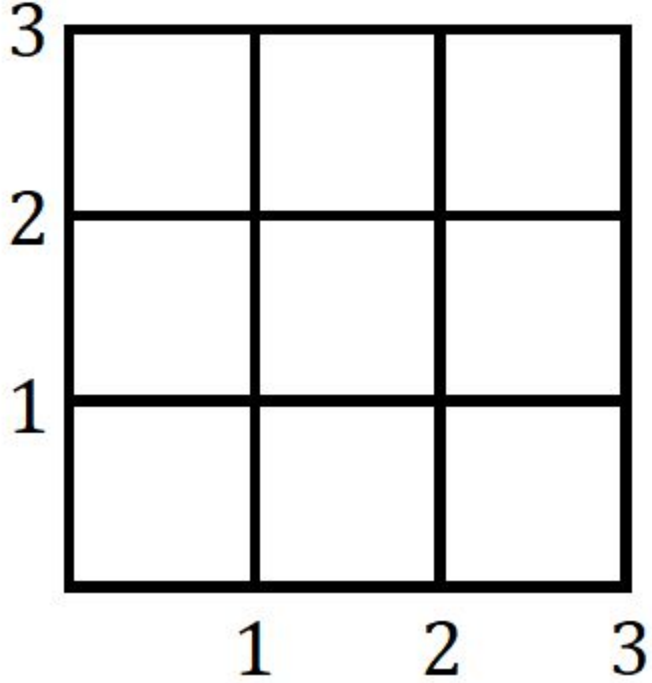
Cartesian Product of Sets

We can write $\{1, 2, 3\}^2$ to take the Cartesian Product of a set with itself.

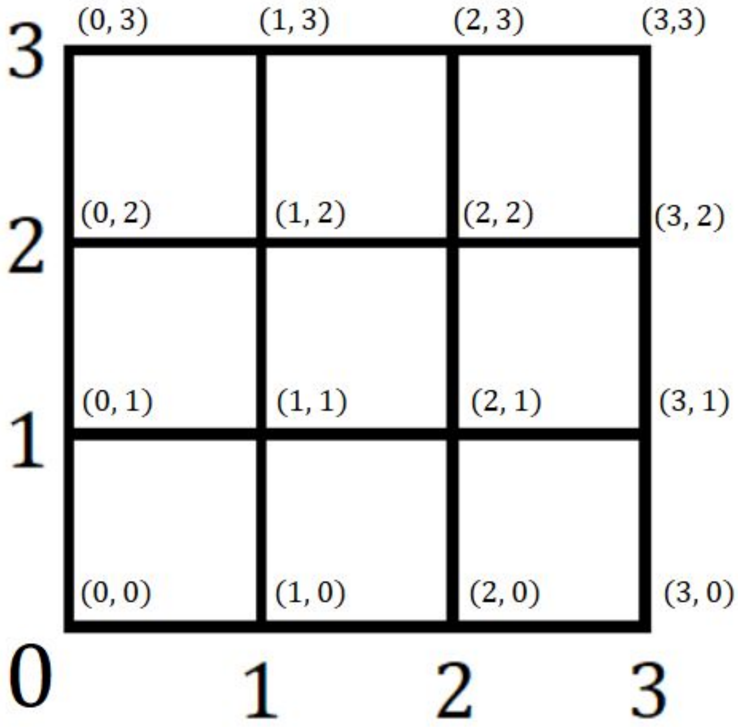
Your Turn: What is $\{1, 2, 3\}^2$?

Answer: $\{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

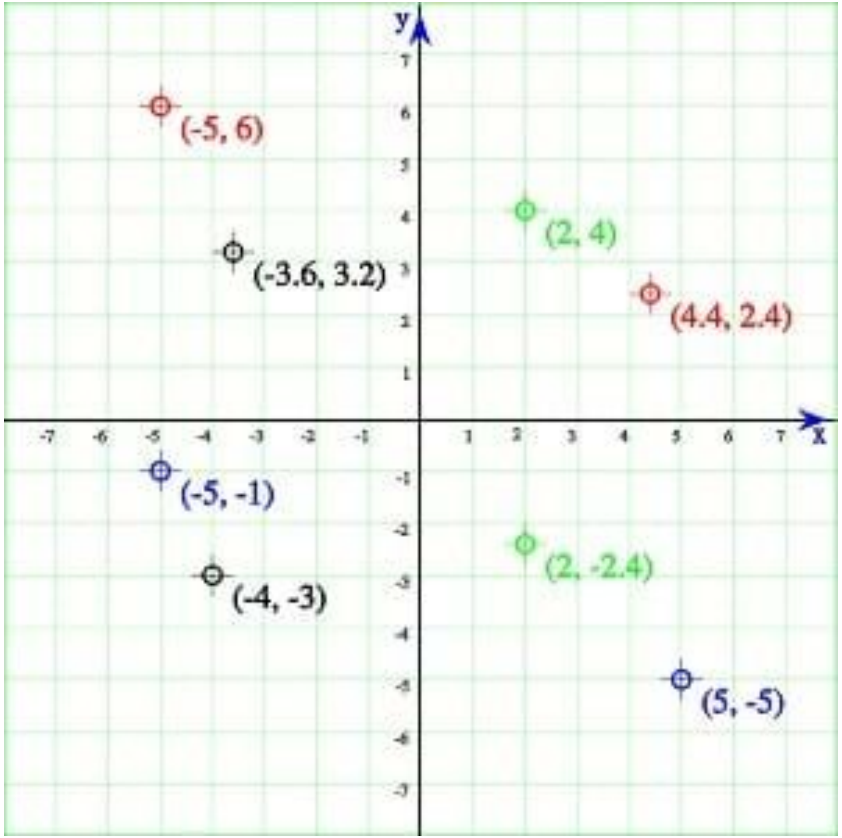
$$\{0, 1, 2, 3\}^2$$



$$\{0, 1, 2, 3\}^2$$



Cartesian Product of Sets



$\mathbb{R} \times \mathbb{R}$: The coordinate plane

Cartesian Product of Sets

Your Turn: What is $\{1, 2\} \times \{2, 3\} \times \{1, 3\}$?

Answer:

Cartesian Product of Sets

Your Turn: What is $\{1, 2\} \times \{2, 3\} \times \{1, 3\}$?

Answer: $\{(1, 2, 1), (1, 2, 3), (1, 3, 1), (1, 3, 3), (2, 2, 1), (2, 2, 3), (2, 3, 1), (2, 3, 3)\}$

Cartesian Product of Sets

Your Turn: What is $\{1\} \times \{1\} \times \{1, 0\}$?

Answer:

Cartesian Product of Sets

Your Turn: What is $\{1\} \times \{1\} \times \{1, 0\}$?

Answer: $\{(1, 1, 1), (1, 1, 0)\}$

Cartesian Product of Sets

Your Turn: What is $\{1, 2\} \times \{3, 4\} \times \{\}$?

Answer:

Cartesian Product of Sets

Your Turn: What is $\{1, 2\} \times \{3, 4\} \times \{\}$?

Answer: $\{\}$

Cartesian Product of Sets

Your Turn: What is $\{1, 2\}^0$?

Cartesian Product of Sets

Your Turn: What is $\{1, 2\}^0$?

Answer: $\{()\}$

Cartesian Product of Sets

Your Turn: What is $\{1, 2\}^0$?

Answer: $\{()\}$

(we want $S^0 \times S = S$)

Propositions

A proposition is a statement that is either true or false

Examples of Proposition

This sentience is a proposition.

Sets cannot have duplicates.

It snowed last night.

2+2 = 3

Examples of things that aren't Proposition

How was your day today?

What is a number?

Be quiet!

Propositions

A proposition is a statement that is either true or false

Examples of Proposition

(Eggs are blue) = *p*

(I am a human) = *q*

(2 + 3 = 5) = *r*

Examples of things that aren't Proposition

What are you doing Friday?

What is 3 + 3?

Sit down!

Propositions

A proposition is a statement that is either true or false

When dealing with propositions, we abstract away difficulties of defining, and we can just give them letters (define variables), like p

Propositions

A proposition, *p,* is a statement that is either true or false. "True" or "False" is considered the "truth value" of *p*.

https://www.cs.virginia.edu/luther/2102/F2020/symbols.html

Concept	Java/C	Python	This class	Bitwise	Other
true	`true`	`True`	$\top$ or 1	`-1`	T, tautology
false	`false`	`False`	$\bot$ or 0	`0`	F, contradiction

Propositions

A proposition is a statement that is either true or false

We can combine and relate propositions with *connectives:*

Propositions

A proposition is a statement that is either true or false

We can combine and relate propositions with *connectives:*

- $\vee$ is "or"
- $\wedge$ is "and"
- $\neg$ is "not"

Looks Familiar?

We can modify, combine and relate propositions with *connectives:*

- $\vee$ is "or"
- $\wedge$ is "and"
- $\neg$ is "not"

$$\boxed{} = \{x \in U \mid x \in S \wedge x \notin T\}$$

Looks Familiar?

We can modify, combine and relate propositions with *connectives:*

- $\vee$ is "or"
- $\wedge$ is "and"
- $\neg$ is "not"

$$S \setminus T = \{x \in U \mid x \in S \wedge x \notin T\}$$

Looks Familiar?

We can modify, combine and relate propositions with *connectives:*

- $\vee$ is “or”
- $\wedge$ is “and”
- $\neg$ is “not”

$$\square = \{x \in U \mid x \in S \vee x \in T\}$$

Looks Familiar?

We can modify, combine and relate propositions with *connectives:*

- $\lor$ is "or"
- $\land$ is "and"
- $\neg$ is "not"

$$S \cup T = \{x \in U \mid x \in S \lor x \in T\}$$

Looks Familiar?

We can modify, combine and relate propositions with *connectives:*

- $\vee$ is "or"
- $\wedge$ is "and"
- $\neg$ is "not"

$$\square = \{x \in U \mid x \in S \wedge x \in T\}$$

Looks Familiar?

We can modify, combine and relate propositions with *connectives:*

- $\vee$ is "or"
- $\wedge$ is "and"
- $\neg$ is "not"

$$S \cap T = \{x \in U \mid x \in S \wedge x \in T\}$$

Looks Familiar?

We can modify, combine and relate propositions with *connectives:*

- $\vee$ is "or"
- $\wedge$ is "and"
- $\neg$ is "not"

Set theory is a branch of mathematical logic. So it makes sense to use logical language and symbols to describe sets.

“Not” operator

How to define:

Make a truth table

“Not” operator

p	$\neg p$
T	F
F	T

“And” operator

“And” operator

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

“Or” operator

"Or" operator

p	q	$p \lor q$
T	T	T
T	F	T
F	T	T
F	F	F

"Implies" operator

If p, then q

The conditional $p \to q$ can be expressed by different sentences, some of them are listed below:

- p implies q
- p is a sufficient condition for q
- q is a necessary condition for p
- q follows from p
- p only if q

"Implies" operator

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T