Agenda

- **Quiz Friday:** practice
- Cartesian Product of 3 or more sets
- Set builder: duplicate elements
- Logical Operator Definitions
  - Truth Table Example 1
  - Truth Table Example 2
  - Truth Table Example 3

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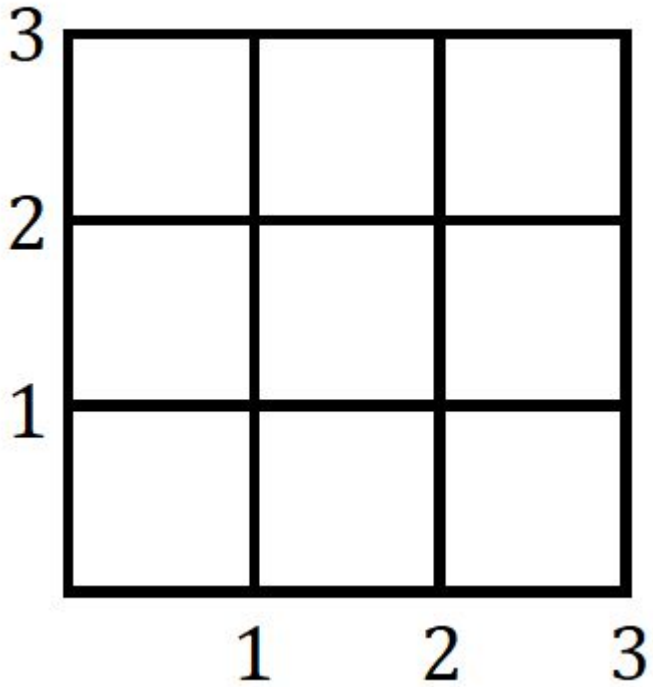
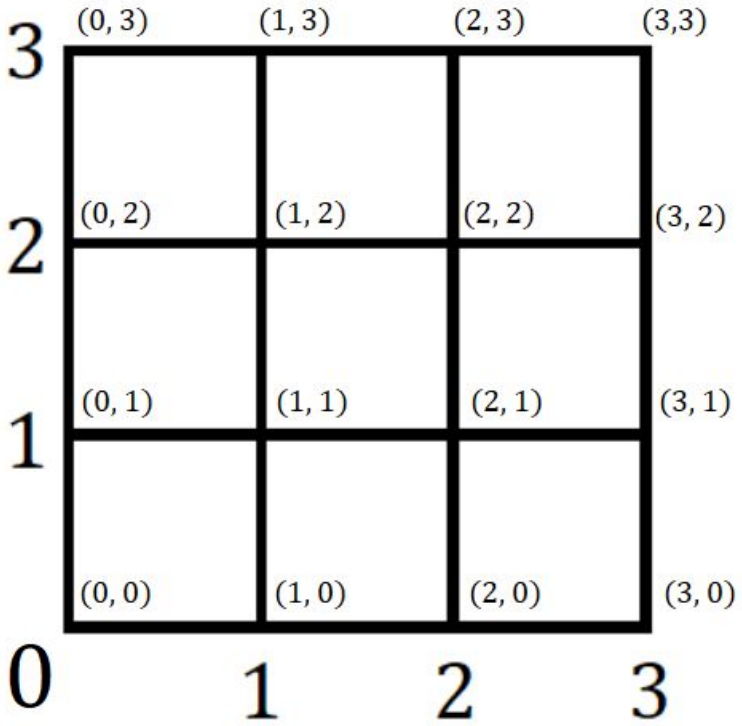
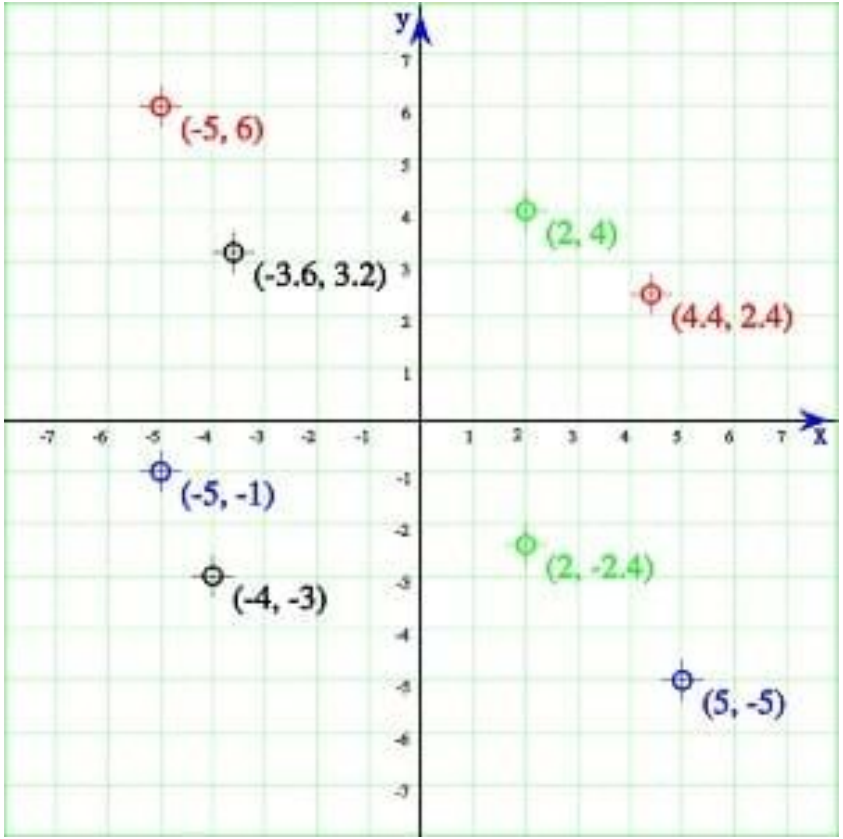
$$\{0, 1, 2, 3\}^2$$



$$\{0, 1, 2, 3\}^2$$



Cartesian Product of Sets



$\mathbb{R} \times \mathbb{R}$: The coordinate plane

Cartesian Product of Sets

Your Turn: What is $\{1, 2\} \times \{2, 3\} \times \{1, 3\}$?

Answer:

Cartesian Product of Sets

Your Turn: What is $\{1, 2\} \times \{2, 3\} \times \{1, 3\}$?

Answer: $\{(1, 2, 1), (1, 2, 3), (1, 3, 1), (1, 3, 3), (2, 2, 1), (2, 2, 3), (2, 3, 1), (2, 3, 3)\}$

Cartesian Product of Sets

Your Turn: What is $\{1\} \times \{1\} \times \{1, 0\}$?

Answer:

Cartesian Product of Sets

Your Turn: What is $\{1\} \times \{1\} \times \{1, 0\}$?

Answer: $\{(1, 1, 1), (1, 1, 0)\}$

Cartesian Product of Sets

Your Turn: What is $\{1, 2\} \times \{3, 4\} \times \{\}$?

Answer:

Cartesian Product of Sets

Your Turn: What is $\{1, 2\} \times \{3, 4\} \times \{\}$?

Answer: $\{\}$

Cartesian Product of Sets

Your Turn: What is $\{1, 2\}^0$?

Cartesian Product of Sets

Your Turn: What is $\{1, 2\}^0$?

Answer: $\{()\}$

Cartesian Product of Sets

Your Turn: What is $\{1, 2\}^0$?

Answer: $\{()\}$

(we want $S^0 \times S = S$)

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Set-Builder -- duplicate elements

Question 78 (see above)

$$\left\{ \{a, b\} \;\middle|\; (a \in A) \wedge \big(b \in \{4, 8\}\big) \right\}$$

Mulitquestion Consider the following sets: $A = \{2, 4, 8\}, B = \{1, 2, 4\}, C = \mathcal{P}\big(\{1, 2\}\big)$

Evaluate each expression

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Propositions

A proposition, *p*, is a statement that is either true or false. “True” or “False” is considered the “truth value” of *p*.

https://www.cs.virginia.edu/luther/2102/F2020/symbols.html

Concept	Java/C	Python	This class	Bitwise	Other
true	`true`	`True`	$\top$ or 1	`-1`	T, tautology
false	`false`	`False`	$\bot$ or 0	`0`	F, contradiction

Propositions

A proposition is a statement that is either true or false

We can combine and relate propositions with *connectives:*

“Not” operator

How to define:

Make a truth table

“Not” operator

p	$\neg p$
T	F
F	T

“And” operator

		And
P	***Q***	$P \wedge Q$
F	F	F
F	T	F
T	F	F
T	T	T

“Or” operator

		Or
P	***Q***	$P \vee Q$
F	F	F
F	T	T
T	F	T
T	T	T

“Implies” operator

		Implies
P	***Q***	***P*→*Q***
F	F	T
F	T	T
T	F	F
T	T	T

“Xor” operator

		Xor
P	***Q***	$P \oplus Q$
F	F	F
F	T	T
T	F	T
T	T	F

“Bi-implication” operator

		Bi-implies
P	***Q***	$P \leftrightarrow Q$
F	F	T
F	T	F
T	F	F
T	T	T

		Or	*And*	*Implies*	*Xor*	*Bi-implies*
P	***Q***	$P \vee Q$	$P \wedge Q$	$P \rightarrow Q$	$P \oplus Q$	$P \leftrightarrow Q$
F	F	F	F	T	F	T
F	T	T	F	T	T	F
T	F	T	F	F	T	F
T	T	T	T	T	F	T

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What if we want to combine logical operators for longer expressions?

Ex: $\neg (P \wedge Q)$

P	Q	¬(P ∧ Q)
T	T	
T	F	
F	T	
F	F	

First fill in the known values

P	Q	¬(P ∧ Q)
T	T	
T	F	
F	T	
F	F	

First fill in the known values

P	Q	¬(P ∧ Q)
T	T	
T	F	
F	T	
F	F	

Apply the ∧ rule for the parentheses

P	Q	¬(P ∧ Q)
T	T	
T	F	
F	T	
F	F	

First fill in the known values

P	Q	¬(P ∧ Q)
T	T	T
T	F	
F	T	
F	F	

Apply the ∧ rule for the parentheses

P	Q	¬(P ∧ Q)
T	T	
T	F	
F	T	
F	F	

First fill in the known values

P	Q	¬(P ∧ Q)
T	T	T
T	F	F
F	T	
F	F	

Apply the ∧ rule for the parentheses

P	Q	¬(P ∧ Q)
T	T	
T	F	
F	T	
F	F	

First fill in the known values

P	Q	¬(P ∧ Q)
T	T	T
T	F	F
F	T	F
F	F	

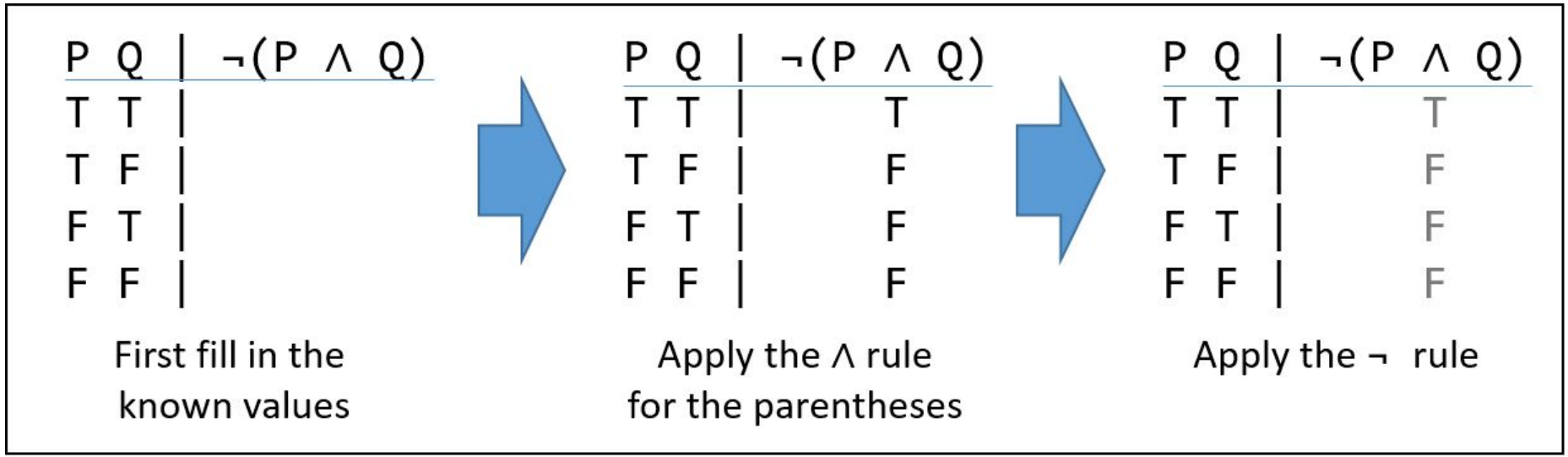
Apply the ∧ rule for the parentheses

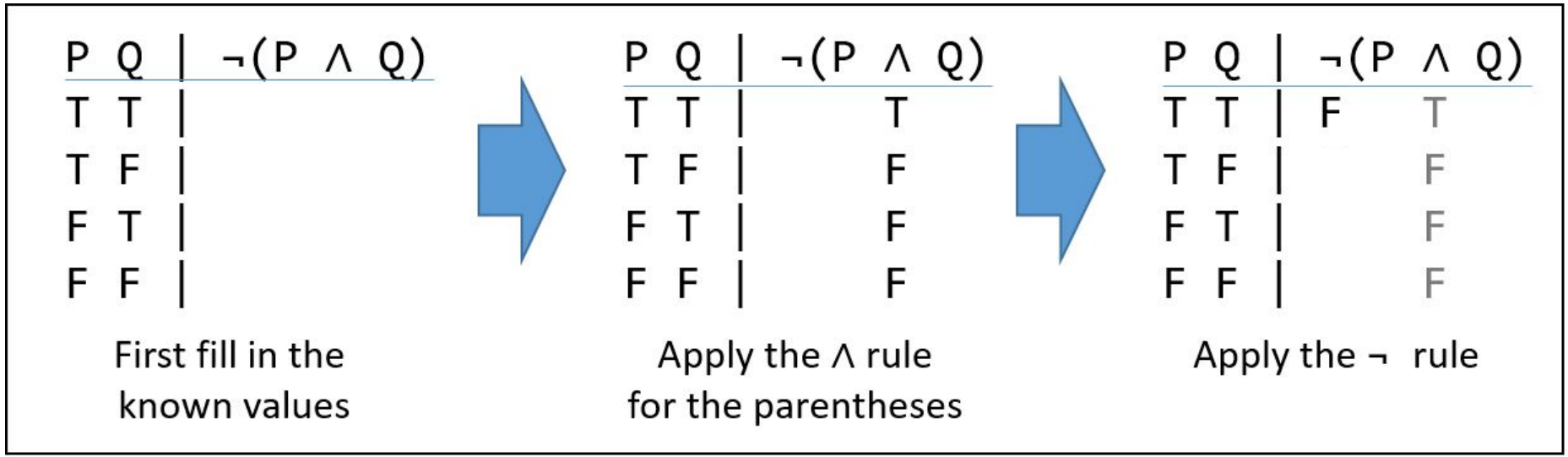
P	Q	¬(P ∧ Q)
T	T	
T	F	
F	T	
F	F	

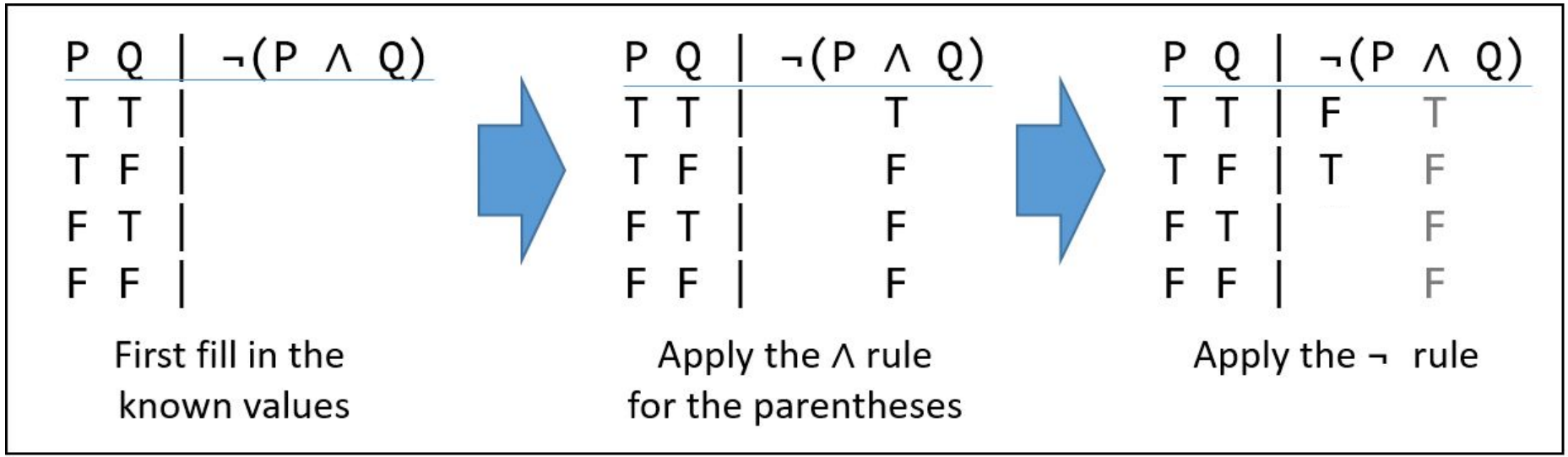
First fill in the known values

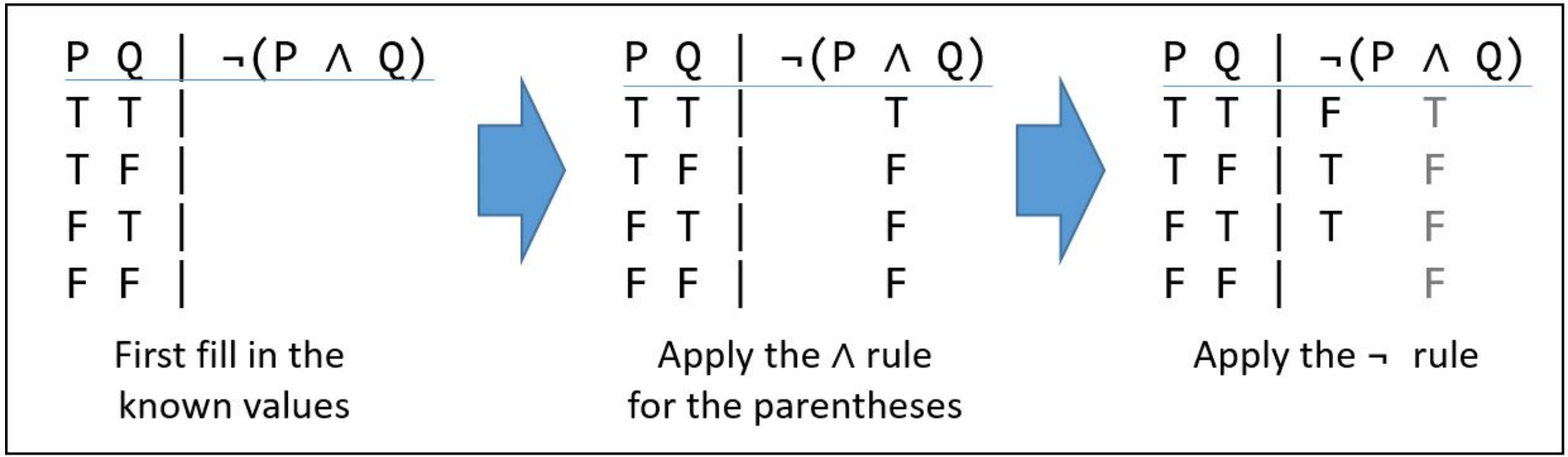
P	Q	¬(P ∧ Q)
T	T	T
T	F	F
F	T	F
F	F	F

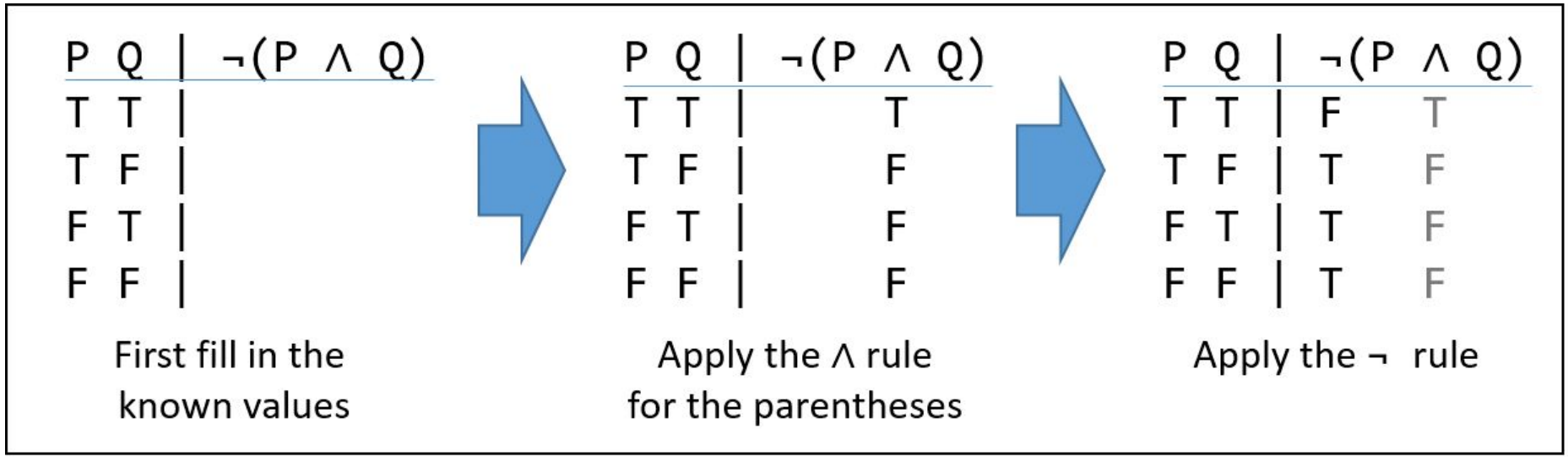
Apply the ∧ rule for the parentheses

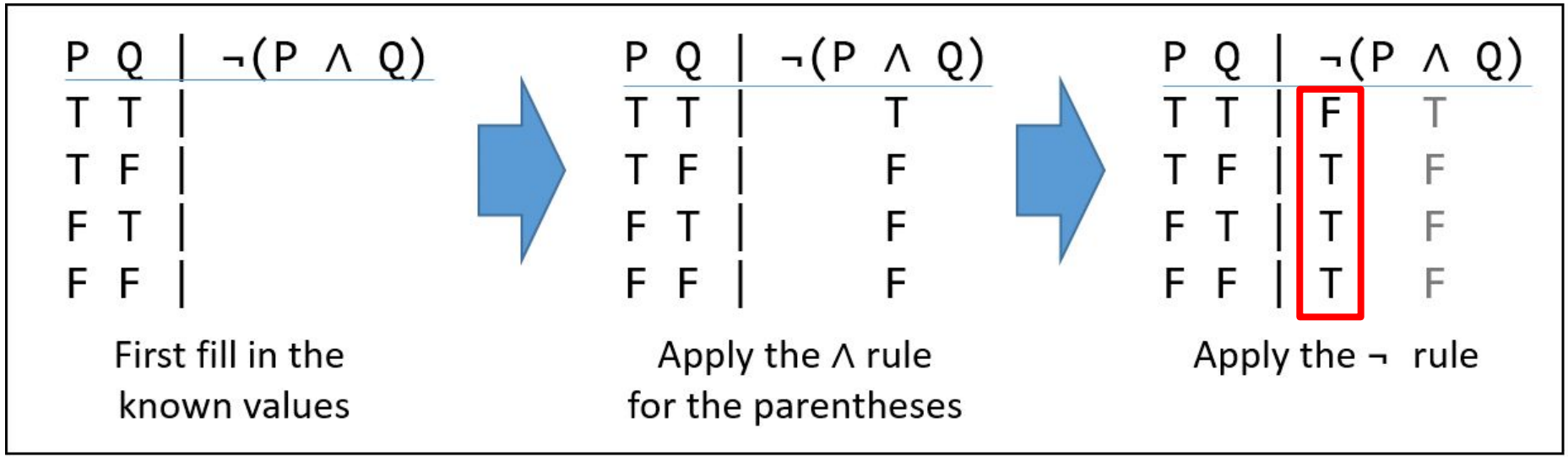

P Q | ¬(P ∧ Q)
T T |
T F |
F T |
F F |
First fill in the known values
P Q | ¬(P ∧ Q)
T T | T
T F | F
F T | F
F F | F
Apply the ∧ rule for the parentheses
P Q | ¬(P ∧ Q)
T T | T
T F | F
F T | F
F F | F
Apply the ¬ rule


P Q | ¬(P ∧ Q)
T T |
T F |
F T |
F F |
First fill in the known values
P Q | ¬(P ∧ Q)
T T | T
T F | F
F T | F
F F | F
Apply the ∧ rule for the parentheses
P Q | ¬(P ∧ Q)
T T | F T
T F | F
F T | F
F F | F
Apply the ¬ rule


P Q | ¬(P ∧ Q)
T T |
T F |
F T |
F F |
First fill in the known values
P Q | ¬(P ∧ Q)
T T | T
T F | F
F T | F
F F | F
Apply the ∧ rule for the parentheses
P Q | ¬(P ∧ Q)
T T | F T
T F | T F
F T | F
F F | F
Apply the ¬ rule


P Q | ¬(P ∧ Q)
T T |
T F |
F T |
F F |
First fill in the known values
P Q | ¬(P ∧ Q)
T T | T
T F | F
F T | F
F F | F
Apply the ∧ rule for the parentheses
P Q | ¬(P ∧ Q)
T T | F T
T F | T F
F T | T F
F F | F
Apply the ¬ rule


P Q | ¬(P ∧ Q)
T T |
T F |
F T |
F F |
First fill in the known values
P Q | ¬(P ∧ Q)
T T | T
T F | F
F T | F
F F | F
Apply the ∧ rule for the parentheses
P Q | ¬(P ∧ Q)
T T | F T
T F | T F
F T | T F
F F | T F
Apply the ¬ rule


P Q | ¬(P ∧ Q)
T T |
T F |
F T |
F F |
First fill in the known values
P Q | ¬(P ∧ Q)
T T | T
T F | F
F T | F
F F | F
Apply the ∧ rule for the parentheses
P Q | ¬(P ∧ Q)
T T | F T
T F | T F
F T | T F
F F | T F
Apply the ¬ rule

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Question 123

☆ 0

Consider the expression "$(P \rightarrow Q) \leftrightarrow (Q \rightarrow P)$". This full expression has the same truth value as

A. ○ $P \oplus Q$

B. ○ $P \vee Q$

C. ○ $P \wedge Q$

D. ○ $P \rightarrow Q$

E. ○ $P \leftrightarrow Q$

F. ○ P

G. ◉ Q

▶ Key:

https://kytos.cs.virginia.edu/cs2102/quizzes/review.php?qid=1-1

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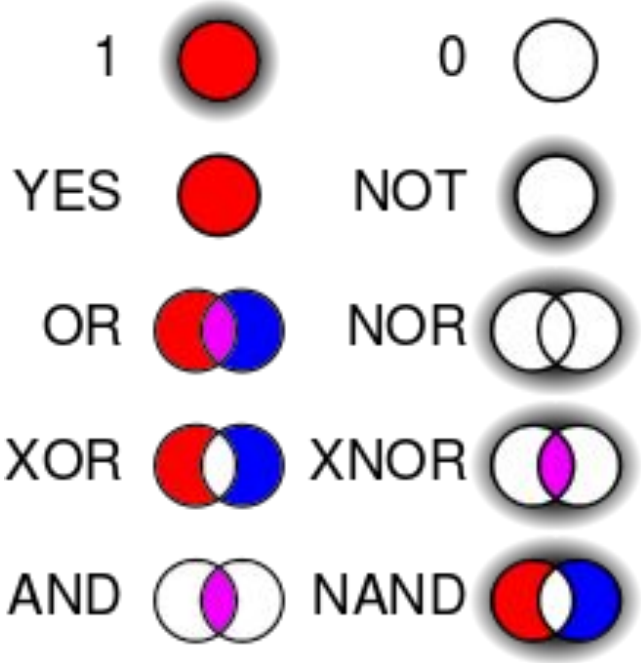
What is the truth table for:

$$(P \vee Q) \rightarrow (\neg R)$$


1
0
YES
NOT
OR
NOR
XOR
XNOR
AND
NAND