

Feb 22nd Slides

# Boolean Algebra

# ~~Boolean~~ Algebra

Prove

$$3(x + y) = 3x + 3y$$

# ~~Boolean~~ Algebra

Prove

$x$

$y$

$$3(x + y) = 3x + 3y$$

# ~~Boolean~~ Algebra

Prove

$x$

$y$

$0$

$0$

$$3(x + y) = 3x + 3y$$

$$3(0 + 0) = 3(0) + 3(0)$$

# ~~Boolean~~ Algebra

Prove

$x$

$y$

0

0

1

0

$$3(x + y) = 3x + 3y$$

$$3(0 + 0) = 3(0) + 3(0)$$

$$3(1 + 0) = 3(1) + 3(0)$$

# ~~Boolean~~ Algebra

Prove

$x$

$y$

0

0

1

0

2

0

$$3(x + y) = 3x + 3y$$

$$3(0 + 0) = 3(0) + 3(0)$$

$$3(1 + 0) = 3(1) + 3(0)$$

$$3(2 + 0) = 3(1) + 3(0)$$

# ~~Boolean~~ Algebra

Prove

$x$

$y$

0

0

1

0

2

0

No.....

$$3(x + y) = 3x + 3y$$

$$3(0 + 0) = 3(0) + 3(0)$$

$$3(1 + 0) = 3(1) + 3(0)$$

$$3(2 + 0) = 3(2) + 3(0)$$



“Is equivalent to”



# Double Negation

Prove

$$\neg\neg P \equiv P$$

# Double Negation

Prove

$$\neg\neg P \equiv P$$

$\neg\neg P$

|  
Given

# Double Negation

Prove

$\neg\neg P$   
 $P$

$\neg\neg P \equiv P$

Given  
double negation

# Double Negation

Prove

$$\equiv \begin{array}{l} \neg\neg P \\ P \end{array}$$

$$\neg\neg P \equiv P$$

Given  
double negation

# Simplification

Prove

$$\begin{array}{l} P \vee \perp \\ \equiv P \end{array}$$

$$P \vee \perp \equiv P$$

Given  
Simplification

# Simplification

Prove

$$P \equiv P \wedge (P \leftrightarrow T)$$

# Simplification

Prove

$$P \equiv P \wedge (P \leftrightarrow T)$$

P | Given



# Simplification

Prove

$$P \equiv P \wedge (P \leftrightarrow T)$$

$\equiv$

|              |  |                |
|--------------|--|----------------|
| $P$          |  | Given          |
| $P \wedge P$ |  | Simplification |

# Simplification

Prove

$$P \equiv P \wedge (P \leftrightarrow T)$$

|          |                                  |                |
|----------|----------------------------------|----------------|
|          | $P$                              |                |
| $\equiv$ | $P \wedge P$                     | Given          |
| $\equiv$ | $P \wedge (P \leftrightarrow T)$ | Simplification |
|          |                                  | Simplification |

# Definition of Implication

Prove

$$A \rightarrow (B \oplus A) \equiv \neg A \vee (B \oplus A)$$

# Definition of Implication

*Scratch work:*  $A \rightarrow B \equiv \neg A \vee B$  where  $A = P$  and  $B = (Q \oplus P)$

Prove

$$P \rightarrow (Q \oplus P) \equiv \neg P \vee (Q \oplus P)$$

# Definition of Implication

*Scratch work:*  $A \rightarrow B \equiv \neg A \vee B$  where  $A = P$  and  $B = (Q \oplus P)$

Prove

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$$\begin{aligned} & P \rightarrow (Q \oplus P) \\ \equiv & \neg P \vee (Q \oplus P) \end{aligned}$$

Given

Definition of Implication

# Definition of Implication

Prove

$$P \rightarrow (Q \oplus P) \equiv \neg P \vee (Q \oplus P)$$

$$\begin{aligned} & P \rightarrow (Q \oplus P) \\ \equiv & \neg P \vee (Q \oplus P) \end{aligned}$$

Given

Definition of Implication

# Boolean Algebra

**Associative Property:** you can move parentheses around the operator

Example:  $(2+3)+5 = 2+(3+5)$

Counterexample:  $(2-3)-5 \neq 2-(3-5)$

# Boolean Algebra

Which symbols are associative/commutative?

$\neg$

$\vee$

$\wedge$

$\oplus$

$\leftrightarrow$

$\rightarrow$



# Associativity

Prove

$$(P \wedge Q) \wedge (A) \equiv P \wedge (Q \wedge (A))$$

# Associativity

Prove  
( $P \wedge Q$ )  $\wedge$  ( $R \vee Q$ )  
 $\equiv$  ( $P \wedge (Q \wedge (R \vee Q))$ )

$$(P \wedge Q) \wedge (R \vee Q) \equiv P \wedge (Q \wedge (R \vee Q))$$

|          |                                  |               |
|----------|----------------------------------|---------------|
| $\equiv$ | $(P \wedge Q) \wedge (R \vee Q)$ | Given         |
|          | $P \wedge (Q \wedge (R \vee Q))$ | Associativity |

# Boolean Algebra

**Commutative Property:** you can swap their operands' position

Example:  $2+3 = 3+2$

Counterexample:  $2-3 \neq 3-2$

# Boolean Algebra

Which symbols are associative/commutative?

$\neg$

$\vee$

$\wedge$

$\oplus$

$\leftrightarrow$

$\rightarrow$

# Commutativity

Prove

$$(P \vee Q) \vee (R \vee Q) \equiv (P \vee Q) \vee R$$

# Commutativity

Prove

$$(P \vee Q) \vee (R \vee Q) \equiv (P \vee Q) \vee R$$

|          |                              |                |
|----------|------------------------------|----------------|
|          | $(P \vee Q) \vee (R \vee Q)$ | Given          |
| $\equiv$ | $(P \vee Q) \vee (Q \vee R)$ | Commutativity  |
| $\equiv$ | $(P \vee (Q \vee Q)) \vee R$ | Associativity  |
| $\equiv$ | $(P \vee Q) \vee R$          | Simplification |

# DeMorgan's Law

Prove

$$P \vee \neg(Q \wedge \neg R) \equiv P \vee (\neg Q \vee R)$$

# DeMorgan's Law

Prove

$$P \vee \neg(Q \wedge \neg R) \equiv P \vee (\neg Q \vee R)$$

|          |                                   |                 |
|----------|-----------------------------------|-----------------|
|          | $P \vee \neg(Q \wedge \neg R)$    | Given           |
| $\equiv$ | $P \vee (\neg Q \vee \neg\neg R)$ | DeMorgan's Law  |
| $\equiv$ | $P \vee (\neg Q \vee R)$          | Double Negation |



# Distributive Law

Prove  $(P \vee \neg Q) \wedge (P \vee \neg R) \equiv P \vee (\neg Q \wedge \neg R)$   
)

# Distributive Law

$$A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C) \quad A = P \quad B = \neg Q \quad C = \neg R$$

Prove  $(P \vee \neg Q) \wedge (P \vee \neg R) \equiv P \vee (\neg Q \wedge \neg R)$   
)

# Distributive Law

$$A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C) \quad A = P \quad B = \neg Q \quad C = \neg R$$

Prove  $(P \vee \neg Q) \wedge (P \vee \neg R) \equiv P \vee (\neg Q \wedge \neg R)$   
)

$$\begin{array}{l|l} (P \vee \neg Q) \wedge (P \vee \neg R) & \text{Given} \\ \equiv P \vee (\neg Q \wedge \neg R) & \text{Distributive Law} \end{array}$$