Mar 15th Slides

Elizabeth Orrico

What is a Function?

1. **What can a function output?**
   Numbers
   Booleans
   Vectors
   Anything and everything
   String

2. What can be inputs to a function?
Same thing as output

3. How can a function be defined?
formula/equation
Code-- subroutine
algorithm

What is a Function?

A function assigns an element of one SET to another SET…

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$$f : A \to B$$

Why a SET and not a SEQUENCE?

What is a Function?

The notation:

$$f : A \rightarrow B$$

indicates that f **is a function with domain,** A, and **codomain,** $\boldsymbol{B}$**.** The familiar notation "$\boldsymbol{f(a) = b}$" indicates that f assigns the element $\boldsymbol{b \in B}$ to a specific argument $\boldsymbol{a \in A}$**.**

Here b would be called the **value** of f at **argument** a**.**

Domain

A function *need not* be defined for every element in its domain.
For example, if we consider $f_1(x) : \mathbb{R} \rightarrow \mathbb{R}$

$$f_1(x) = 1/x^2$$

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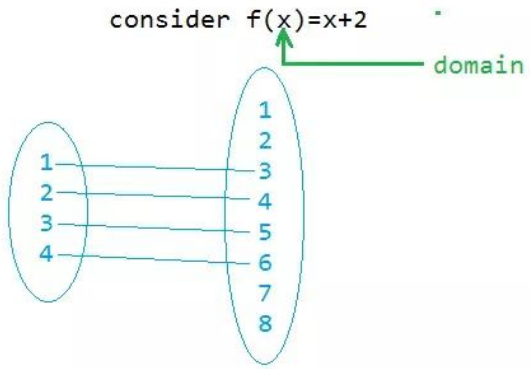
If there are domain elements for which a function is not defined, it is a *partial function*.

Meanwhile, a *total function* ???

Codomain

A function *need not* be able to return every element of its codomain...



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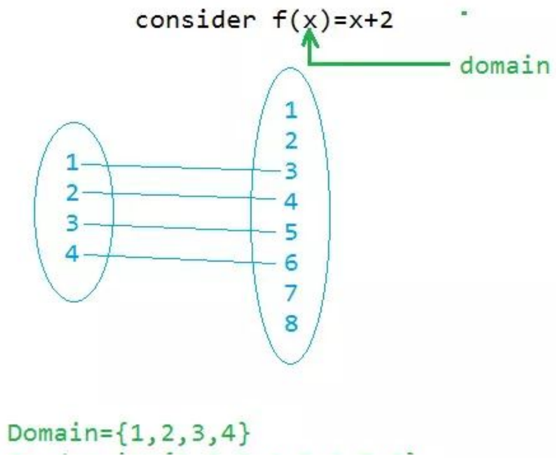

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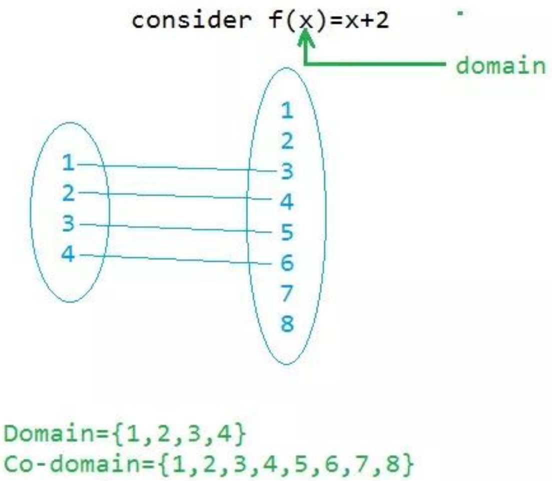

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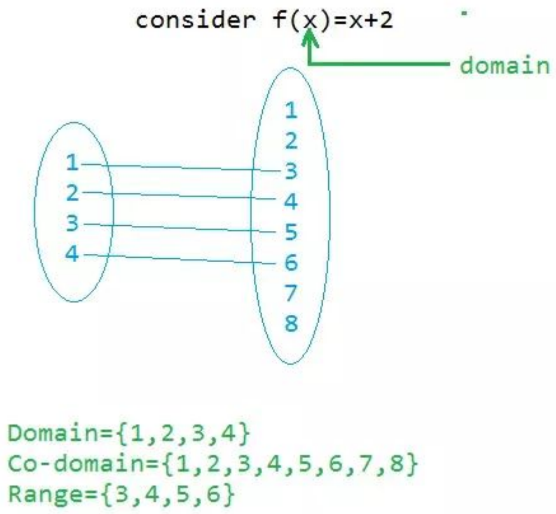


Range $\subseteq$ Codomain

Codomain

Surjective, onto : Codomain = Range

What does this mean in English?

$$\forall b \in B,\ \exists a \in A,\ f(a) = b.$$

Codomain

Surjective, onto : Codomain = Range

$$\forall b \in B,\ \exists a \in A,\ f(a) = b.$$

Surjective or not surjective?

$f_1(x) : \mathbb{R} \rightarrow \mathbb{R}$

$f_1(x) = x^2$

Codomain

Surjective, onto : Codomain = Range

$$\forall b \in B,\ \exists a \in A,\ f(a) = b.$$

Surjective or not surjective?

$f_1(x)\ :\ \mathbb{R} \rightarrow [0,\inf)$

$f_1(x) = x^2$

Injective

Every input maps to a different output!

$$\forall\ x, y \in D\ .\ \ (\ x \neq y\ ) \rightarrow (\ f(x) \neq f(y)\ )$$

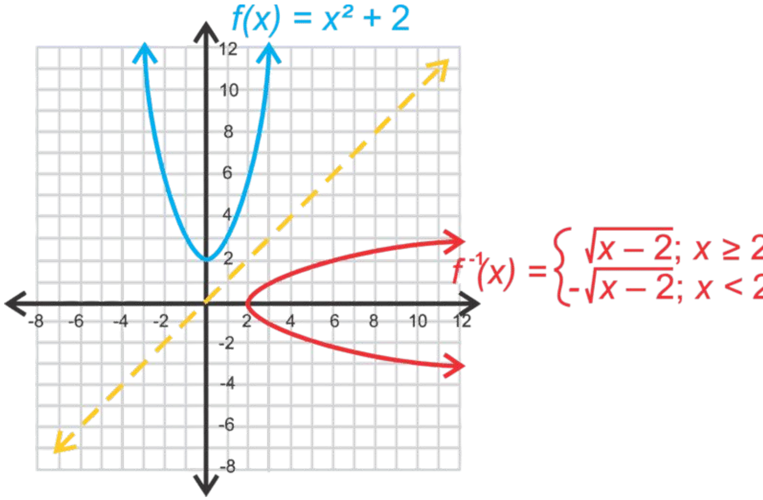
Is a parabola injective?

Injective

Every input maps to a different output!

$$\forall\ x, y \in D\ .\ \ (\, x \neq y\,) \rightarrow (\, f(x) \neq f(y)\,)$$

https://www.desmos.com/calculator



Bijective (or invertible or correspondence)

Must be **surjective, and injective**! (oh my)

Surjective: $\forall b \in B,\ \exists a \in A,\ f(a) = b.$

Injective: $\forall a_1 \in A,\ \forall a_2 \in A,\ (f(a_1) = f(a_2)) \implies (a_1 = a_2).$

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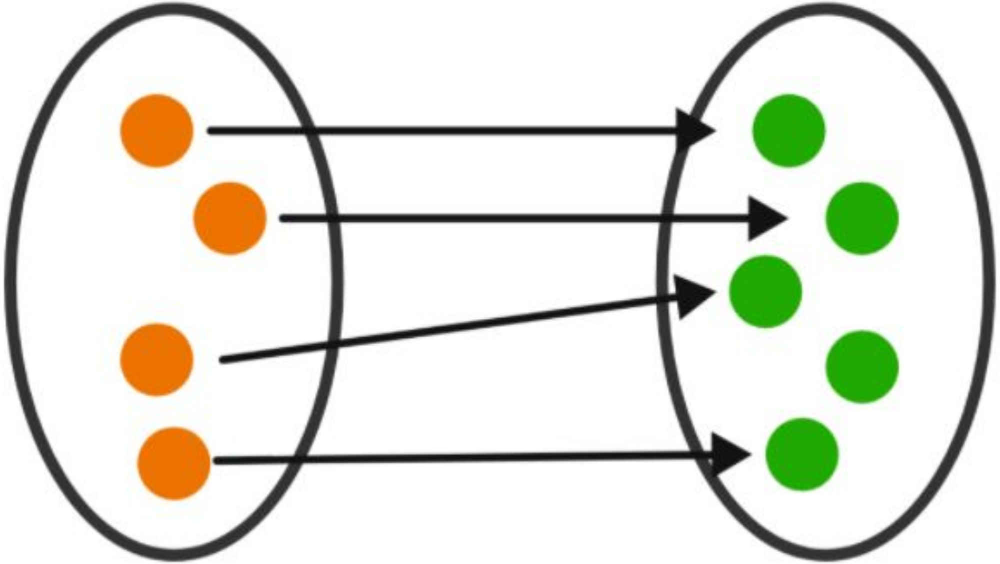

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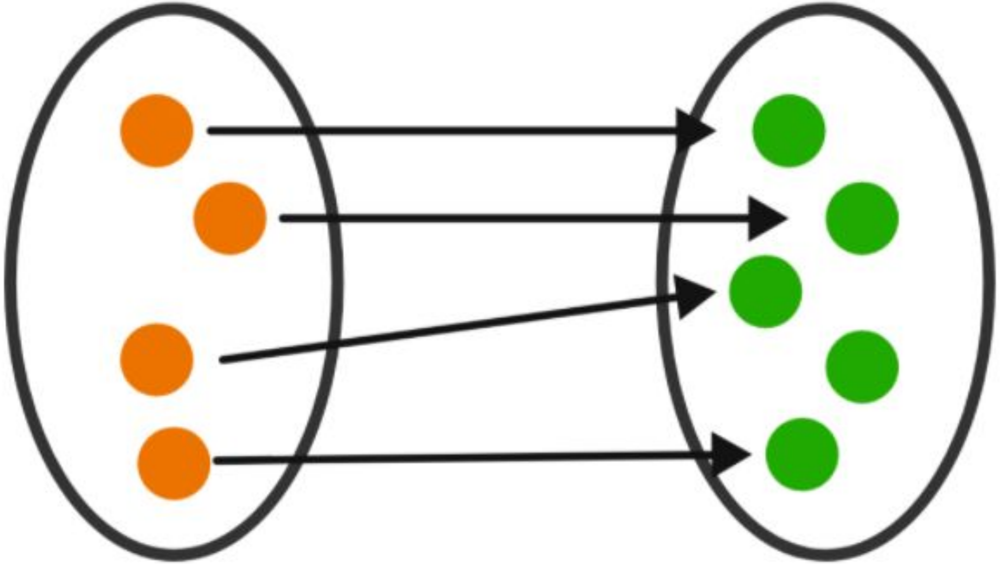
Injection (One-to-One)



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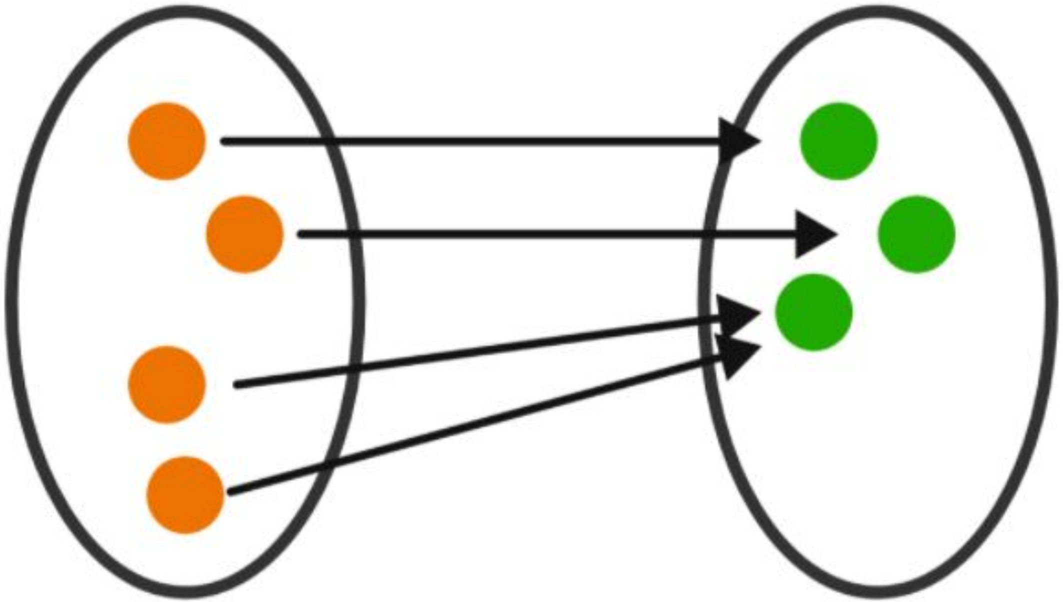

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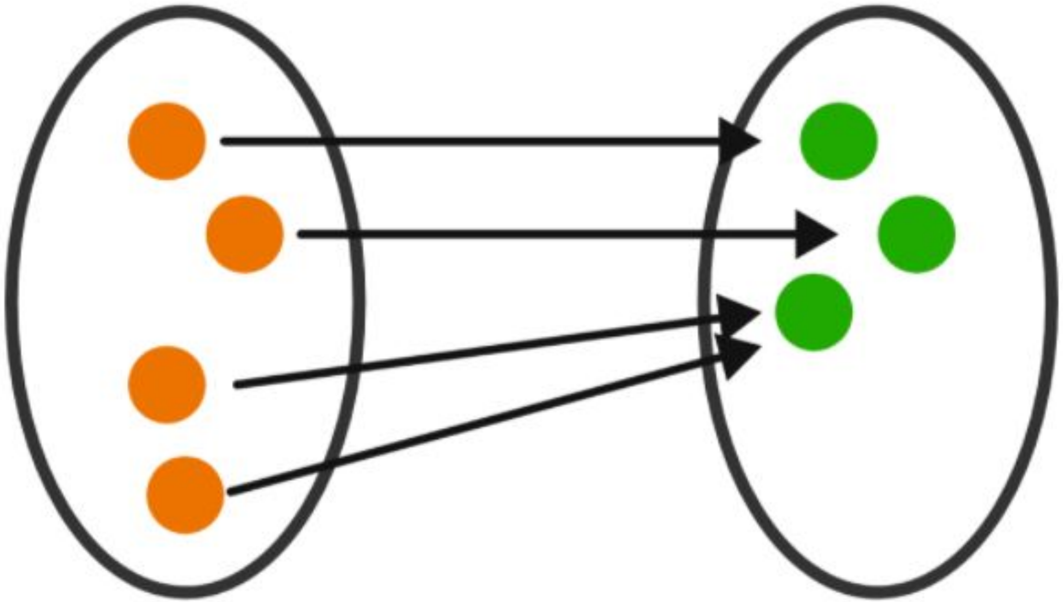
Surjection (Onto)



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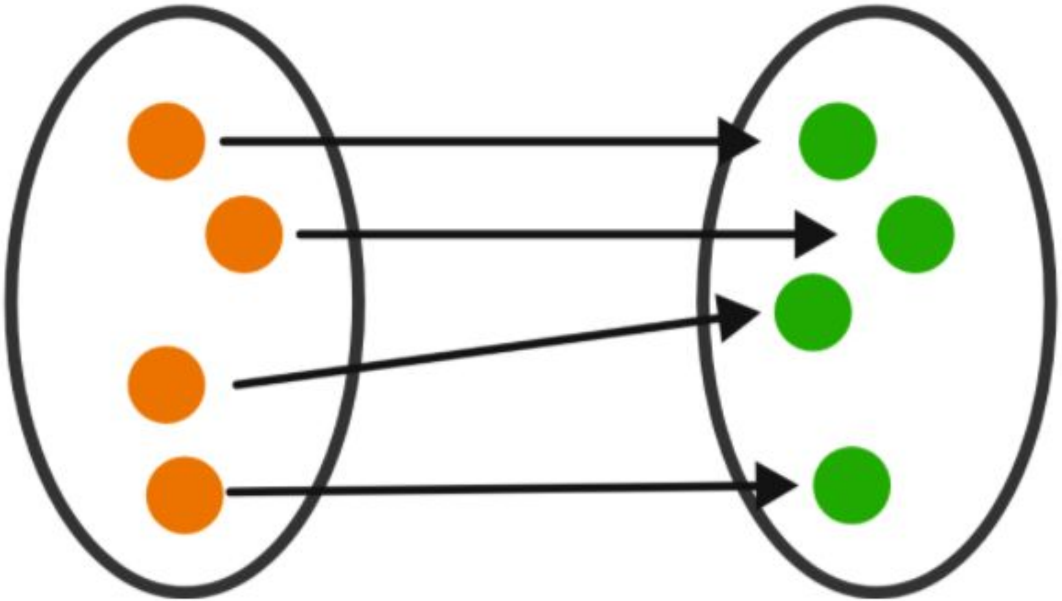

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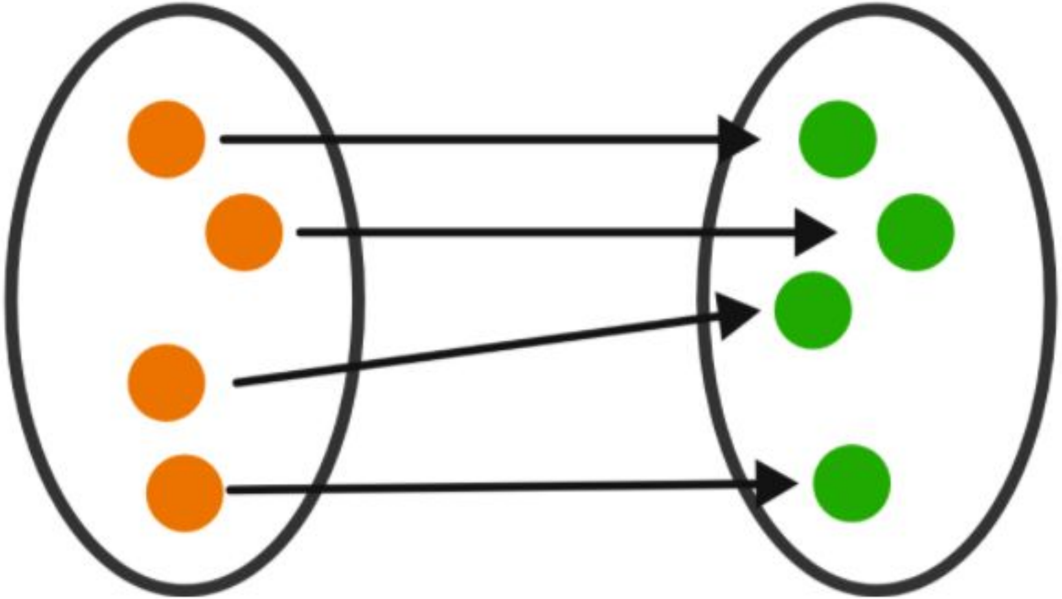
Bijection (One-to-One and Onto)



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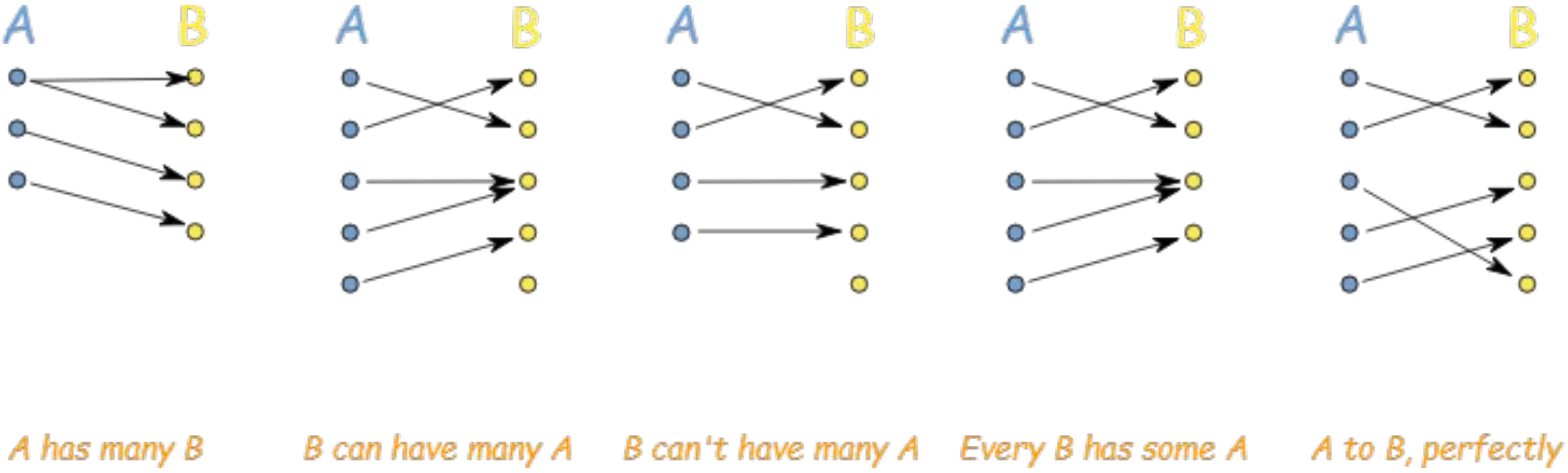

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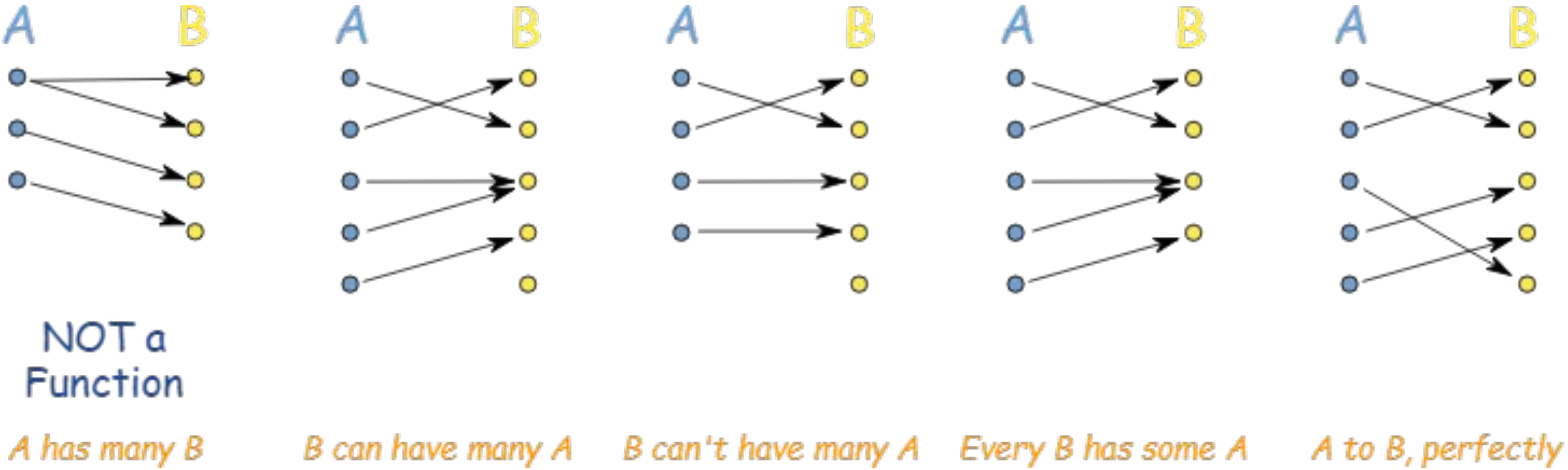

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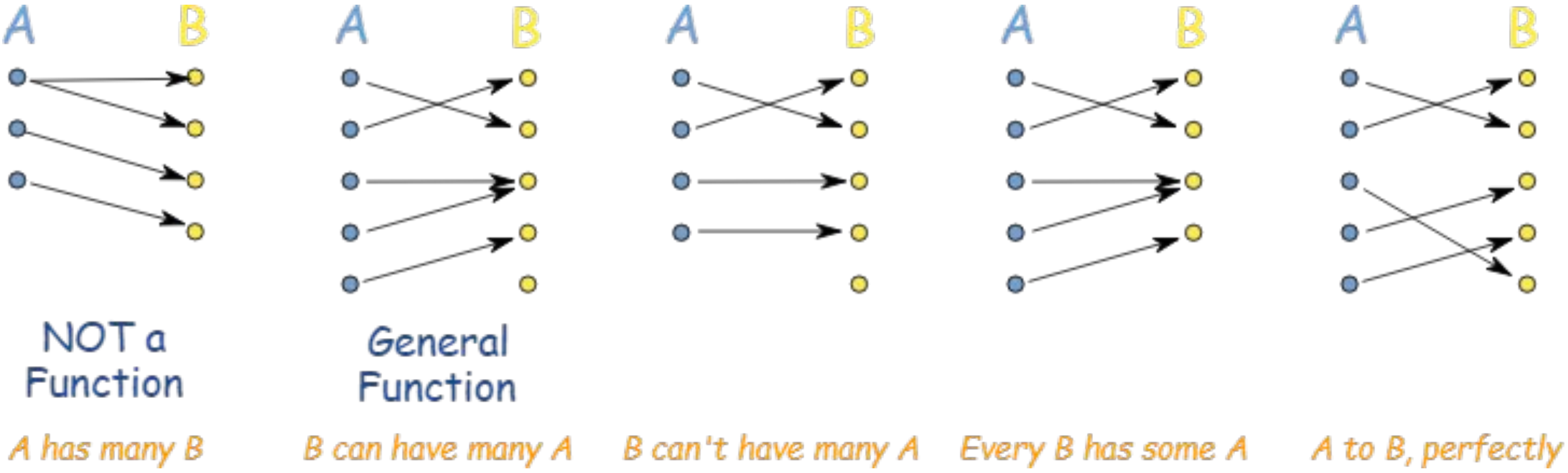

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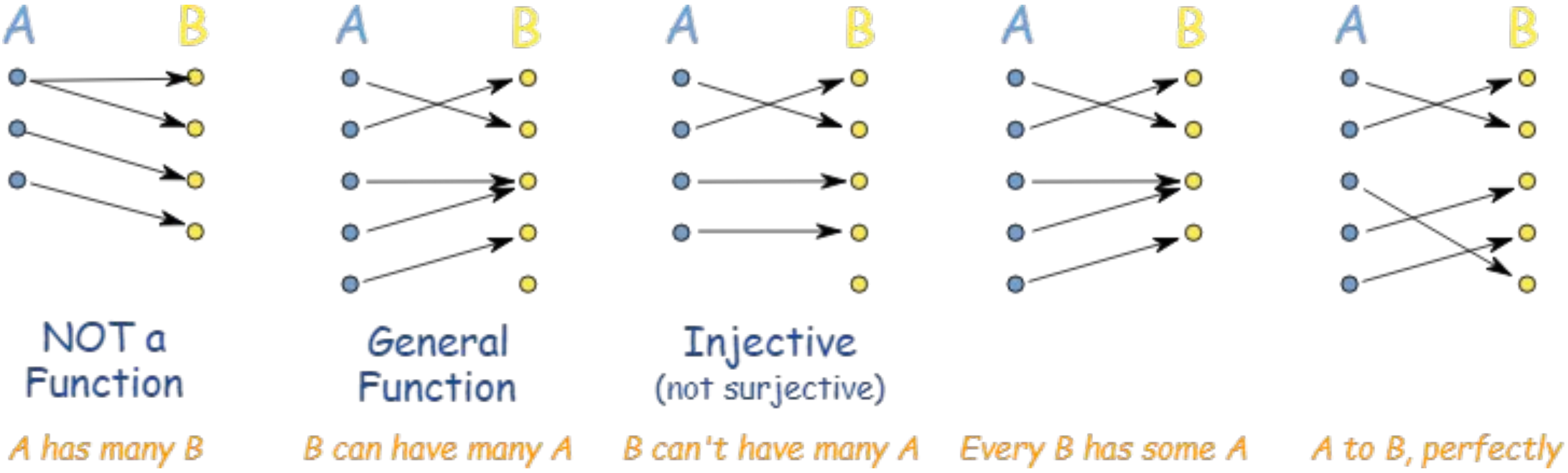

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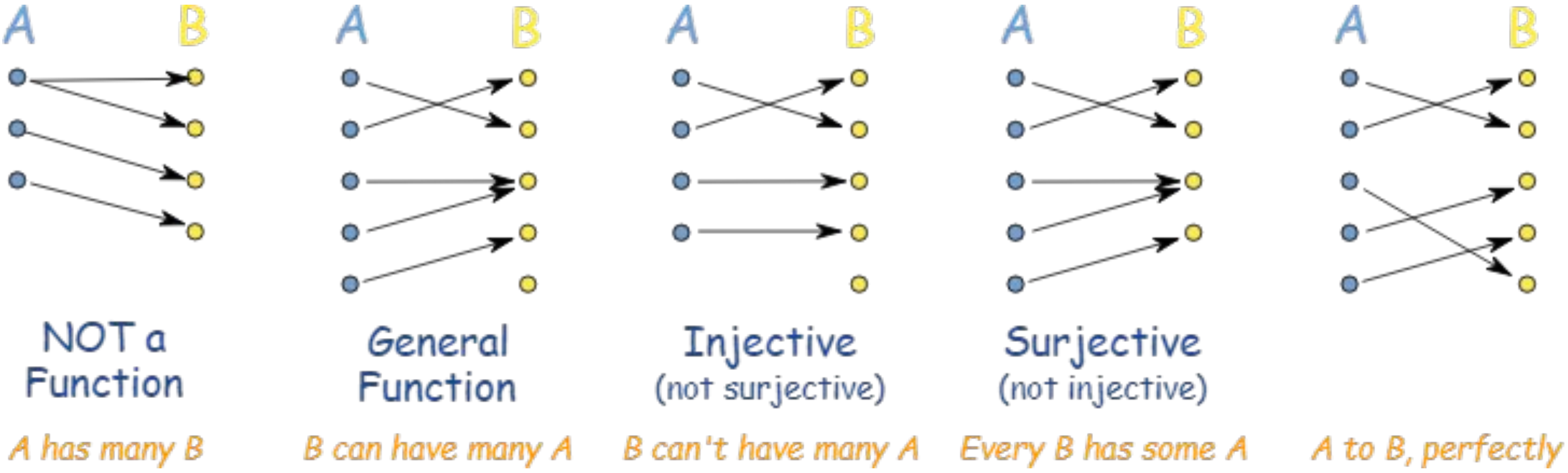

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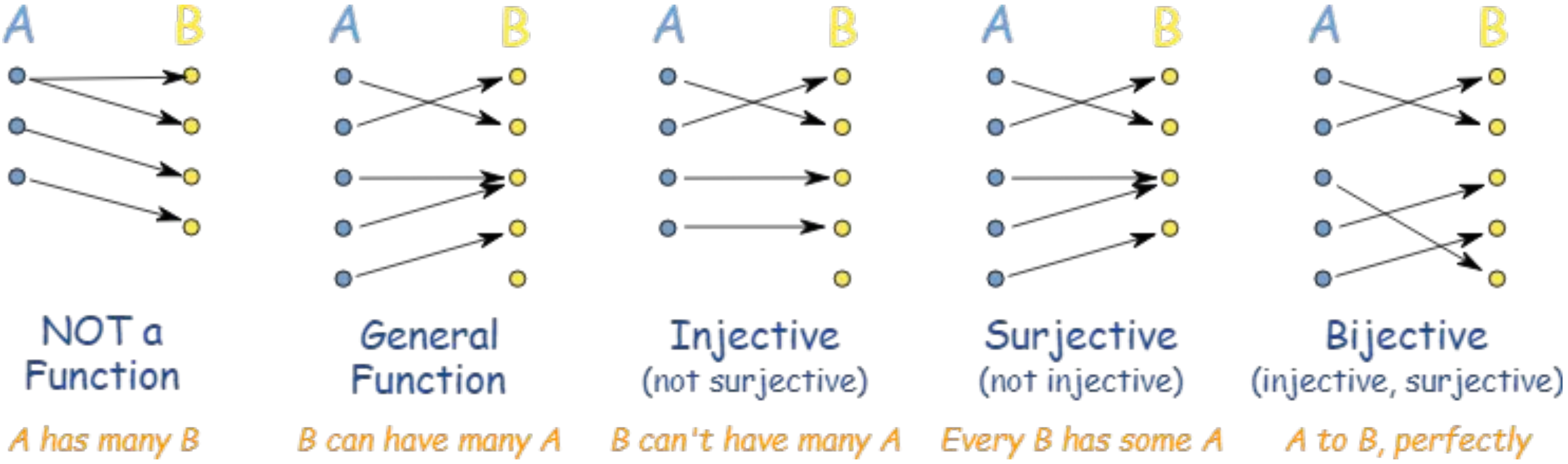

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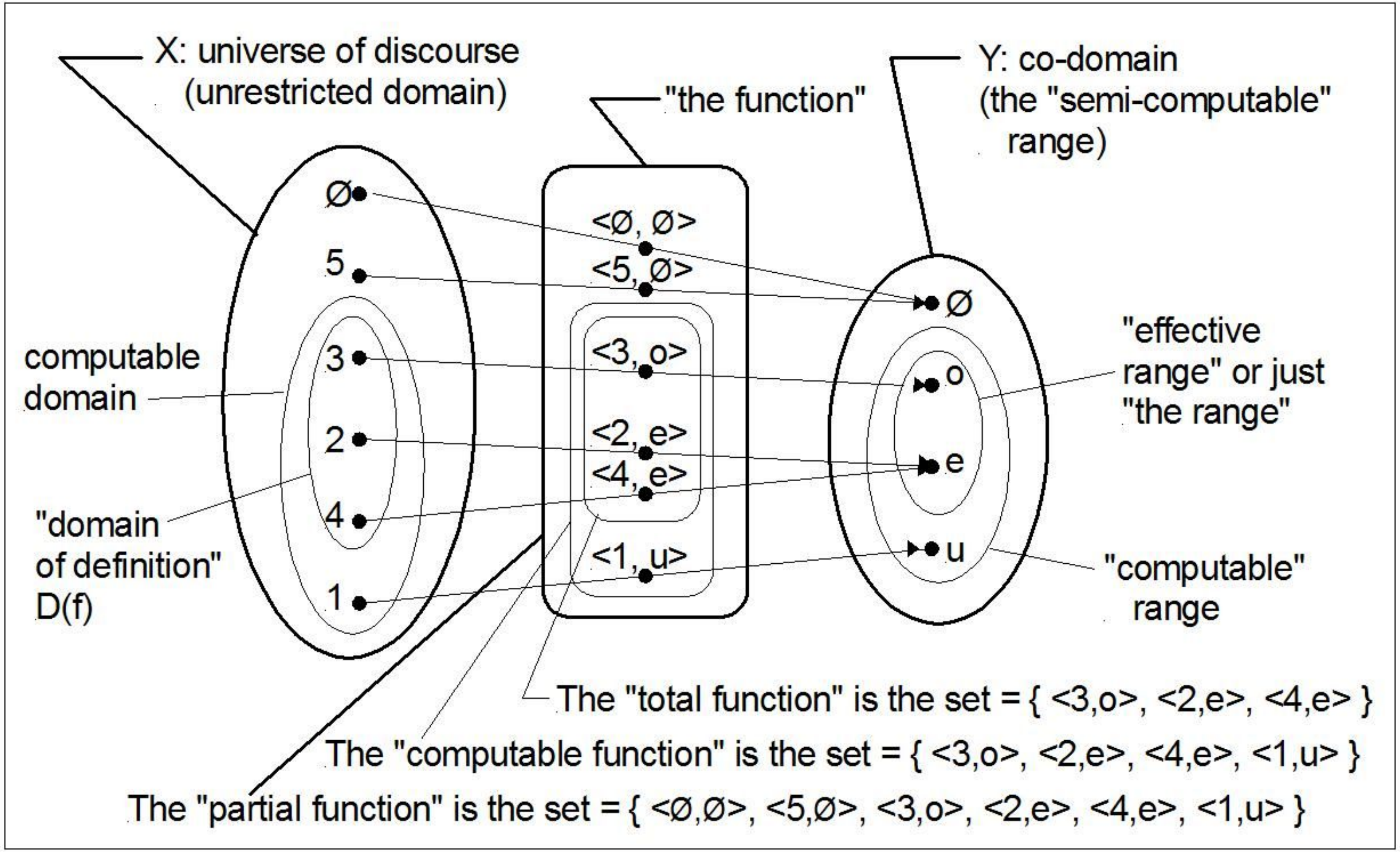
{T, F}^2 =

{ (T, T), (T, F), (F, T), (F, F) }


X: universe of discourse
(unrestricted domain)
"the function"
Y: co-domain
(the "semi-computable"
range)
Ø
5
3
2
4
1
computable
domain
"domain
of definition"
D(f)
<Ø, Ø>
<5, Ø>
<3, o>
<2, e>
<4, e>
<1, u>
Ø
o
e
u
"effective
range" or just
"the range"
"computable"
range
The "total function" is the set = { <3,o>, <2,e>, <4,e> }
The "computable function" is the set = { <3,o>, <2,e>, <4,e>, <1,u> }
The "partial function" is the set = { <Ø,Ø>, <5,Ø>, <3,o>, <2,e>, <4,e>, <1,u> }