

Nov 4th Slides

Elizabeth Orrico

CS CAREER EXPLORATION

Thursday, Nov 5th & 12th
7-8 PM EST

Nov 5th:

Mobile App Development

Back-end

Product Management

Data Science

Nov 12th:

Grad School

Video Game Development

Cybersecurity

<PRESENTED BY: ACM X WICS X GWC>



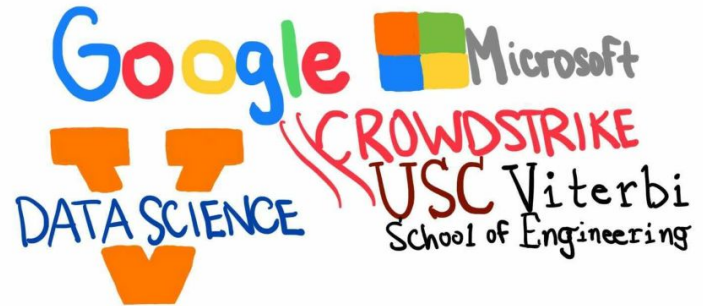
CS CAREER EXPLORATION PANEL

ACM@UVA girls who code
WICS @UVA

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Sets vs Sequences

Both can:

- Contain anything
- Can have a sequence of sequences, set of sets, sequence of sets, etc
- Cannot be modified

Sets:

- no duplicates
- no order
- has cardinality

Sequences:

- can have duplicates
- has order
- has length

Lists, Arrays, Ordered pairs, Tuples, etc!

Cartesian Product of Sets

Ordered Pair: An ordered pair is a pair of objects where one element is designated first and the other element is designated second, denoted (a, b) . (is an example of a sequence with 2 elements)

Cartesian Product: The Cartesian product of two sets A and B , denoted $A \times B$, is the set of all possible ordered pairs where the elements of A are first and the elements of B are second.

In set-builder notation, $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$.

Cartesian Product of Sets

$$\{1, 2\} \times \{3, 4, 5\}$$

$$= \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}$$

Cartesian Product of Sets

$$|\{1, 2\} \times \{3, 4, 5\}|$$

$$= |\{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}|$$

$$= 6$$

Cartesian Product of Sets

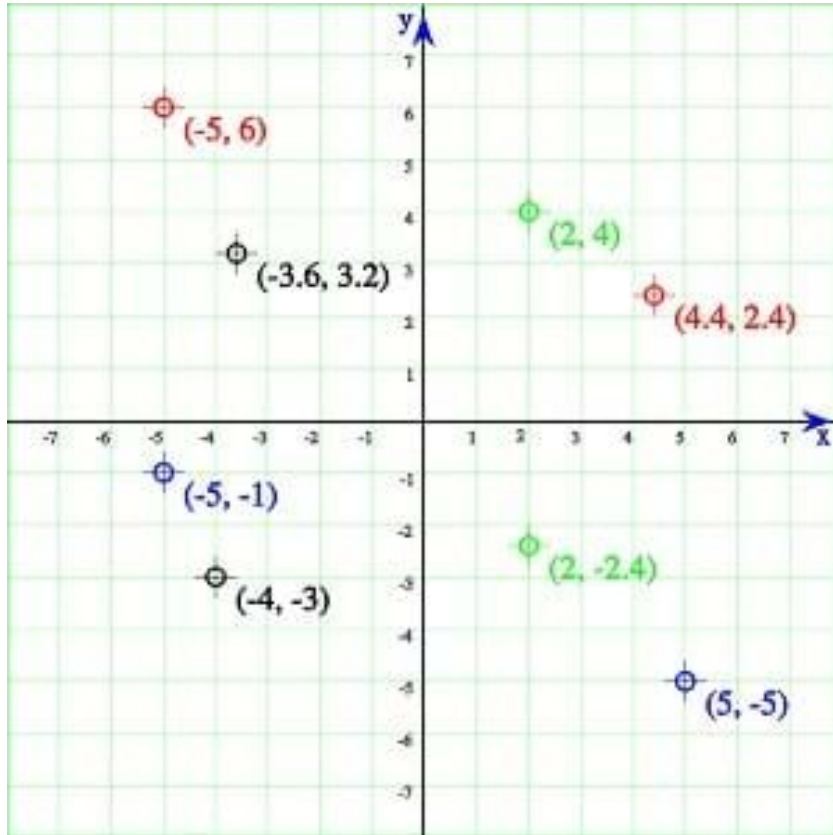
Your Turn: What is $\{1, 2\} \times \{2, 3\}$?

Cartesian Product of Sets

Your Turn: What is $\{1, 2\} \times \{2, 3\}$?

Answer: $\{(1, 2), (1, 3), (2, 2), (2, 3)\}$

Cartesian Product of Sets



$\mathbb{R} \times \mathbb{R}$: The
coordinate plane

Characters

Letters of an alphabet: abcdefghijklmnopqrstuvwxyz

When characters are in a **sequence**, we call it a **string**

$(a, b) \Rightarrow ab$

Kleene Star *

Counting Character and Number Combinations

Imagine a state that requires default license plates to have 3 characters followed by 4 numbers:

How many possible plates?



How many possible plates?

$L = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$

$D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$



$L \times L \times L \times D \times D \times D \times D = \text{Set of every plate}$

How many possible plates?

$L = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$

$D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$



$L \times L \times L \times D \times D \times D \times D = \text{Set ev'ry plate seq}$
 $26 \times 26 \times 26 \times 10 \times 10 \times 10 \times 10 = \# \text{ of possible plates}$

$= 175,760,000$

How many sequences of digits of length 4?

$$\underline{\quad} \times \underline{\quad} \times \underline{\quad} \times \underline{\quad} = \underline{\hspace{2cm}}$$

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$$\begin{array}{ccccccc} \underline{\quad} & \times & \underline{\quad} & \times & \underline{\quad} & \times & \underline{\quad} & = & \underline{\hspace{2cm}} \\ \mathbf{10} & \times & \mathbf{10} & \times & \mathbf{10} & \times & \mathbf{10} & = & \mathbf{10,000} \end{array}$$

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(Numbers 0-9,999: makes sense)

How many sequences of digits of length 4
with no repeats?

$$\underline{\quad} \times \underline{\quad} \times \underline{\quad} \times \underline{\quad} = \underline{\hspace{2cm}}$$

How many sequences of digits of length 4
with no repeats?

$$\frac{\quad}{\quad} \times \frac{\quad}{\quad} \times \frac{\quad}{\quad} \times \frac{\quad}{\quad} = \frac{\quad}{\quad}$$
$$10 \times 9 \times 8 \times 7 = 5,040$$

Side-note: Product Notation and Factorials

$$\prod_{i=1}^6 i^2 = (1)(4)(9)(16)(25)(36)$$

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$$\prod_{i=1}^6 i^2 = (1)(4)(9)(16)(25)(36)$$

$$\prod_{i=1}^5 i = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 5!$$

How can we represent $10 \times 9 \times 8 \times 7$ as Factorials?

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$$\frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{6 \times 5 \times 4 \times 3 \times 2 \times 1} =$$

$$\prod_{i=1}^5 i = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 5!$$

How can we represent $10 \times 9 \times 8 \times 7$ as Factorials?

$$\frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{10!}{6!} = \frac{10!}{(10-4)!}$$

$$\prod_{i=1}^5 i = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 5!$$

COUNTING

***n* options for *k* positions**

n^k : All possible sequences

$\frac{n!}{(n-k)!}$: All possible permutations

$\frac{n!}{k!(n-k)!}$: All possible combinations (SETS)

Permutation: A shuffling, re-ordering of sequence

$${}^n P_k = \frac{n!}{(n-k)!}$$

We're just looking at **how many sequences** with **unique elements (order matters)**

For an individual permutation, we've settled on the k elements *and the ordering we want to include*.

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How could we **remove** these shufflings, so we're just looking at **how many sets** with **unique elements**?

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How many permutations of the same k elements are possible?

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How could we **remove** these shufflings, so we're just looking at **how many sets** with **unique elements**?

How many permutations of the same k elements are possible?

ie, for $k = 2$: (1, 2), (2, 1)

Example: How many orderings of 4 elements (5, 8, 1, 3) are there?

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____, _____, _____, _____

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$$\frac{\quad}{4} \times \frac{\quad}{3} \times \frac{\quad}{2} \times \frac{\quad}{1} \quad \text{or} \quad k!$$

Example: How many orderings of 4 elements (5, 8, 1, 3) are there?

$$\underline{\quad}, \underline{\quad}, \underline{\quad}, \underline{\quad}$$
$$4 \times 3 \times 2 \times 1 \quad \text{or} \quad k!$$

So, to remove all the different orderings counted, divide the permutations-- by $k!$

Combinations: How many possible **sets** (no repeats, order doesn't matter)

$$\binom{n}{k} = C_k^n = {}_n C_k$$

“n choose k”

Question 18: How many sequences are there with Exactly 8 characters taken from the 26 lower-case ASCII letters and 10 ASCII digits, with no characters repeated twice back-to-back (i.e., abadaeag is OK but abcddefg is not)

What's the number of options? **n=36**

What's the number of positions? **k=8**

$$\begin{array}{l} (\text{_e_}, \text{_l_}, \text{_i_} \dots \dots) \\ 36 \times 35 \times 35 \dots \dots = 36 * 35^7 \end{array}$$

$$10^4 = 10,000$$

$$5,040$$

$$10!/((10-4)!*4!) = 210$$

Question 19: Exactly 8 characters taken from the 26 lower-case ASCII letters and 10 ASCII digits, with no repeated characters (i.e., neither abcdaefg nor abcddefg are OK)

$$36! / (36-8)!$$

Ask to solve: Are there repeats allowed? Do we count different orderings with the same options (permutations)?

Question 29: How many 21-element subsets of a 31-element set are there?

$$n = 31$$

$$k = 21$$

N choose k = combination $\rightarrow 31!/(31-21)!*21!$

Ask to solve: Are there repeats allowed? Do we count different orderings with the same options (permutations)?

Question 29: How many 21-element subsets of a 31-element set are there?

Ask to solve: Are there repeats allowed? Do we count different orderings with the same options (permutations)?