Nov 9th Slides

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https://docs.google.com/document/d/1JCzR8CS7mk3ghgiyB3zn0OFL6oYA1FQXjDiB_-V1zAA/edit?usp=sharing

What is a Function?

1. **What can a function output?**
   Numbers
   Booleans
   Vectors
   Anything and everything
   String

2. What can be inputs to a function?
Same thing as output

3. How can a function be defined?
formula/equation
Code-- subroutine
algorithm

What is a Function?

A function assigns an element of one SET to another SET…

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$$f : A \to B$$

Why a SET and not a SEQUENCE?

What is a Function?

The notation:

$$f : A \rightarrow B$$

indicates that f **is a function with domain,** A, and **codomain,** $\boldsymbol{B}$**.** The familiar notation "$\boldsymbol{f(a) = b}$" indicates that f assigns the element $\boldsymbol{b \in B}$ to a specific argument $\boldsymbol{a \in A}$**.**

Here b would be called the **value** of f at **argument** a**.**

Domain

A function *need not* be defined for every element in its domain.
For example, if we consider $f_1(x) : \mathbb{R} \rightarrow \mathbb{R}$

$$f_1(x) = 1/x^2$$

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If there are domain elements for which a function is not defined, it is a *partial function*.

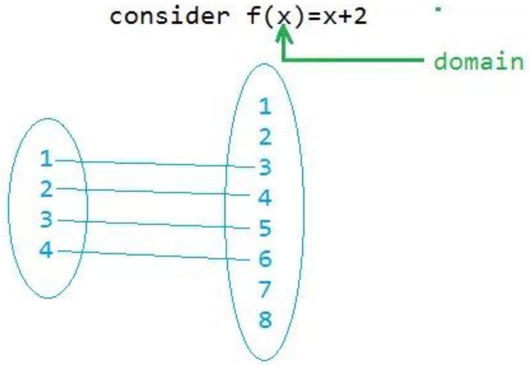
$f_1(x) : \mathbb{R} \rightarrow [0,\text{inf})$

$f_1(x) = x\text{^}2$

Meanwhile, a *total function* ???

Codomain

A function *need not* be able to return every element of its codomain...



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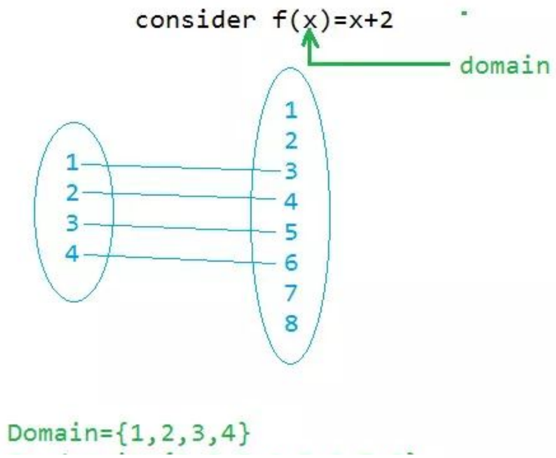

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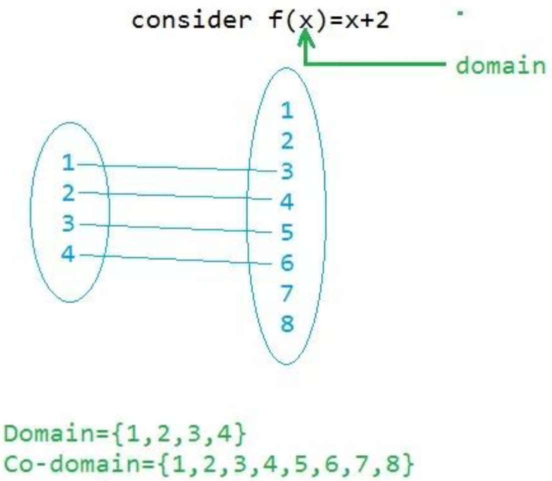

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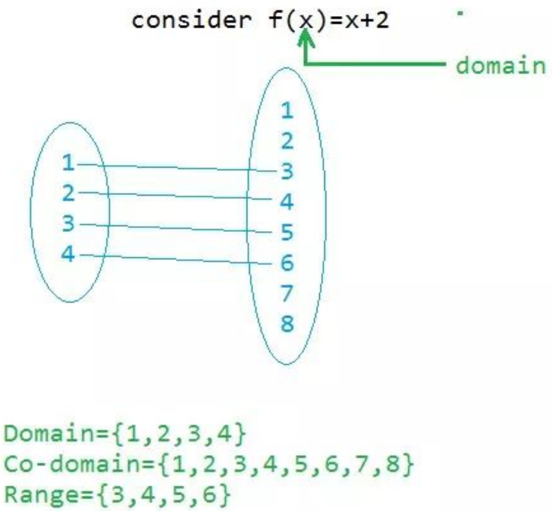


Range ⊆ Codomain

Codomain

Surjective, onto : Codomain = Range

Kind of the "reverse" of "total"

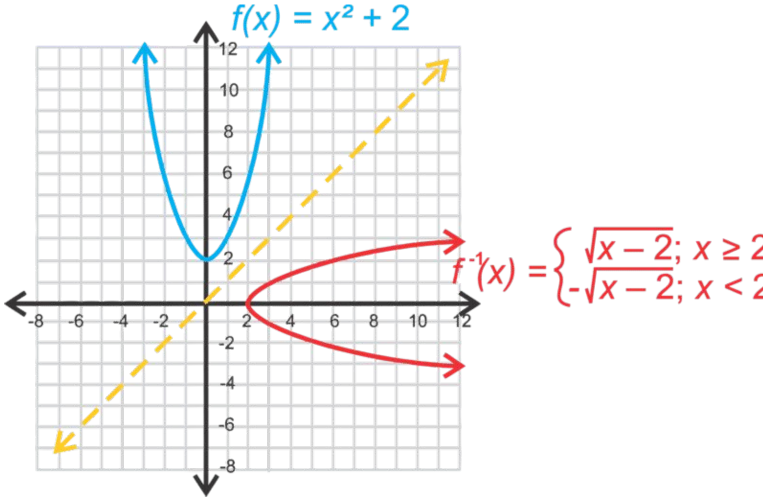
$$\forall b \in B,\ \exists a \in A,\ f(a) = b.$$

Injective

Every input maps to a different output!

$\forall\ x, y \in D\ .\ \ (\ x \neq y\ ) \rightarrow (\ f(x) \neq f(y)\ )$

think parabola NOT INJECTIVE



Bijective (or invertible or correspondence)

Must be **total, and surjective, and injective**! (oh my)

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Injective: $\forall a_1 \in A,\ \forall a_2 \in A,\ (f(a_1) = f(a_2)) \implies (a_1 = a_2).$

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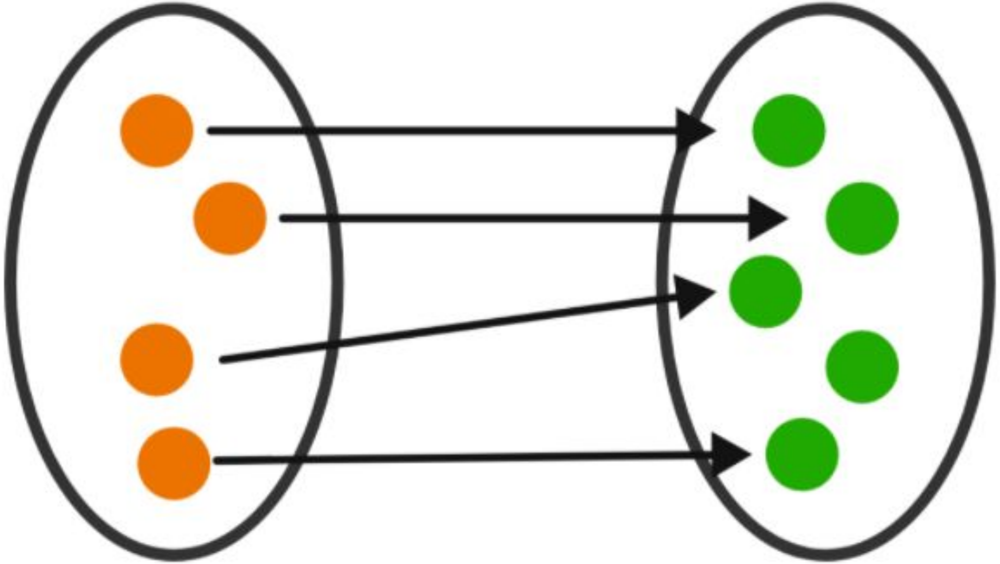

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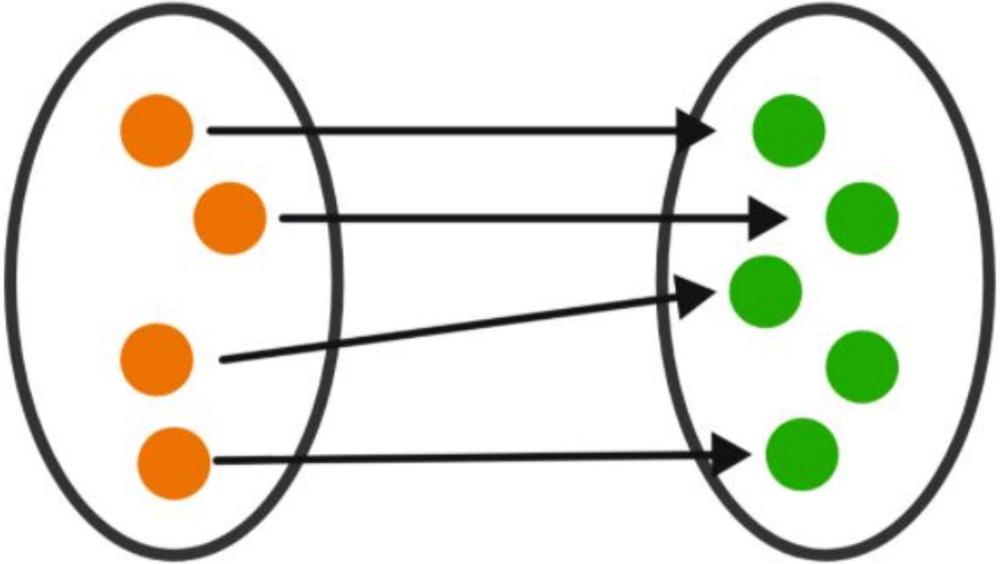
Injection (One-to-One)



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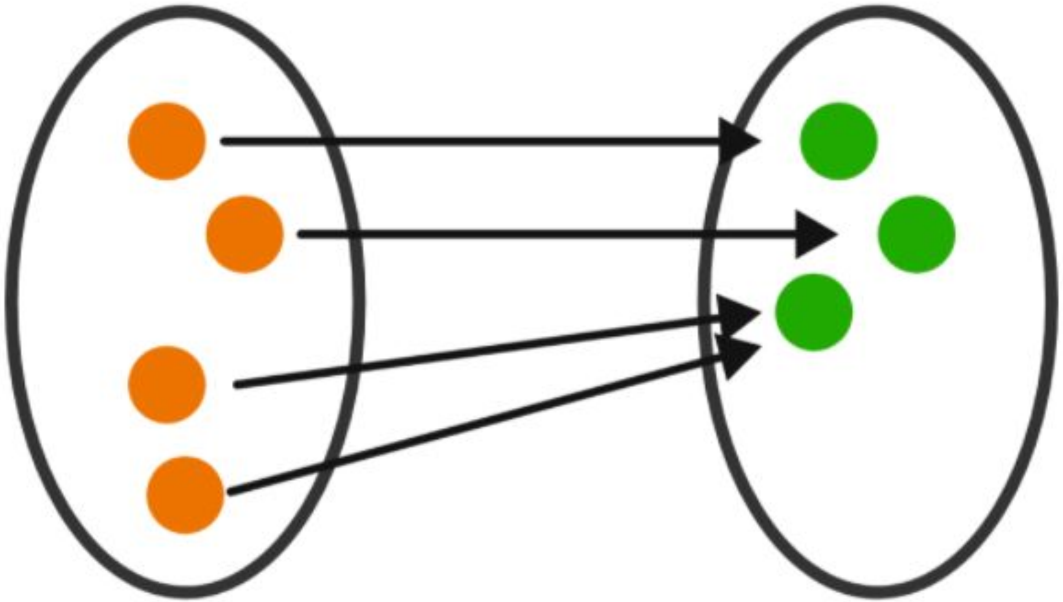

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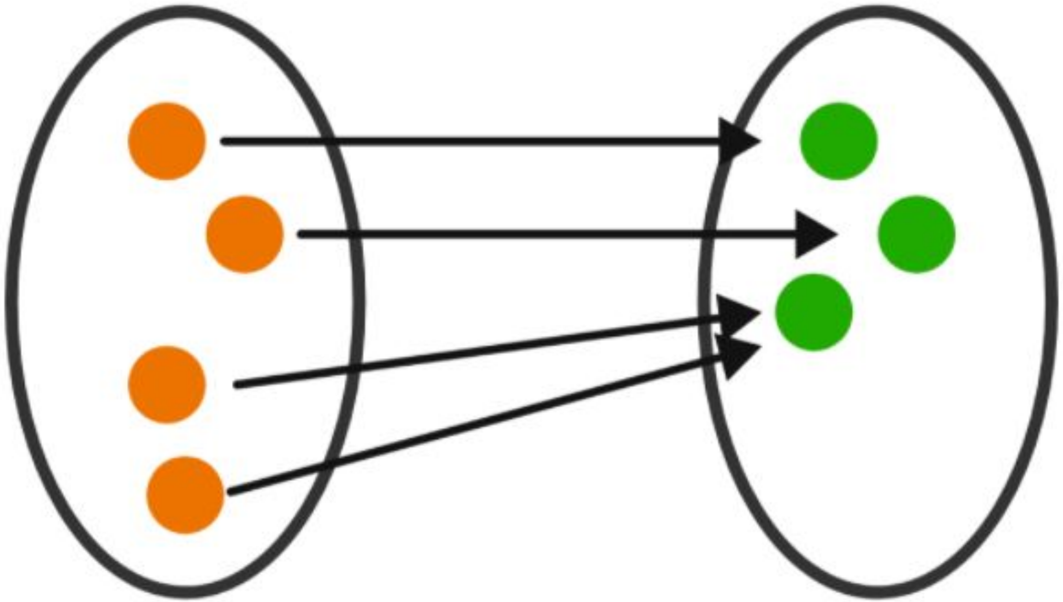
Surjection (Onto)



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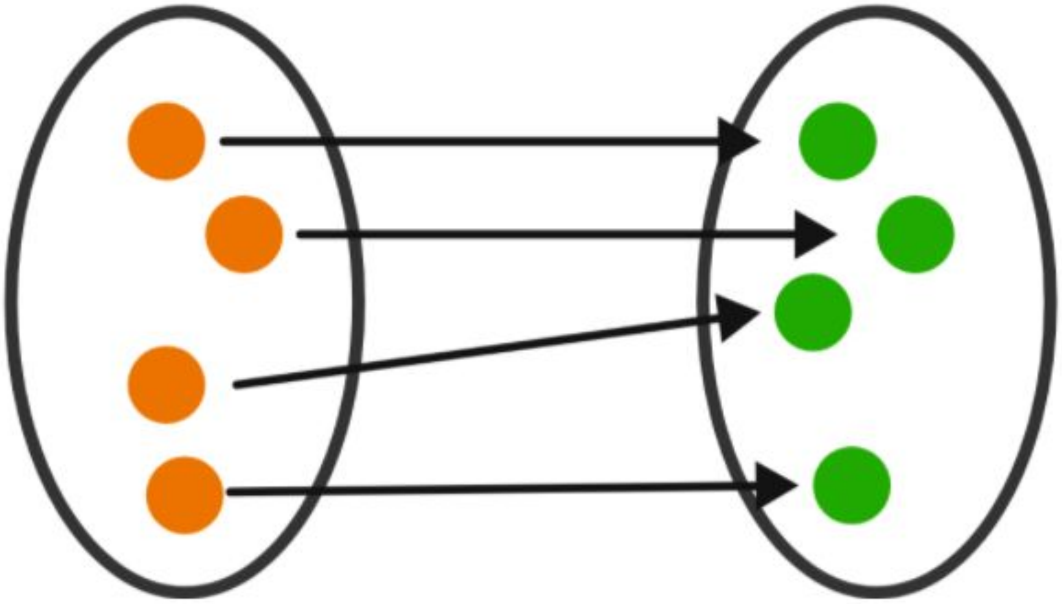

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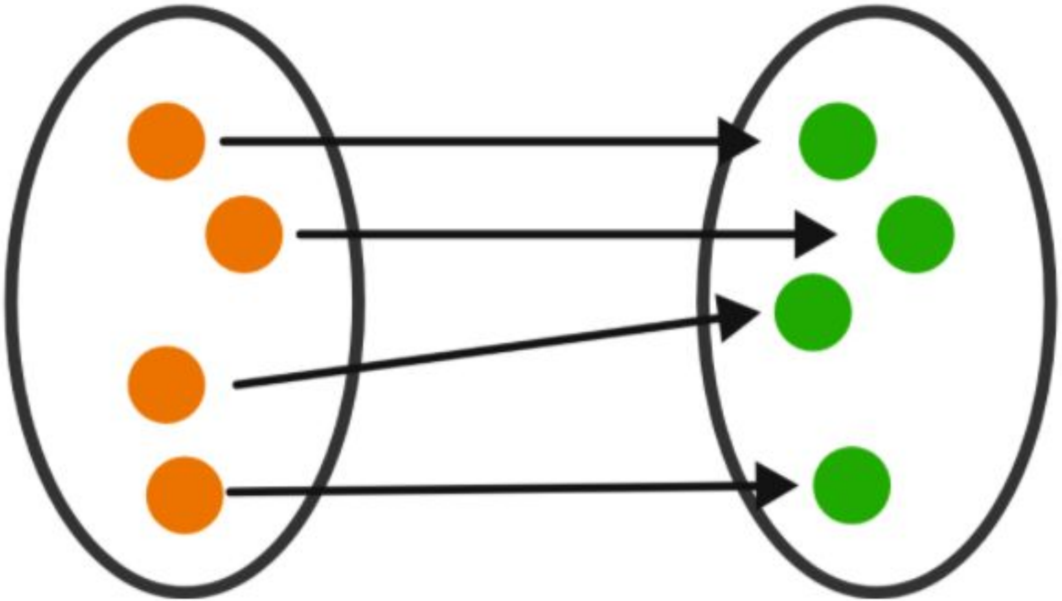
Bijection (One-to-One and Onto)



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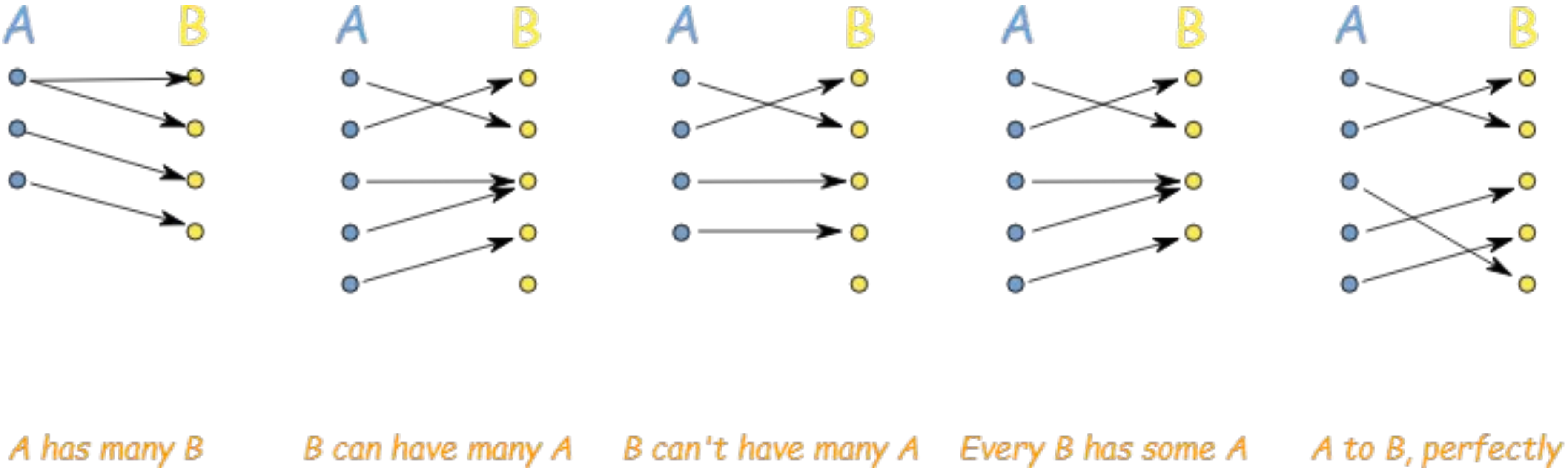

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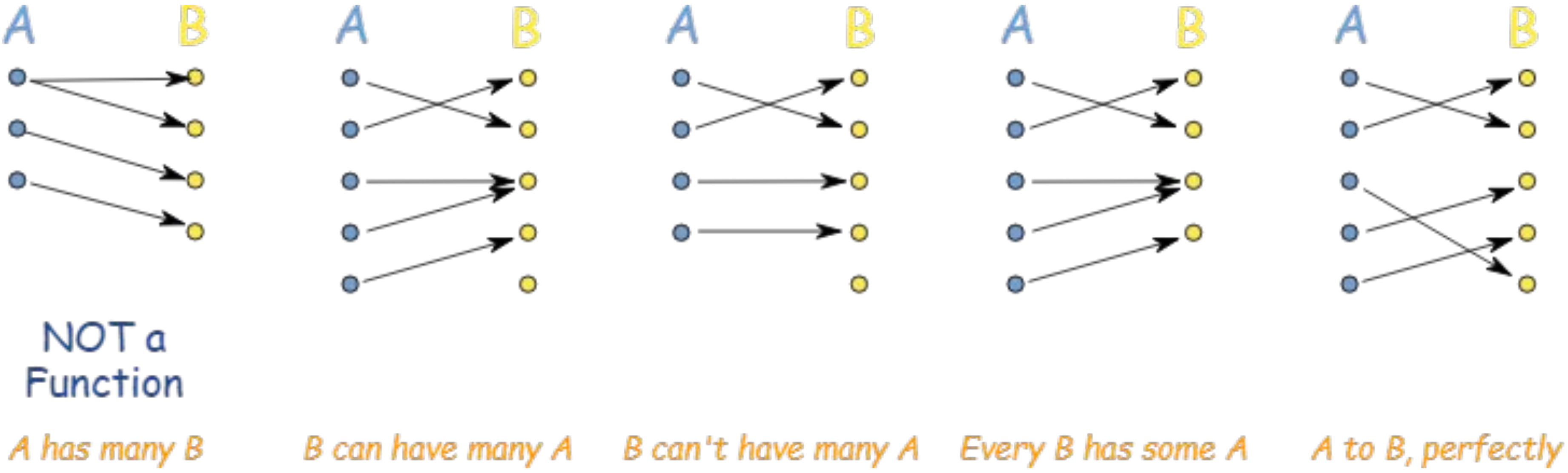

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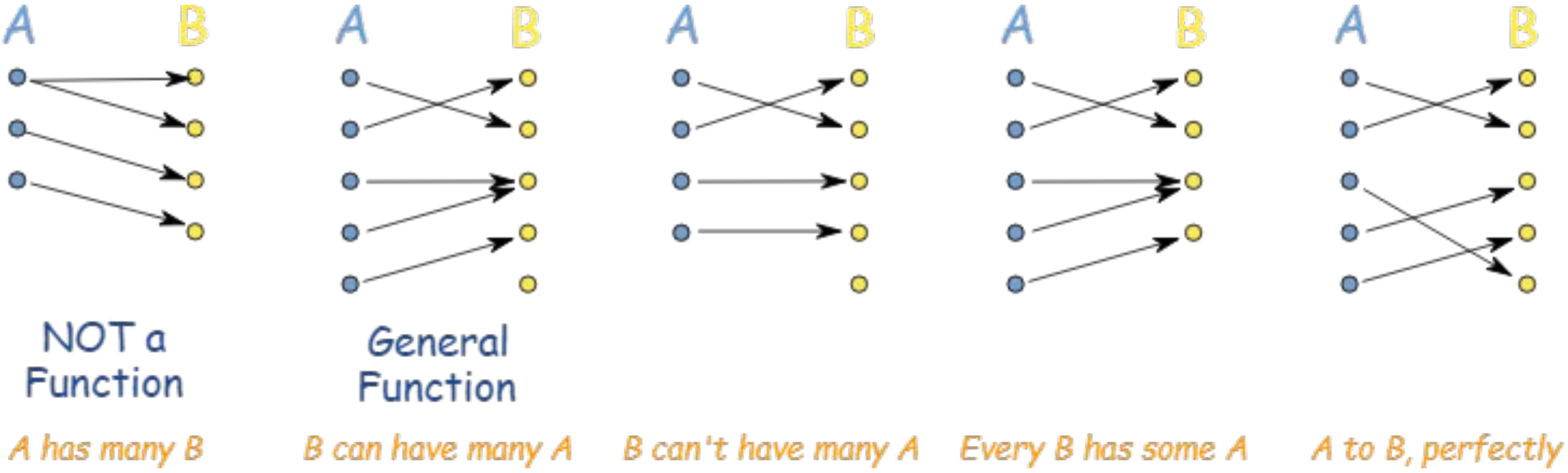

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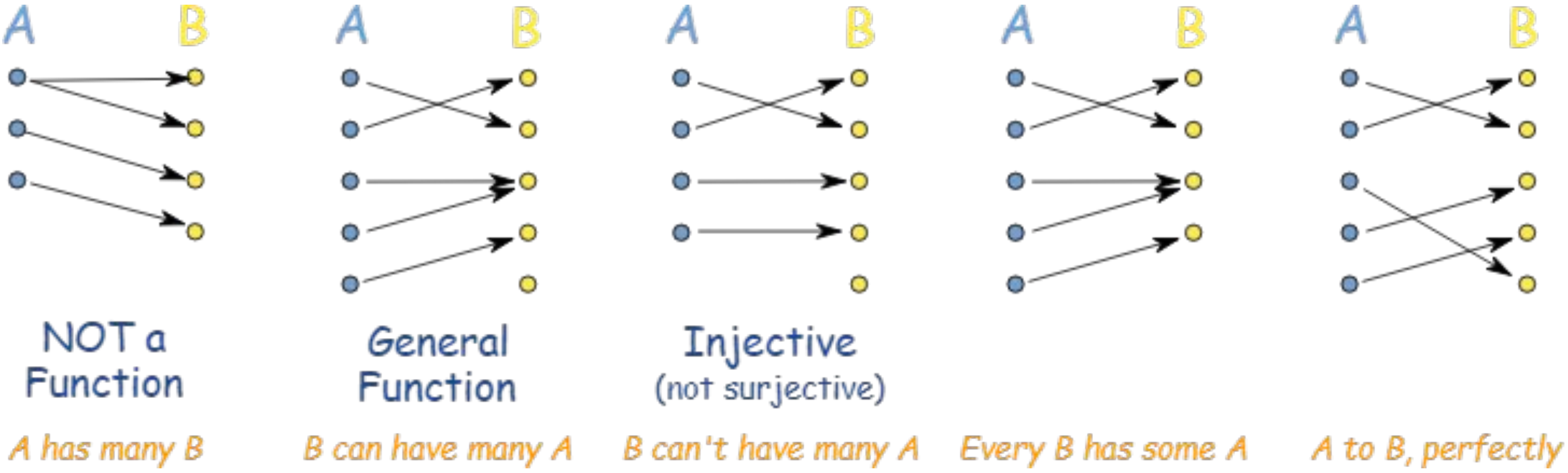

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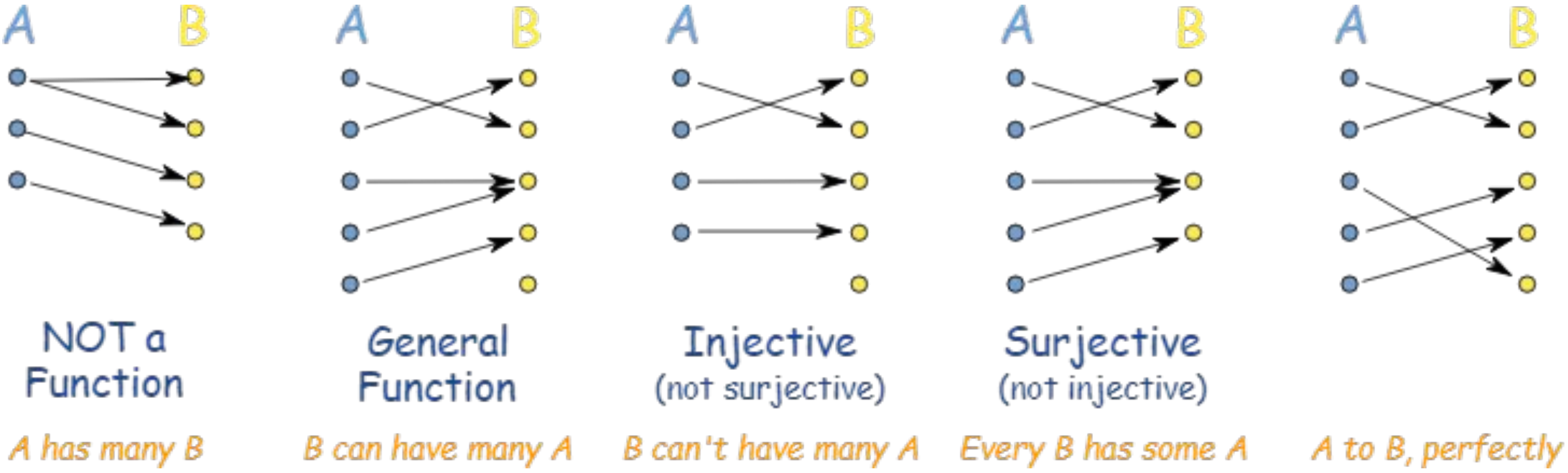

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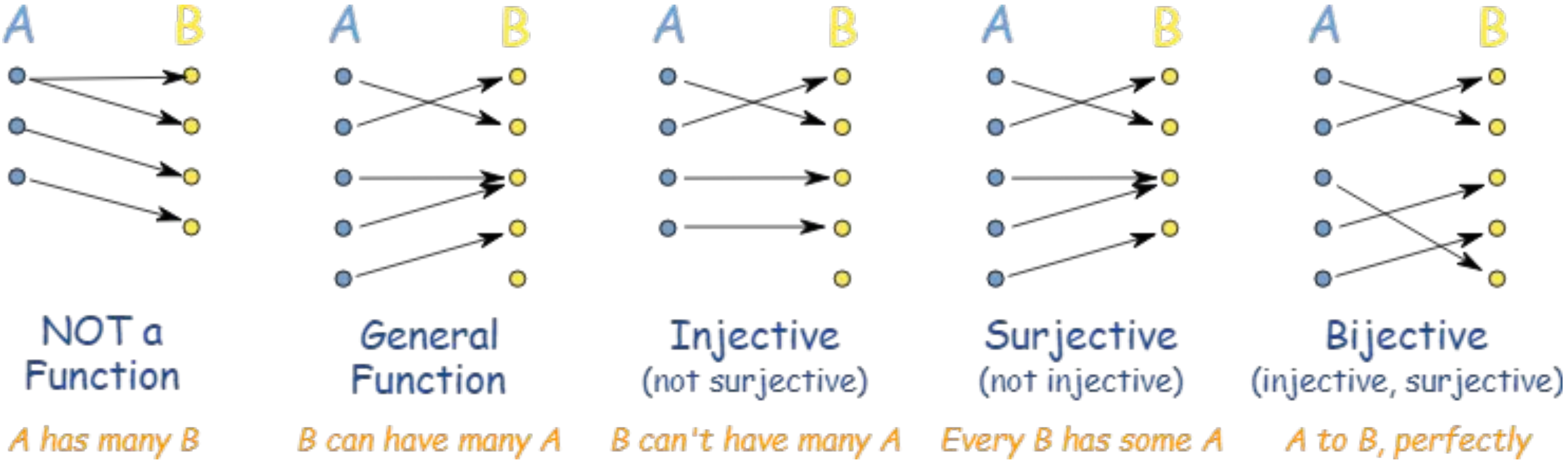

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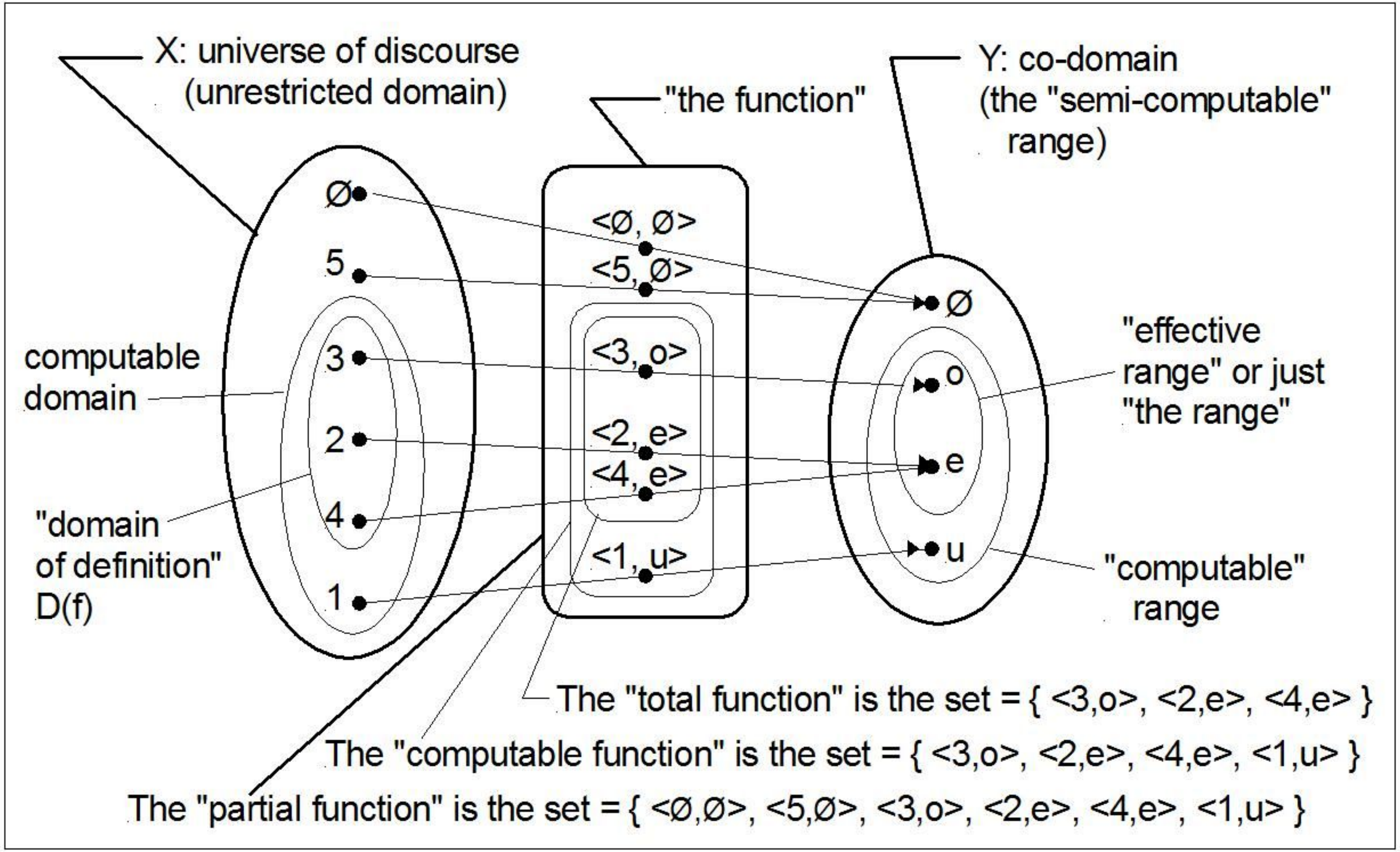
Definition 4.4.2. A binary relation, R, is:

- a *function* when it has the [≤ 1 arrow **out**] property.
- *surjective* when it has the [≥ 1 arrows **in**] property. That is, every point in the righthand, codomain column has at least one arrow pointing to it.
- *total* when it has the [≥ 1 arrows **out**] property.
- *injective* when it has the [≤ 1 arrow **in**] property.
- *bijective* when it has both the [$= 1$ arrow **out**] and the [$= 1$ arrow **in**] property.



X: universe of discourse (unrestricted domain)
"the function"
Y: co-domain (the "semi-computable" range)
Ø
5
3
2
4
1
<Ø, Ø>
<5, Ø>
<3, o>
<2, e>
<4, e>
<1, u>
Ø
o
e
u
computable domain
"domain of definition" D(f)
"effective range" or just "the range"
"computable" range
The "total function" is the set = { <3,o>, <2,e>, <4,e> }
The "computable function" is the set = { <3,o>, <2,e>, <4,e>, <1,u> }
The "partial function" is the set = { <Ø,Ø>, <5,Ø>, <3,o>, <2,e>, <4,e>, <1,u> }