

# Oct 14th Slides

Elizabeth Orrico

# What's a factor?

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That is, for some integer ***k***

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1, 2, 4, -4, -2, -1

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From this point forward, we will only be referring to the positive integers though.

# What's a prime

A prime is a number greater than 1 that is divisible only by 1 and itself.

Is 2 a prime number?

# “Fundamental theorem of Arithmetic”

Aka Unique Factorization thm

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$$60 = 2 \cdot 2 \cdot 3 \cdot 5$$

^The one and only way of expressing 60 as a set of prime factors

“Prime factorization” -> multiplicity of factor 2 is 2 (of 5 is 1)

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$$60 = 2^2 * 5 * 3$$



# Greatest Common Divisor

A **common divisor** of  $a$  and  $b$  is a number that divides them both.

The **greatest common divisor** of  $a$  and  $b$  is the biggest number that divides them both.

$$\text{gcd}(60, 8) = ???$$

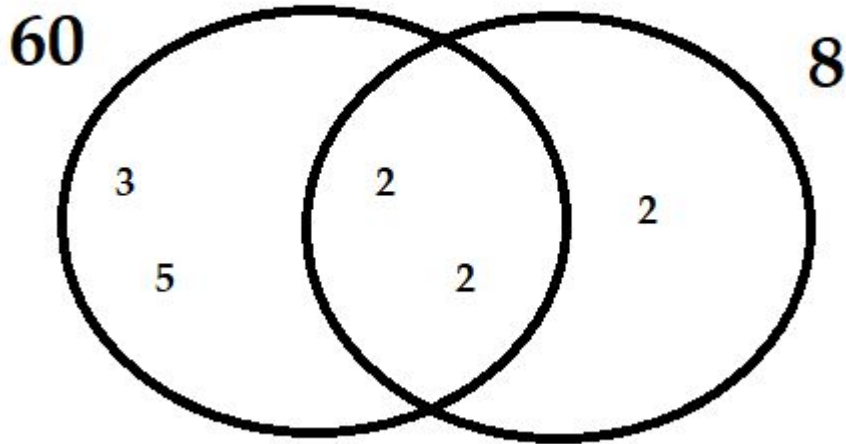
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$$\text{gcd}(60, 8) = 4$$

“intersect”



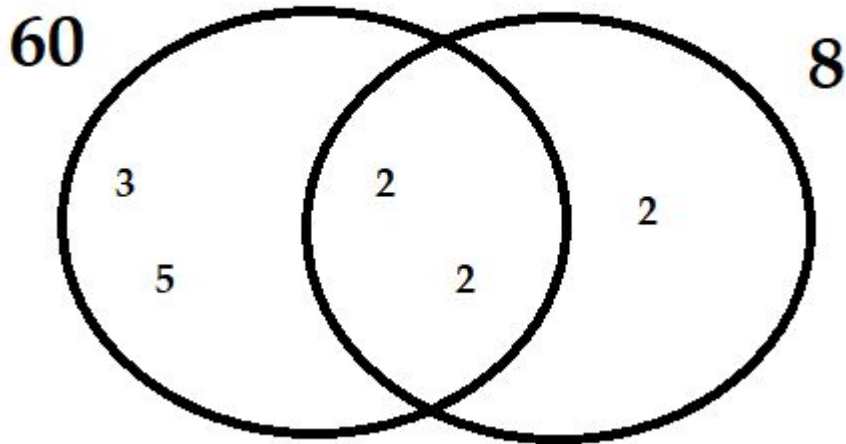
# Least Common Multiple

A **common multiple** of a and b is a number that they can both divide

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“Union”



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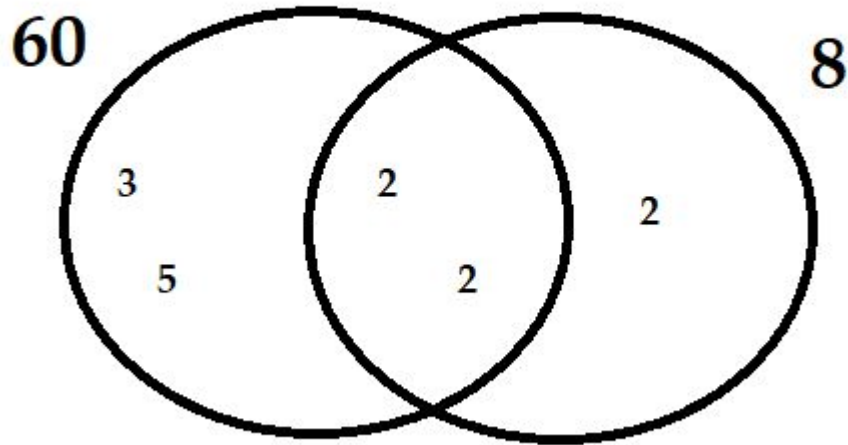
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(union)

# Relatively Prime

Two positive integers greater than 1 are **relatively prime** iff  $\gcd(x, y) = 1$

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**Goldbach's conjecture:** Every even integer greater than 2 is the sum of two primes.

**Collatz conjecture:** Start with any positive integer  $n$ . Then, if the number is even, divide by 2. If it's odd, multiply by 3 and add 1. Repeat. The conjecture is that no matter what value of  $n$ , the sequence will always reach 1.

# Proof by contradiction

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**Theorem: Not A**

**We proceed by contradiction.**

Assume A to prove this”

...

A proves FALSE

**Therein lies the contradiction.**

**Therefore**

**Not A**

*Often how we prove something is impossible in computing*

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# Proof by contradiction -- Informal

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One side has zero factors, one has at least 1?? Contradiction

# Proof by contradiction -- Formal

Thm:  $\frac{1}{2}$  is not an element of the natural numbers

We proceed by contradiction

Assume  $\frac{1}{2} \in \mathbb{N}$ .

Then,  $\exists x \in \mathbb{N}. x = \frac{1}{2}$

By algebra, that means  $2x = 1$ .

By the fundamental theorem of arithmetic, both sides of the equation are equal, so 1 and  $2x$  must have the same unique prime factorization.

But the factors of  $2x$  include 2, and the factors of 1 do not. Therein lies the contradiction.

Therefore, [Because this assumption led to a contradiction,]  $\frac{1}{2} \notin \mathbb{N}$