

Oct 26th Slides

Elizabeth Orrico

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A pattern!

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$$8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36$$

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What if the room had 20 people? 100 people? 1000 people?
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limits above and below tell us where to start and stop summing

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$$\sum_{i=1}^3 2i = 2(1) + 2(2) + 2(3)$$

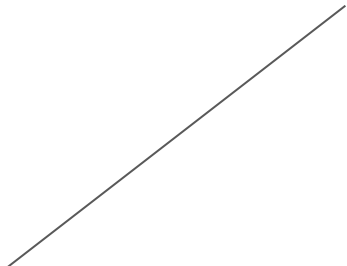
Your turn

To represent adding a sequence of numbers together, we can often use **summation notation**.

$$\sum_{i=2}^4 3i = ???$$

Problem

The second way to what we're summing over is to give a **set**

$$\sum_{i \in \mathbb{N}} \frac{1}{3^i}$$


Problem

We can switch between the upper and lower limits and sets as follows:

$$\sum_{i=2}^5 3i = \sum_{i \in A} 3i$$

Where

$$A = \{ x \in \mathbb{Z} \mid (x \geq 2) \wedge (x \leq 5) \}$$

Your turn

Given this definition of upper and lower limits, what could this be?

$$\sum_{i=2}^1 3i = ???$$

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Given this definition of upper and lower limits, what could this be?

$$\sum_{i=2}^1 3i = ???$$

$$\{ i \in \mathbb{Z} \mid (i \geq 2) \wedge (i \leq 1) \}$$

Your turn

The second way to what we're summing over is to give a **set**

$$\sum_{i=2}^1 3i = 0$$

By convention

$$\{ i \in \mathbb{Z} \mid (i \geq 2) \wedge (i \leq 1) \}$$

Your turn

What about....?

$$\sum_{i=2}^2 3i = ???$$

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What about....?

$$\sum_{i=2}^2 3i = ???$$

$$\{ i \in \mathbb{Z} \mid (i \geq 2) \wedge (i \leq 2) \} = \{2\}$$

Your turn

What about....?

$$\sum_{i=2}^2 3i = 3(2) = 6$$

Where

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$$\sum_{i=1}^{9-1} i = \sum_{i=1}^8 i$$

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n people are in a room. Each person shakes hands with each other person in the room. How many handshakes occurred total?

How would we express this problem as a summation?

Remember, answer was $(n-1)+(n-2)\dots+2+1$

$$\sum_{i=1}^{n-1} i$$

Proof by Induction

Structure:

Thm: “ $P(x)$ is true for all the natural numbers”

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$P(x-1)$ can be used to show $P(x)$ is true

By the principle of induction, $P(x)$ is true for all the natural numbers.

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$$\sum_{i=0}^n 2 = 2n$$

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Just another 2. Doing some math, we get:

$2(x-1) + 2 = 2x - 2 + 2 = 2x$. This proves the case $n = x$ for some arbitrary x .

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By induction on n , we have proven that this summation is equal to $2n$ for any value of n in the natural numbers.

Problem:

How many handshakes?

Prove this by induction

$$\sum_{i=0}^{n-1} i = \frac{n(n-1)}{2}$$

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$$\sum_{i=0}^{n-1} i = \frac{n(n-1)}{2}$$

Base Case:

This summation evaluated when $n=1$ is 0.

Inductive Step:

Assume summation evaluated at $n = x$ will be equal to $(x)(x-1)/2$.

Then, considering $n = x + 1$ will add one more term to the sum: *An additional $x + 1$.*

Doing some math, we get:

$(x)(x-1)/2 + x + 1 = (x^2 - x + 2x + 2)/2 = (x^2 + x + 2)/2$ which factors to $(x+1)(x+2)/2$. **This proves that if the summation holds for the case of $n = x$, then it will also hold for the case $n = x + 1$ for some arbitrary x .**

By induction on n , we have proven that this summation is equal to $2n$ for any value of n in the natural numbers.