Oct 26th Slides

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A pattern!

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8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = _________

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A pattern!

$8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36$

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What if the room had 20 people? 100 people? 1000 people?
Representing and solving becomes tedious math.

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limits above and below tell us where to start and stop summing

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To represent adding a sequence of numbers together, we can often use **summation notation.**

$$\sum_{i=1}^{3} 2i = 2(1) + 2(2) + 2(3)$$

Your turn

To represent adding a sequence of numbers together, we can often use **summation notation.**

$$\sum_{i=2}^{4} 3i = ???$$

Problem

The second way to what we're summing over is to give a **set**

$$\sum_{i \in \mathbb{N}} \frac{1}{3^i}$$

Problem

We can switch between the upper and lower limits and sets as follows:

$$\sum_{i=2}^{5} 3i = \sum_{i \in A} 3i$$

$$\textit{Where}$$

$$A = \{ x \in \mathbb{Z} \mid (x \geq 2) \land (x \leq 5) \}$$

Your turn

Given this definition of upper and lower limits, what could this be?

$$\sum_{i=2}^{1} 3i = ???$$

Your turn

Given this definition of upper and lower limits, what could this be?

$$\sum_{i=2}^{1} 3i = ???$$

$$\{\, i \in \mathbb{Z} \,|\, (i \geq 2) \wedge (\, i \leq 1) \,\}$$

Your turn

The second way to what we're summing over is to give a **set**

$$\sum_{i=2}^{1} 3i = 0$$

By convention

$$\{\, i \in \mathbb{Z} \mid (i \geq 2) \wedge (\, i \leq 1) \,\}$$

Your turn

What about….?

$$\sum_{i=2}^{2} 3i = ???$$

Your turn

What about….?

$$\sum_{i=2}^{2} 3i = ???$$

$$\{\, i \in \mathbb{Z} \,|\, (i \geq 2) \wedge (\, i \leq 2) \,\} = \{2\}$$

Your turn

What about….?

$$\sum_{i=2}^{2} 3i = 3(2) = 6$$

$$Where$$

$$\{\, i \in \mathbb{Z} \,|\, (i \geq 2) \wedge (\, i \leq 2) \,\} = \{2\}$$

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$$\sum_{i=1}^{9-1} i = \sum_{i=1}^{8} i$$

Problem

n people are in a room. Each person shakes hands with each other person in the room. How many handshakes occurred total?

How would we express this problem as a summation?

Remember, answer was (n-1)+(n-2)...+2+1

$$\sum_{i=1}^{n-1} i$$

Proof by Induction

Structure:

Thm: “P(x) is true for all the natural numbers”

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P(x-1) can be used to show P(x) is true

By the principle of induction, P(x) is true for all the natural numbers.

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Prove this by induction

$$\sum_{i=0}^{n} 2 = 2n$$

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Inductive Step:

Assume summation evaluated at n = (x-1) = 2(x-1).

Then, considering n=x will add one more term to the sum:

Just another 2.

Prove this by induction

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Base Case:

This summation evaluated when n=0 is 0.

Inductive Step:

Assume summation evaluated at $n = (x\text{-}1) = 2(x\text{-}1)$.
Then, considering n=x will add one more term to the sum:
Just another 2. Doing some math, we get:
$2(x\text{-}1) + 2 = 2x - 2 + 2 = 2x$. This proves the case $n = x$ for some arbitrary x.

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Just another 2. Doing some math, we get:
2(x-1) + 2 = 2x - 2 + 2 = 2x. This proves the case $n = x$ for some arbitrary x.

By induction on n, we have proven that this summation is equal to *2n* for any value of *n* in the natural numbers.

Problem:
How many handshakes?
Prove this by induction

$$\sum_{i=0}^{n-1} i = \frac{n(n-1)}{2}$$

Prove this by induction

$$\sum_{i=0}^{n-1} i = \frac{n(n-1)}{2}$$

Base Case:

This summation evaluated when **n=1** is 0.

Inductive Step:

Assume summation evaluated at ***n = x*** will be equal to (x)(x-1)/2.
Then, considering ***n = x + 1*** will add one more term to the sum: *An additional* ***x + 1***.
Doing some math, we get:
$(x)(x-1)/2 + x + 1 = (x^2 - x + 2x + 2)/2 = (x^2 + x + 2)/2$ which factors to **(x+1)(x+2)/2**. **This proves that if the summation holds for the case of *n = x*, then it will also hold for the case *n = x + 1* for some arbitrary *x*.**

By induction on n, we have proven that this summation is equal to *2n* for any value of *n* in the natural numbers.