Oct 28th Slides

Elizabeth Orrico

Problem

9 people are in a room. Each person shakes hands with each other person in the room. How many handshakes occurred total?

Proof by Induction

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Thm: “P(x) is true for all the natural numbers”

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By the principle of induction, P(x) is true for all the natural numbers.

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Inductive Step:

Assume summation evaluated at n-1 will result in 2(n-1).

Prove for n equal to all positive integers:

$$\sum_{i=1}^{n} 2 = 2n$$

Base Case:

This summation evaluated when n=1 is 2.

Inductive Step:

Assume summation evaluated at n-1 will result in 2(n-1).
Then, considering n will add one more term to the left-hand
Summation: *Just another 2.*

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Base Case:

This summation evaluated when n=1 is 2.

Inductive Step:

Assume summation evaluated at n-1 will result in 2(n-1). Then, considering n will add one more term to the left-hand Summation: *Just another 2.* Add this 2 to the right hand-side gives us: 2(n-1) + 2 = 2n - 2 + 2 = 2n. This proves that if the theorem holds for some arbitrary *n-1*, then it also must hold for *n*.

Prove for n equal to any positive integer:

$$\sum_{i=1}^{n} 2 = 2n$$

Base Case:

This summation evaluated when n=1 is 2.

Inductive Step:

Assume summation evaluated at n-1 will result in 2(n-1). Then, considering n will add one more term to the left-hand Summation: *Just another 2.* Add this 2 to the right hand-side gives us: 2(n-1) + 2 = 2n - 2 + 2 = 2n. This proves that if the theorem holds for some arbitrary *n-1*, then it also must hold for *n.*

By induction on n, we have proven that this summation is equal to *2n* for any value of *n* in the positive integers.

Problem:
How many handshakes?
Prove this by induction

$$\sum_{i=0}^{n-1} i = \frac{n(n-1)}{2}$$

Prove that for all positive integers:

$$\sum_{i=0}^{n-1} i = \frac{n(n-1)}{2}$$

Base Case:

This summation evaluated when **n=1** is 0.

Inductive Step:

Assume summation evaluated at some ***n-1*** will be equal to (n-1)(n-2)/2.
Then, considering ***n = (n-1)+1*** will add one more term to the left-hand side summation: *An additional* ***n-1***.
We add this term to both sides of the equation, which results in:

$= (n-1)(n-2)/2 + n - 1$
$= (n^2 - n + 2)/2 + n - 1$
$= (n^2 - 3n + 2 + 2n - 2)/2$
$= (n^2 - n)/2 = n(n-1)/2$

for the right-hand side. This proves that **if the summation holds for the case of n-1, then it will also hold for the case *n* for some arbitrary *n*.**

You **do not** need to show every algebraic step in your own prose proof (provided here for clarity)

By induction on n, we have proven that this summation is equal to *2n* for any value of *n* in the natural numbers.

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Prove for all the natural numbers:

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Not all the natural numbers!

Prove that for all natural numbers:

Make sure that your base cases cover the entire domain of the variable you're inducting over **that create an empty sum**

$$\sum_{i=1}^{n}(2i-1) = n^2$$

Base Case:

This summation evaluated when **n=0** sums over nothing, so it results in 0.
This summation evaluated when **n=1 is 1,** which is equal to $1^2 = n^2$

Inductive Step:

Assume summation evaluated at ***n-1*** will be equal to $\mathbf{(n-1)^2}$.
Then, considering ***n = (n-1) + 1*** will add one more term to the sum: *An additional* ***2n - 1***.
We add this term to both sides of the equation, which results in:
$(n-1)^2 + 2n - 1 = (n^2 - 2n + 1) + 2n - 1 = n^2$. **This proves that if the summation holds for the case of n-1, then it will also hold for the case *n* for some arbitrary *n.***

By the principle of induction, it's proven that this summation is equal to n^2 for any value of *n* in the natural numbers.