Sept 14th Slides

Agenda

- Opening Notes
- Implication
- Bi-implication
- Boolean Algebra
- Boolean Algebra - Equivalences
- Boolean Algebra - Associative and Commutative properties

Now I’m taking it for granted that you know….

$\neg$

$\vee$

$\wedge$

$\oplus$

As well as….

$\in$ $\subset$ $\subseteq$ $\mathscr{P}(S)$ $|S|$

Implication

P = My animal is a poodle
Q = it is a dog

$$P \rightarrow Q$$

p	q	$p \rightarrow q$
T	T	
T	F	
F	T	
F	F	

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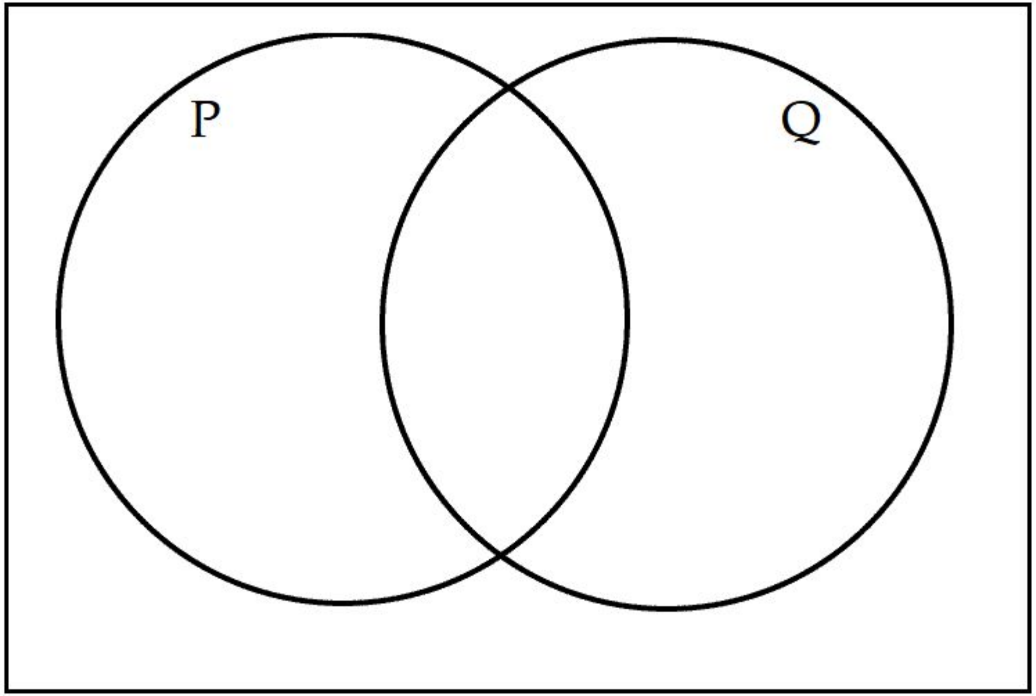
$P \rightarrow Q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

What should be shaded?



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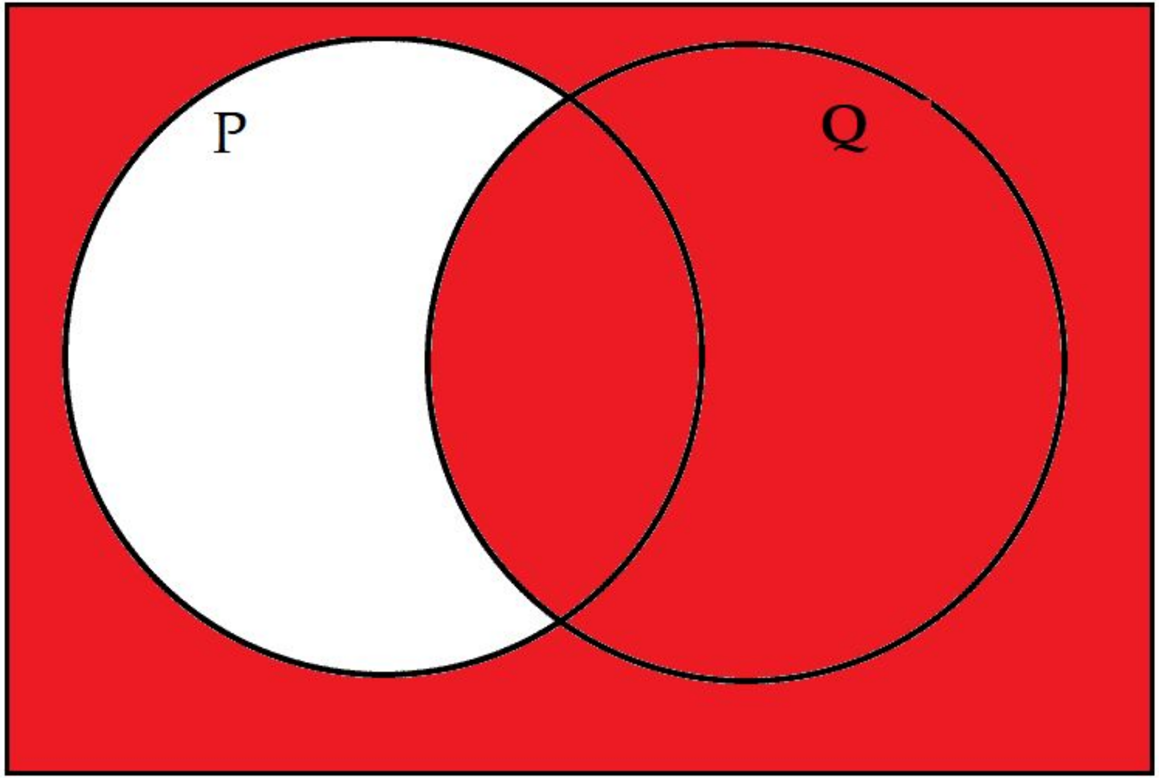

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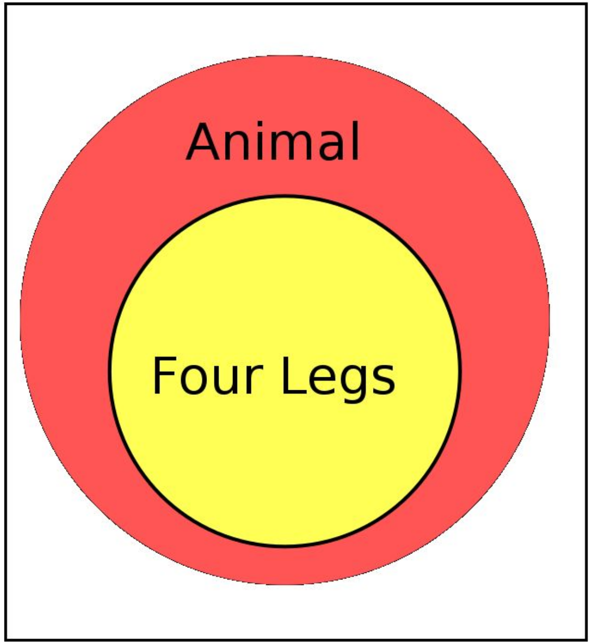
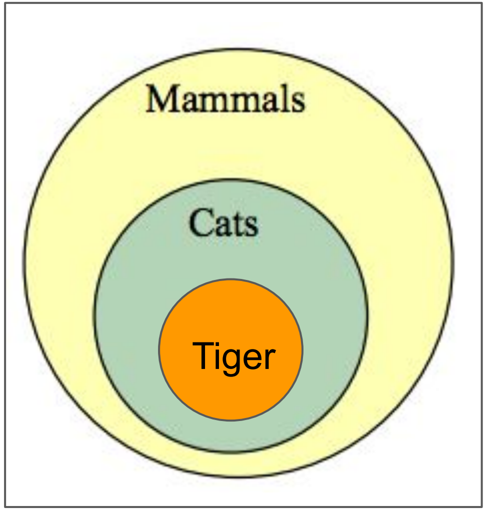




Implication -- Try on your own to write all 4 as implications (you can use assign variables, like y for yoga)

- "Whenever I do yoga, I feel calm"
- “All kangaroos are mammals”
- “If I’m in discrete class, then I’m on zoom today.”
- “I wear a hat if it’s sunny”

Some ways of stating Implication

- p implies q
- p is a sufficient condition for q
- q is a necessary condition for p
- q follows from p
- p only if q

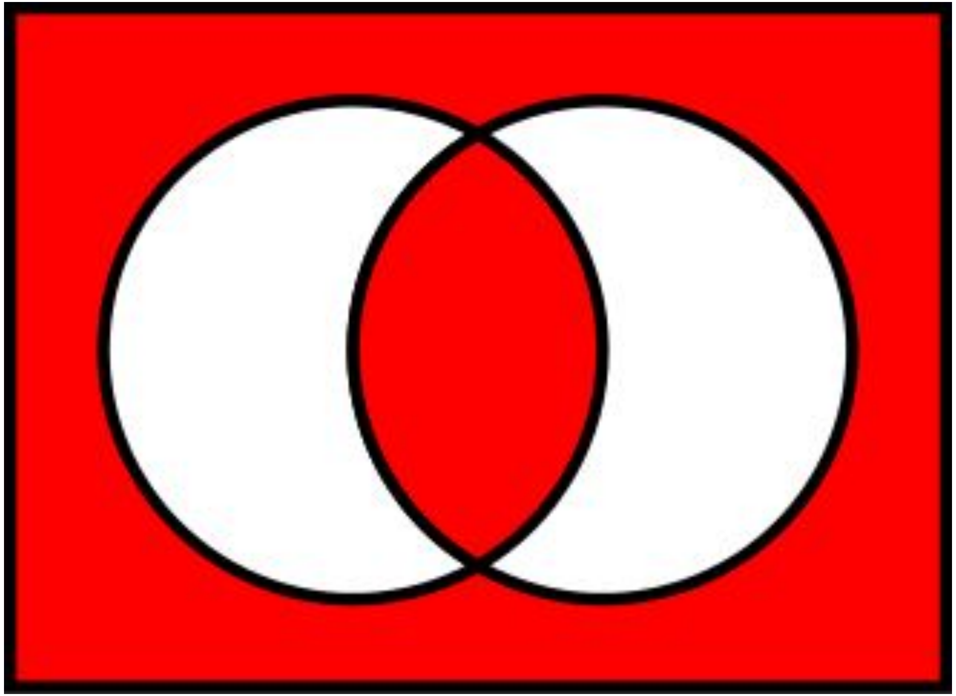
Bi-Implication

I will do laundry if and only if I have only dirty clothes!

L = do laundry   D = only have dirty clothes

L	D	$L \leftrightarrow D$
T	T	__T__
T	F	__F__
F	T	__F__
F	F	__T__

Bi-Implication



What does this look like the inverse of?

Boolean Algebra

Boolean Algebra

Prove $3(x + y) = 3x + 3y$

~~Boolean~~ Algebra

Prove	x	y	$3(x + y)$	$=$	$3x + 3y$

~~Boolean~~ Algebra

Prove	x	y	$3(x + y)$	$=$	$3x + 3y$
	0	0	$3(0 + 0)$	$=$	$3(0) + 3(0)$

~~Boolean~~ Algebra

Prove	x	y	$3(x + y)$	$=$	$3x + 3y$
	0	0	$3(0 + 0)$	$=$	$3(0) + 3(0)$
	1	0	$3(1 + 0)$	$=$	$3(1) + 3(0)$

~~Boolean~~ Algebra

Prove

x	y	$3(x + y)$	=	$3x + 3y$
0	0	3(0 + 0)	=	3(0) + 3(0)
1	0	3(1 + 0)	=	3(1) + 3(0)
2	0	3(2 + 0)	=	3(1) + 3(0)

~~Boolean~~ Algebra

Prove	x	y	$3(x + y) = 3x + 3y$
	0	0	$3(0 + 0) = 3(0) + 3(0)$
	1	0	$3(1 + 0) = 3(1) + 3(0)$
	2	0	$3(2 + 0) = 3(2) + 3(0)$

No……...

Boolean Algebra -- Simplify without using a truth table

$$\neg\neg P$$

$$P \wedge \bot$$

$$P \wedge \top$$

$$P \vee \bot$$

$$P \vee \top$$

Boolean Algebra -- Simplify without using a truth table

$\neg\neg P$	P
$P \wedge \bot$	$\bot$
$P \wedge \top$	P
$P \vee \bot$	P
$P \vee \top$	$\top$

Boolean Algebra -- Choose a few to reason out!

simplified	$\rightarrow$	$\leftrightarrow$	$\oplus$	$\wedge$	$\vee$
P	$\top \rightarrow P$ $\neg P \rightarrow P$	$\top \leftrightarrow P$	$\bot \oplus P$	$\top \wedge P$ $P \wedge P$	$\bot \vee P$ $P \vee P$
$\neg P$	$P \rightarrow \bot$ $P \rightarrow \neg P$	$\bot \leftrightarrow P$	$\top \oplus P$		
$\top$	$\bot \rightarrow P$ $P \rightarrow \top$ $P \rightarrow P$	$P \leftrightarrow P$	$P \oplus \neg P$		$\top \vee P$ $P \vee \neg P$
$\bot$		$P \leftrightarrow \neg P$	$P \oplus P$	$\bot \wedge P$ $P \wedge \neg P$	

Boolean Algebra

Associative Property: you can add and remove parentheses around them

Example: $(2+3)+5 = 2+(3+5)$

Counterexample: $(2-3)-5 \neq 2-(3-5)$

Boolean Algebra

Commutative Property: you can swap their operands' position

Example: $2+3 = 3+2$

Counterexample: $2-3 \neq 3-2$

Boolean Algebra

Which symbols are associative/commutative?

$\neg$ $\vee$ $\wedge$ $\oplus$ $\leftrightarrow$ $\rightarrow$