

**Sept 4 Slides**

# Sept 4 Slides

1. Opening Notes
2. Power Sets
3. Disjoint Sets
4. Set-builder Notation
5. Properties/Laws of sets  
[More practice]

## Other Notations

 $\mathcal{P}(X)$  $\mathcal{P}(X)$  $\mathcal{P}(X)$  $\mathcal{P}(X)$  $\bar{A}$  $A'$  $\tilde{A}$  $A^{\sim}$  $A^c$

# Power sets

## 4.1.3 Power Set

The set of all the subsets of a set,  $A$ , is called the *power set*,  $\text{pow}(A)$ , of  $A$ . So

$$B \in \text{pow}(A) \quad \text{IFF} \quad B \subseteq A.$$

For example, the elements of  $\text{pow}(\{1, 2\})$  are  $\emptyset$ ,  $\{1\}$ ,  $\{2\}$  and  $\{1, 2\}$ .

# Power sets -- Break-outs

- 1.) What is the power-set of  $\{\}$ ? Cardinality?
- 2.) What is the power set of  $\{a, b, c\}$ ? Cardinality?
- 3.) What is the power set of  $\{W, X, Y, Z\}$ ?  
Cardinality?

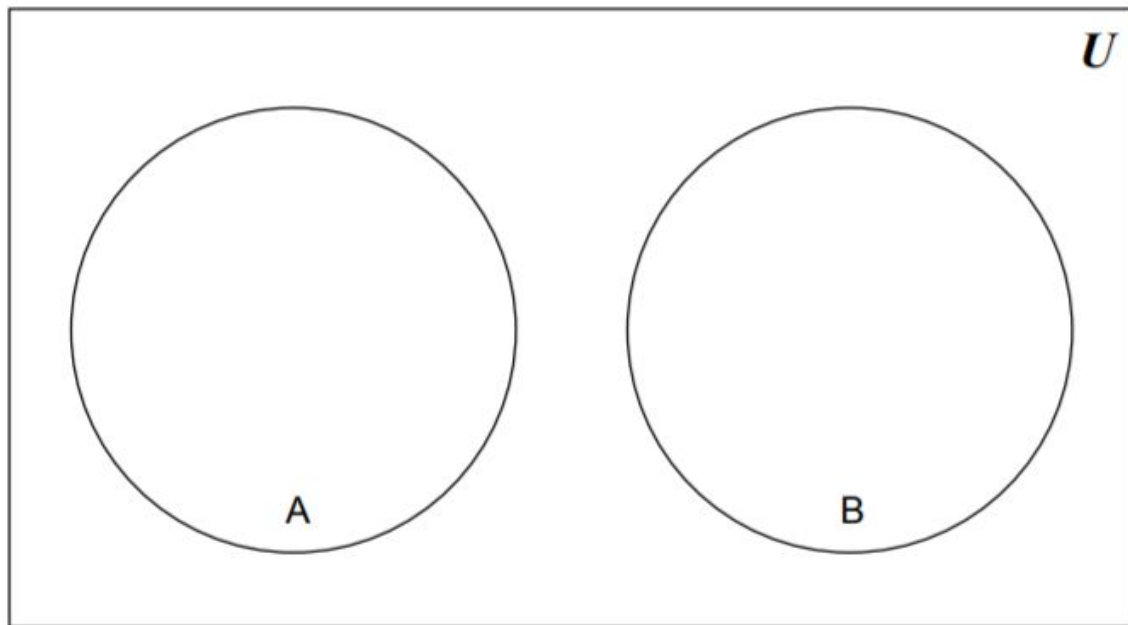
**Can we see a rule/pattern to  
determine the cardinality of a  
powerset?**

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$$|\mathcal{P}(X)| = 2^{|X|}$$

# Disjoint Sets

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- Formal definition for disjoint sets: *two sets are disjoint if their intersection is the empty set*
- Further examples:
  - $\{1, 2, 3\}$  and  $\{3, 4, 5\}$  are not disjoint

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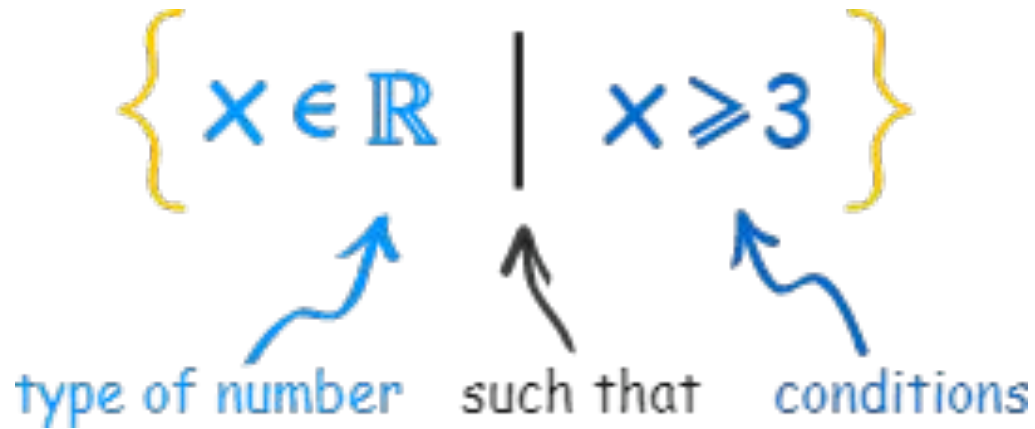
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    - Their intersection is the empty set
  - $\emptyset$  and  $\emptyset$  are disjoint!
    - Their intersection is the empty set

# Set-Builder Notation

$$\{ x \in \mathbb{R} \mid x \geq 3 \}$$

type of number    such that    conditions

The diagram shows the set-builder notation  $\{ x \in \mathbb{R} \mid x \geq 3 \}$ . Below the expression, three labels are provided: "type of number" with a blue arrow pointing to  $\mathbb{R}$ , "such that" with a black arrow pointing to the vertical bar  $\mid$ , and "conditions" with a blue arrow pointing to  $x \geq 3$ .

# Set-Builder Notation

**The set of**      **The natural numbers**

$$E = \{x \in N \mid x > 2\} =$$

**in**      **such that**

The diagram illustrates the components of the set-builder notation  $E = \{x \in N \mid x > 2\}$ . The phrase "The set of" (purple) points to the expression  $x \in N$ . The phrase "The natural numbers" (green) points to the symbol  $N$ . The word "in" (green) points to the membership symbol  $\in$ . The phrase "such that" (blue) points to the condition  $x > 2$ .

# Set-Builder Notation

**The set of**      **The natural numbers**

$$E = \{x \in N \mid x > 2\} = \{3, 4, 5, 6, \dots\}$$

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The diagram illustrates the components of the set-builder notation  $E = \{x \in N \mid x > 2\} = \{3, 4, 5, 6, \dots\}$ . It shows how the mathematical symbols correspond to English words: 'The set of' (purple) points to the membership symbol  $\in$ ; 'The natural numbers' (green) points to the set  $N$ ; 'in' (green) points to the membership symbol  $\in$ ; and 'such that' (blue) points to the condition  $x > 2$ .

# Set-Builder Notation

Let's *formalize* our set operators in “set-builder notation”

## Quick Side-Note:

-We will need to link together multiple “conditions” with “and’s”, “not’s” and “or’s”

## Special symbols:

$\vee$  is “or” (notice similarity to  $\cup$ )

$\wedge$  is “and” (notice similarity to  $\cap$ )

$\neg$  is “not”



## Set-Builder Notation -- My turn!

**Intersection**  $S \cap T$ : the elements that belong both to  $S$  and to  $T$ .

$$S \cap T =$$

---

### For Reference:

$\vee$  is “or”

$\wedge$  is “and”

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$$S \cap T = \{x \in U \mid x \in S\}$$

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## Set-Builder Notation -- Your turn!

**Union**  $S \cup T$ : the elements that belong either to  $S$  or to  $T$  (or both).

$$S \cup T =$$

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## Set-Builder Notation -- Your turn!

**Union**  $S \cup T$ : the elements that belong either to  $S$  or to  $T$  (or both).

$$S \cup T = \{x \in U \mid x \in S \vee x \in T\}$$

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## Set-Builder Notation -- Your turn!

**Difference**  $S \setminus T$ : the elements that belong to  $S$  but not to  $T$ .

$$S \setminus T =$$

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## Set-Builder Notation -- Your turn!

**Difference**  $S \setminus T$ : the elements that belong to  $S$  but not to  $T$ .

$$S \setminus T = \{x \in U \mid x \in S \wedge x \notin T\}$$

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## Set-Builder Notation -- Your turn!

**Complement**  $\bar{S}$ : elements (of the universe) that don't belong to  $S$ .

$$\bar{S} =$$

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$$\bar{S} = \{x \in U \mid x \notin S\}$$

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