Cost of Sorts and Asymptotic Growth

(and addition!)

One-Slide Summary

- **g is in O(f)** iff there exist positive constants c and n_0 such that $g(n) \le cf(n)$ for all $n \ge n_0$.
- If g is in O(f) we say that f is an **upper bound** for g.
- We use **Omega Ω** for lower bounds and **Theta Θ** for tight bounds.
- To **prove** that g is in O(f) you must find the constants c and n_0.
- We can add two numbers with electricity.

Outline

- Sorting: timing and costs
- Big Oh: upper bound
- Big Omega: lower bound
- Big Theta: tight bound

- Time Permitting:
- Adding Two Numbers With Electricity

Sorting Cost

Recall “simple sorting”: Find best card, take it out and set it aside, repeat.

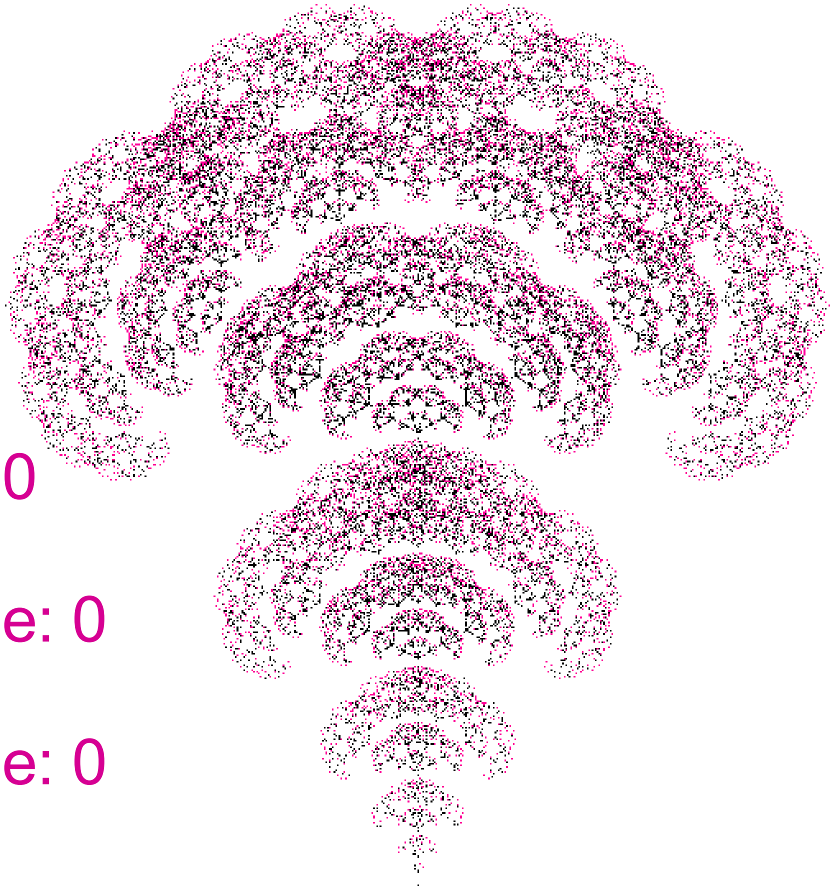
- What grows?
  - n = the number of elements in lst
- How much work are the pieces?

  find-best: work scales as n (increases by one)

  delete: work scales as n (increases by one)
- How many times does sort evaluate find-best and delete? n
- Total cost: scales as n^2

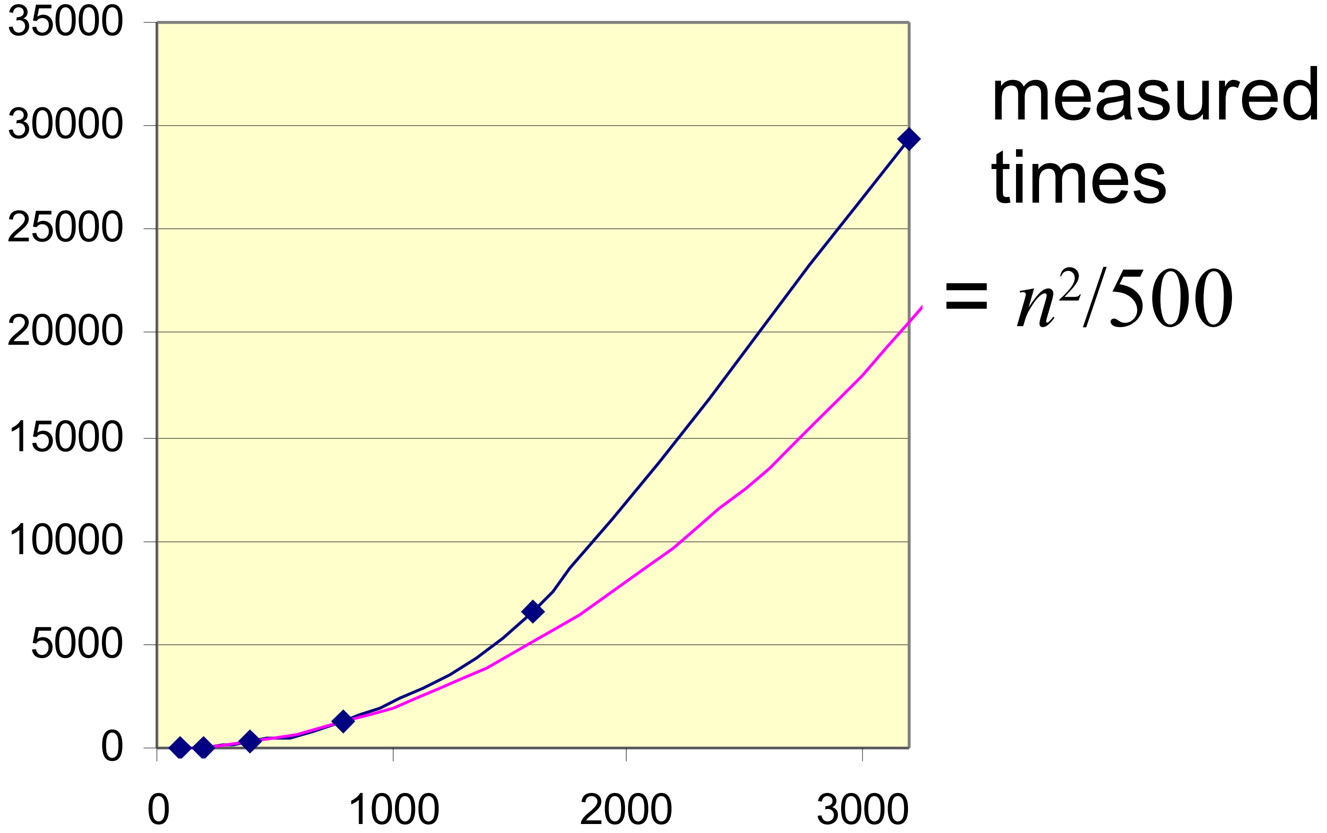
Timing Sort

```
> (time (sort < (revintsto 100)))
cpu time: 20 real time: 20 gc time: 0
> (time (sort < (revintsto 200)))
cpu time: 80 real time: 80 gc time: 0
> (time (sort < (revintsto 400)))
cpu time: 311 real time: 311 gc time: 0
> (time (sort < (revintsto 800)))
cpu time: 1362 real time: 1362 gc time: 0
> (time (sort < (revintsto 1600)))
cpu time: 6650 real time: 6650 gc time: 0
```



Cherry Blossom
by Ji Hyun Lee, Wei Wang

Timing Sort



35000
30000
25000
20000
15000
10000
5000
0
0
1000
2000
3000
measured times
= $n^2/500$

Sorting Cost

```
(define (sort lst cf)
  (if (null? lst) lst
      (let ((best (find-best lst cf)))
        (cons best (sort (delete lst best) cf)))))
(define (find-best lst cf)
   (if (= 1 (length lst)) (car lst)
        (pick-better cf (car lst) (find-best (cdr lst) cf))))
```

If we double the length of the list, the amount of work *approximately* quadruples: there are twice as many applications of find-best, and each one takes twice as long!

Growth Notations

- $g \in O(f)$ ("Big-Oh")

  g grows **no faster than** f (f is upper bound)

- $g \in \Theta(f)$ ("Theta")

  g grows **as fast as** f (f is tight bound)

- $g \in \Omega(f)$ ("Omega")

  g grows **no slower than** f (f is lower bound)

Which one would we most like to know?

Meaning of O ("big Oh")

g is in $O(f)$ iff:

There are positive constants c and n_0 such that

$$g(n) \le cf(n)$$

for all $n \ge n_0$.

O Examples

g is in $O(f)$ iff there are positive constants c and n_0 such that $g(n) \leq cf(n)$ for all $n \geq n_0$.

Is n in $O(n^2)$?

Is $10n$ in $O(n)$?

Is n^2 in $O(n)$?

O Examples

g is in $O(f)$ iff there are positive constants c and n_0 such that $g(n) \leq cf(n)$ for all $n \geq n_0$.

Is n in $O(n^2)$? Yes, $c = 1$ and $n_0=1$ works.

Is $10n$ in $O(n)$? Yes, $c = 10$ and $n_0=1$ works.

Is n^2 in $O(n)$? No, no matter what c we pick, $cn^2 > n$ for big enough n ($n > c$)

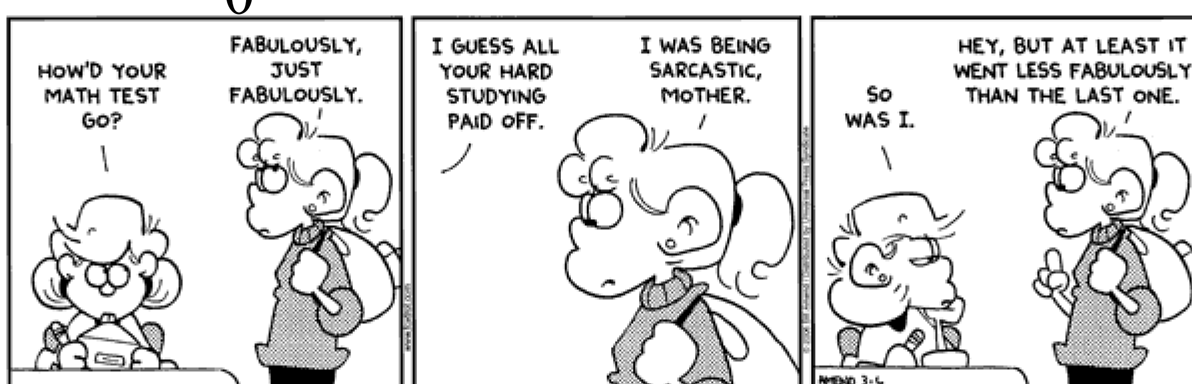
Ω ("Omega"): Lower Bound

g is in $O(f)$ iff there are positive constants c and n_0 such that $g(n) \le cf(n)$ for all $n \ge n_0$.

g is in $\Omega(f)$ iff there are positive constants c and n_0 such that

$$g(n) \ge cf(n)$$

for all $n \ge n_0$.



Proof Techniques

- Theorem:

  There exists a polygon with four sides.

- Proof:

It is a polygon.
It has four sides.
QED.

What kind of proof is this?

Proof by Construction

- We can prove a "there exists an *X* with property *Y*" theorem, but showing an *X* that has property *Y*
- $O(f)$ means "**there are** positive constants c and n_0 such that $g(n) \le cf(n)$ **for all** $n \ge n_0$
- So, to prove g is in $O(f)$ we need to find c and n_0 and show that $g(n) \le cf(n)$ **for all** $n \ge n_0$

Dis-Proof by Construction

- To prove g is **not** in $O(f)$:
- $O(f)$ means: there are positive constants c and n_0 such that $g(n) \le cf(n)$ for all $n \ge n_0$
- So, to prove g is **not** in $O(f)$ we need to find a way given **any** c and n_0, to find an $n \ge n_0$ such that $g(n) > cf(n)$.

Curious why $\neg\exists$**c.**$\exists$**n$_0$.P(c,n$_0$) =** $\forall$**c.**$\forall$**n$_0$.**$\neg$**P(c,n$_0$)** ?
Discrete math!

Liberal Arts Trivia: Archaeology

- In archaeology, this term is used to describe the exposure, processing and recording of archaeological remains. A related subconcept is stratification: relationships exist between different events in the same location or context. Sedimentary layers are deposited in a time sequence, with the oldest on the bottom and the youngest on top.

Liberal Arts Trivia: Creative Writing

- This technique is one of the four rhetorical modes of discourse, along with argumentation, description and narration. Its purpose is to inform, explain, analyze or define. When done poorly, it is sometimes referred to as a “information dump” or “plot dump”.

Growth Notations

- $g \in O(f)$ ("Big-Oh")

  g grows **no faster than** f (upper bound)

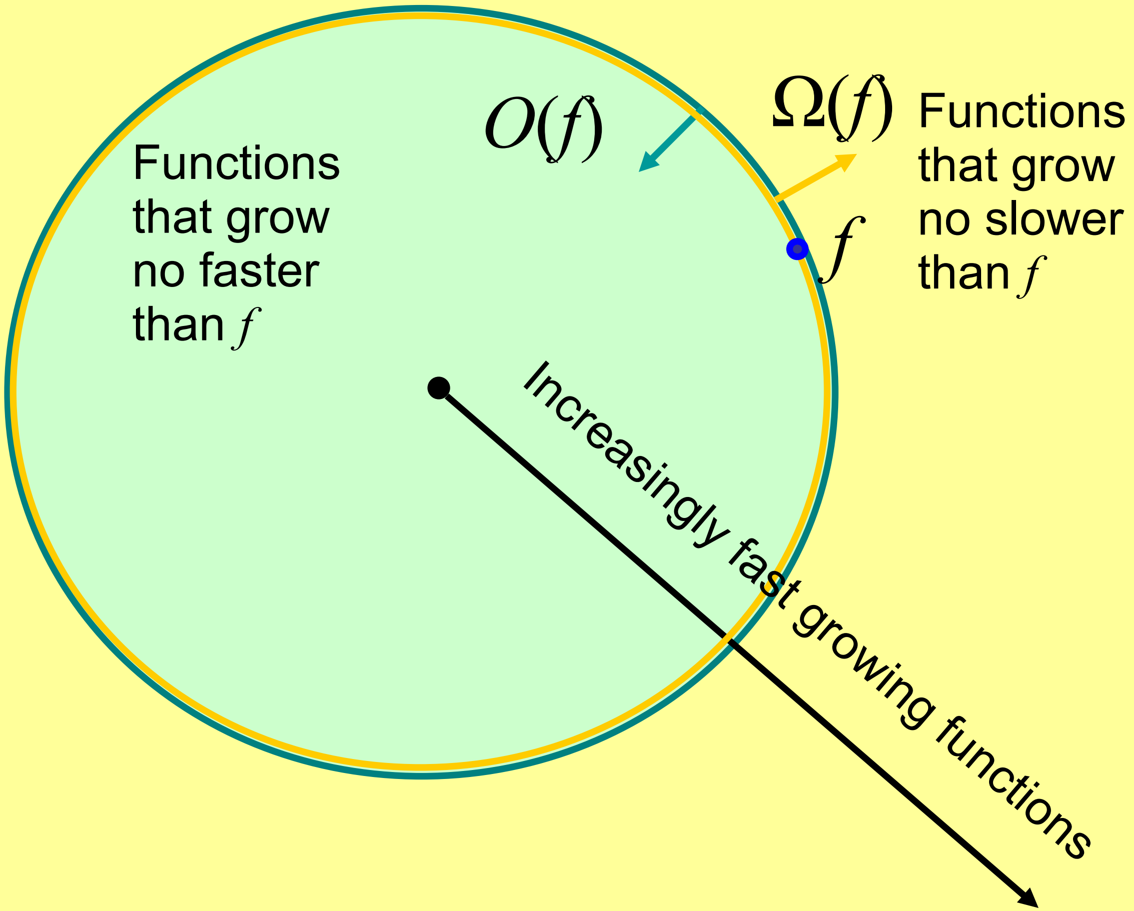
- $g \in \Theta(f)$ ("Theta")

  g grows **as fast as** f (tight bound)

- $g \in \Omega(f)$ ("Omega")

  g grows **no slower than** f (lower bound)

The Sets $O(f)$ and $\Omega(f)$



O and Ω Examples

g is in $O(f)$ iff there are positive constants c and n_0 such that $g(n) \leq cf(n)$ for all $n \geq n_0$.

g is in $\Omega(f)$ iff there are positive constants c and n_0 such that $g(n) \geq cf(n)$ for all $n \geq n_0$.

- n is in $\Omega(n)$
  - ?
- $10n$ is in $\Omega(n)$
  - ?
- Is n^2 in $\Omega(n)$?
  - ?

- n is in $O(n)$
  - ?
- $10n$ is in $O(n)$
  - ?
- n^2 is **not** in $O(n)$
  - ?

O and Ω Examples

g is in $O(f)$ iff there are positive constants c and n_0 such that $g(n) \leq cf(n)$ for all $n \geq n_0$.

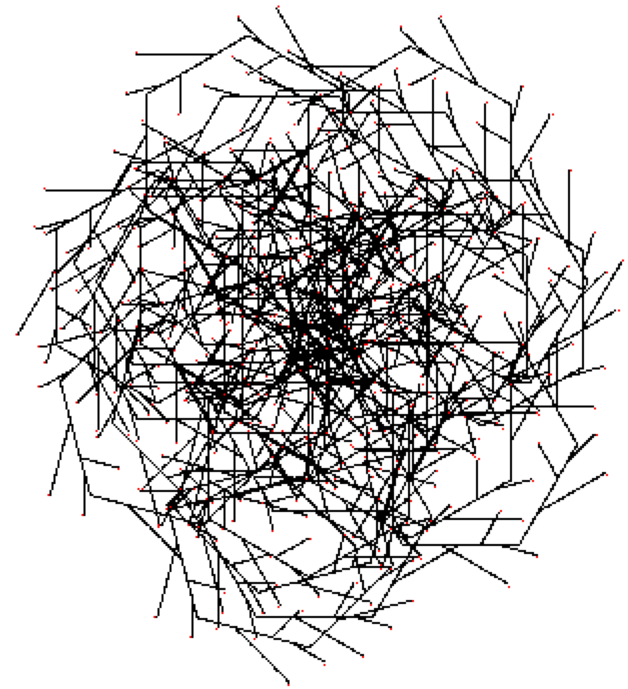
g is in $\Omega(f)$ iff there are positive constants c and n_0 such that $g(n) \geq cf(n)$ for all $n \geq n_0$.

- n is in $\Omega(n)$
  - Yes, pick $c = 1$
- $10n$ is in $\Omega(n)$
  - Yes, pick $c = 1$
- Is n^2 in $\Omega(n)$?
  - Yes! (pick $c = 1$)

- n is in $O(n)$
  - Yes, pick $c = 1$
- $10n$ is in $O(n)$
  - Yes, pick $c = 10$
- n^2 is **not** in $O(n)$
  - Pick $n > c$

Homework

- Read Couse Book Chp 7
  - ... by Friday
- Problem Set 4 Out
  - Start now!
  - (Exam Prep)



The Mask
by Zachary Pruckowski, Kristen Henderson