

## Class 3: Rules of Evaluation

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## Menu

- Questions from Notes
  - Computing photomosaics, non-recursive languages, hardest language elements to learn
- Scheme's Rules of Evaluation
  - (break: Survey Responses)

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If it takes 60 seconds to compute a photomosaic for Problem Set 1 today on a typical PC, estimate how long it will take cs1120 students in 2012 to compute the same photomosaic? How long will it take in 2015?

> (/ (\* (- 2012 2009) 12) 18)

2

> (/ 60 (\* 2 2))

15

> (/ (\* (- 2015 2009) 12) 18)

4

> (/ 60 (\* 2 2 2 2))

15/4

> (exact->inexact (/ 60 (\* 2 2 2 2)))

3.75

Difference in years \* 12 = number of months  
Number of months / 18 = number of doublings  
according to Moore's Law

60 seconds today, 2 doublings by 2012  
15 seconds in 2012

60 seconds today, 4 doublings by 2015  
3.75 seconds in 2015

Reality check:  
Moore's "law" is just  
an "observation".

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Are there any non-recursive natural languages? What would happen to a society that spoke one?

Not for humans at least.

They would run out of original things to say.

Chimps and Dolphins are able to learn non-recursive "languages", but **only humans have learned recursive languages.**

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## Running out of Ideas

"Its all been said before."

Eventually true for a non-recursive language.

Never true for a recursive language.

There is always something original left to say!

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## Language Elements

When learning a foreign language, which elements are hardest to learn?

- Primitives: lots of them, and hard to learn real *meaning*
- Means of Combination
  - Complex, but, all natural languages have similar ones [Chomsky]
 

SOV (45% of all languages)	<i>Sentence ::= Subject Object Verb</i> (Korean)
SVO (42%)	<i>Sentence ::= Subject Verb Object</i> (English)
VSO (9%)	<i>Sentence ::= Verb Subject Object</i> (Welsh)
	"Lladdodd y ddraig y dyn." (Killed the dragon the man.)
OSV (<1%):	Tobati (New Guinea)
<i>Scheme:</i>	<i>Expression ::= (Verb Object)</i>
- Means of Abstraction: few of these, but tricky to learn differences across languages
 

English:	I, we
Tok Pisin (Papua New Guinea):	mi (I), mitupela (he/she and I), mitripela (both of them and I), mipela (all of them and I), yumitupela (you and I), yumitripela (both of you and I), yumipela (all of you and I)
<i>Scheme:</i>	<i>define</i>

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	Pages in Revised <sup>5</sup> Report on the Algorithmic Language Scheme
Primitives	
Means of Combination	
Means of Abstraction	
	<b>48</b> pages total (includes formal specification and examples)

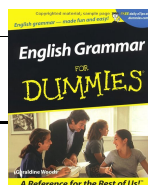
	Pages in Revised <sup>5</sup> Report on the Algorithmic Language Scheme
Primitives	Standard Procedures 18
	Primitive expressions 2
	Identifiers, numerals 1
Means of Combination	Expressions 2
	Program structure 2
Means of Abstraction	Definitions ½
	<b>48</b> pages total (includes formal specification and examples)

	Pages in Revised <sup>5</sup> Report on the Algorithmic Language Scheme	Pages in C++ Language Specification (1998)
Primitives	Standard Procedures 18	
	Primitive expressions 2	
	Identifiers, numerals 1	
Means of Combination	Expressions 2	
	Program structure 2	
Means of Abstraction	Definitions ½	
	<b>48</b> pages total (includes formal specification and examples)	

	Pages in Revised <sup>5</sup> Report on the Algorithmic Language Scheme	Pages in C++ Language Specification (1998)
Primitives	Standard Procedures 18	Standard Procedures 356
	Primitive expressions 2	Primitive expressions 30
	Identifiers, numerals 1	Identifiers, numerals 10
Means of Combination	Expressions 2	Expressions, Statements 197
	Program structure 2	Program Structure 35
Means of Abstraction	Definitions ½	Declarations, Classes 173
	<b>48</b> pages total (includes formal specification and examples)	<b>776</b> pages total (includes no formal specification or examples)

C++ Core language issues list has 948 items!

	Pages in Revised <sup>5</sup> Report on the Algorithmic Language Scheme	English
Primitives	Standard Procedures 18	Morphemes ?
	Primitive expressions 2	Words in Oxford English Dictionary 500,000
	Identifiers, numerals 1	
Means of Combination	Expressions 2	Grammar Rules 100s (?)
	Program structure 2	<i>English Grammar for Dummies</i> Book 384 pages
Means of Abstraction	Definitions ½	Pronouns ~20
	<b>48</b> pages total (includes formal specification and examples)	



## Scheme Grammar

```

Program      ::= ε | ProgramElement Program
ProgramElement ::= Expression | Definition
Definition   ::= (define Name Expression)
Expression   ::= PrimitiveExpression | NameExpression
                | ApplicationExpression
                | ProcedureExpression | IfExpression
PrimitiveExpression ::= Number | true | false
                | PrimitiveProcedure
NameExpression ::= Name
ApplicationExpression ::= (Expression MoreExpressions)
MoreExpressions ::= ε | Expression MoreExpressions
ProcedureExpression ::= (lambda (Parameters) Expression)
Parameters       ::= ε | Name Parameters
IfExpression ::= (if ExpressionPred ExpressionConsequent ExpressionAlt)
    
```

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## Assigning Meanings

```

Program      ::= ε | ProgramElement Program
ProgramElement ::= Expression | Definition
Definition   ::= (define Name Expression)
Expression   ::= PrimitiveExpression | NameExpression
                | ApplicationExpression | ProcedureExpression | IfExpression
PrimitiveExpression ::= Number | true | false | PrimitiveProcedure
NameExpression ::= Name
ApplicationExpression ::= (Expression MoreExpressions)
MoreExpressions ::= ε | Expression MoreExpressions
ProcedureExpression ::= (lambda (Parameters) Expression)
Parameters       ::= ε | Name Parameters
IfExpression ::= (if ExpressionPred ExpressionConsequent ExpressionAlt)
    
```

This grammar generates (nearly) all surface forms in the Scheme language. What do we need to do to know the meaning of every Scheme program?

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## Definitions

```

Program      ::= ε | ProgramElement Program
ProgramElement ::= Expression | Definition
Definition   ::= (define Name Expression)
    
```

A definition associates the **value of its expression** with the name.

(define two 2)

After this definition, the value associated with the name **two** is 2.

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## Expressions and Values

- (Almost) every *expression* has a *value*
  - Have you seen any expressions that don't have values?
- When an expression with a value is *evaluated*, its value is produced

Our goal is to define a meaning function, **Eval**, that defines the value of every Scheme expression:

**Eval(Expression) ⇒ Value**

Today we do this informally with rules in English.

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## Primitive Expressions

```

PrimitiveExpression ::= Number | true | false | PrimitiveProcedure
    
```



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## Evaluation Rule 1: Primitives

If the expression is a *primitive*, it evaluates to its pre-defined value.

```

> 2
2
> true
#t
> +
#<primitive:+>
    
```

Primitives are the smallest units of meaning: they can't be broken down further, you need to know what they mean.

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## Name Expressions

*Expression ::= NameExpression*

*NameExpression ::= **Name***

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## Evaluation Rule 2: Names

A *name* evaluates to the value associated with that name.

> (define two 2)

> two

2

Caveat: this simple rule only works if the value associated with a name never changes (until PS5).

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## Application Expressions

*Expression ::= Application Expression*

*ApplicationExpression*

*::= (Expression MoreExpressions)*

*MoreExpressions ::= ε*

*MoreExpressions ::= Expression MoreExpressions*

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## Evaluation Rule 3: Application

3. To evaluate an application expression:

- Evaluate** all the subexpressions (in any order)
- Apply** the value of the first subexpression to the values of all the other subexpressions.

*(Expression<sub>0</sub> Expression<sub>1</sub> Expression<sub>2</sub> ... )*

## Rules for Application

- Primitives.** If the procedure to apply is a *primitive procedure*, just do it.
- Constructed Procedures.** If the procedure is a *constructed procedure*, **evaluate** the body of the procedure with each parameter name bound to the corresponding input expression value.

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**Eval** and **Apply** are defined in terms of each other.

Without **Eval**, there would be no **Apply**, without **Apply** there would be no **Eval**!



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## Survey Responses: Majors

- 28 Cognitive Science
- 20 Computer Science
- 7 Psychology
- 4 Math
- 3 Economics
- 3 Commerce/Pre-Commerce
- 3 Undeclared
- 2 Physics
- 1 Environmental Sciences, English, Music

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## Survey Responses: PS Partners



For PS2 everyone will be assigned a partner.  
For other problem sets, you'll have different options.

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## Survey Responses: Office Hours

- Scheduling office hours: (**Set Cover Problem**)
  - Input: a set of sets of available times
  - Output: the minimum size set that includes at least one element from each of the input sets

Later in the course, we'll see that this problem is equivalent to the problem of computing an optimal photomosaic!

- My office hours will be:
  - Mondays, 1:30-2:30pm [Olsson 236A]
  - Tuesdays, 10:30-11:30am [Wilsdorf Cafe]

Not a set cover: Everyone who selected at least three possible times can make at least one of these. If you can't make office hours, email to arrange an appointment.

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## Honor Pledge

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## Finishing Scheme Meanings

```

Program ::= ε | ProgramElement Program
ProgramElement ::= Expression | Definition
Definition ::= (define Name Expression)
Expression ::= PrimitiveExpression | NameExpression
               | ApplicationExpression | ProcedureExpression | IfExpression
PrimitiveExpression ::= Number | true | false | PrimitiveProcedure
NameExpression ::= Name
ApplicationExpression ::= (Expression MoreExpressions)
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ProcedureExpression ::= (lambda (Parameters) Expression)
Parameters ::= ε | Name Parameters
IfExpression ::= (if ExpressionPred ExpressionConsequent ExpressionAlt)
    
```

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## Making Procedures

**lambda** means “make a procedure”

*Expression* ::= ProcedureExpression

*ProcedureExpression* ::=

**(lambda (Parameters) Expression)**

*Parameters* ::= ε

*Parameters* ::= **Name** Parameters

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## Evaluation Rule 4: Lambda

A **lambda** expression evaluates to a procedure that takes the given parameters and has the expression as its body.

```
ProcedureExpression ::= (lambda (Parameters) Expression)
Parameters          ::= ε | Name Parameters
```

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## Lambda Example: Tautology Function

```
(lambda      make a procedure
 ()         with no parameters
 true)     with body true
```

```
> ((lambda () true) 1120)
#<procedure>: expects no arguments, given 1: 1120
> ((lambda () true))
#t
> ((lambda (x) x) 1120)
1120
```

Next class we'll follow the evaluation rules through more interesting examples.

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## Evaluation Rule 5: If

*IfExpression*

```
::= (if ExpressionPredicate
      ExpressionConsequent
      ExpressionAlternate)
```

To evaluate an if expression:

- Evaluate  $Expression_{\text{Predicate}}$ .
- If it evaluates to a false value, the value of the if expression is the value of  $Expression_{\text{Alternate}}$ ; otherwise, the value of the if expression is the value of  $Expression_{\text{Consequent}}$ .

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## Completeness of Evaluation Rules

```
Program ::= ε | ProgramElement Program
ProgramElement ::= Expression | Definition
Definition ::= (define Name Expression)
Expression ::= PrimitiveExpression | NameExpression
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PrimitiveExpression ::= Number | true | false | PrimitiveProcedure
NameExpression ::= Name
ApplicationExpression ::= (Expression MoreExpressions)
MoreExpressions ::= ε | Expression MoreExpressions
ProcedureExpression ::= (lambda (Parameters) Expression)
Parameters ::= ε | Name Parameters
IfExpression ::= (if Expressionpred Expressionconsequent Expressionalt)
```

Since we have an evaluation rule for each grammar rule, we can determine the meaning of any Scheme program!

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## Now You Can Write Any Program!

- You know enough now to define a procedure that performs every possible computation!
  - We'll prove this later in the course
- We'll learn some more useful Scheme forms:
  - There are a few more special forms (like **if**)
  - But, none of these are necessary...just helpful
- We have not defined the evaluation rules precisely enough to unambiguously understand all programs (e.g., what does "value associated with a name" mean?)

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## Charge

- PS1 Due at beginning of class Wednesday**
- Read Chapter 4 by Friday
- Now you know enough to produce every computation, the rest is just gravy:
  - More efficient, elegant ways to express computations
  - Ways to analyze the computations

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