University of Virginia 11 November 2011
cs1120: Introduction of Computing
Explorations in Language, Logic, and Machines

Class 34: Unprovability

Upcoming (Remaining!) Schedule of Assignments

- **11-11-11 11:11:11.11** (the paradox is nigh?)
- **after class today: Peter has office hours**
- **before 5pm Monday, 14 November:** commitment for which PS8 you will do (web form)
- **Exam 2 Review Sessions** (Wednesday/Thursday, 16/17 November)
- **Wednesday, 16 November (11:59pm):** Problem Set 7 due (note extension from earlier deadline)
- **18 November:** finish reading Chapter 12 (note we are starting to talk about this today, so if you prefer to read things yourself before seeing them in class, read it earlier), Rice Hall Dedication
- **Monday, 21 November:** PS8, Preliminary Submission
- **Wednesday, 30 November:** Exam 2 due (will be handed out on **Monday, 21 November**)
- **Monday, 5 December (last class):** PS8, Final Submission due
- **Monday, 12 December (1:00pm):** Final Exam due

Problem Set 8

There are three options for Problem Set 8: **Plan J**, **Plan C**, and **Plan W** (see the course site for details). Students who indicated any interest in majoring in Computer Science on PS0 are expected to do Plan J, and must provide a convincing justification for choosing another option. Everyone else is welcome to select any of the three options. You should decide which option you want by Monday (5pm), and indicate this by submitting the on-line form.

Axiomatic System

Set of axioms and inference rules.

Euclid's Elements: 5 axioms (postulates), logical rules for deducing theorems about geometry, math

A **Perfect Axiomatic System** derives **all** true statements (in the domain), and **no** false statements starting from a finite number of axioms and following mechanical inference rules.

What is an *incomplete* axiomatic system?

What is an *inconsistent* axiomatic system?

Why is an *incomplete* axiomatic system preferable to an *inconsistent* one?

Peano's Postulates

$\mathcal{N}$ is the smallest set satisfying these postulates:

P1. 1 is in $\mathcal{N}$.
P2. If x is in $\mathcal{N}$, then its "successor" (succ x) is in $\mathcal{N}$.
P3. There is no x such that (succ x) = 1.
P4. If x is not 1, then there is a y in $\mathcal{N}$ such that (succ y) = x.
P5. If $\mathcal{S}$ is a subset of $\mathcal{N}$, 1 is in S, and the implication
(*x* in $\mathcal{S} \Rightarrow$ (succ x) in $\mathcal{S}$) holds, then $\mathcal{S} = \mathcal{N}$.

Definition of (+ a b):

How can we prove 1+1=2?

Russell's Paradox

S = the set of all sets that are not members of themselves

Is *S* a member of *S*?

Gödel's Statement

G: This statement does not have any proof in the system of *PM*.

What would it mean if *G* is true?

What would it mean if *G* is false?

What does this mean for any complete and consistent axiomatic system?