Lecture 18: The Story So Far

CS150: Computer Science
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Computer Science

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Lecture 18: Mutation

Menu

- Finish insert-sort-tree
- Course roadmap
- Introducing Mutation

A few people have extensions on Exam 1, so no talking about the exam questions until Wednesday.
If you have an extension on Exam 1, don’t read Chapter 9 until you turn in the exam.

Lecture 18: Mutation

insert-one-tree

(define (insert-one-tree cf el tree)
  (if (null? tree)
      (make-tree null el null)
      (if (cf el (get-element tree))
          (make-tree
           (insert-one-tree cf el (get-left tree))
           (get-element tree)
           (get-right tree))
          (make-tree
           (get-left tree)
           (get-element tree)
           (insert-one-tree cf el (get-right tree))))))

Each time we call insert-one-tree, the size of the tree approximately halves if it is well balanced.
Each application is constant time.

The running time of insert-one-tree is in $\Theta(\log n)$ where $n$ is the number of elements in the input tree, which must be well-balanced.

insert-sort-helper

(define (insert-sort-helper cf lst)
  (if (null? lst) null
      (insert-one-tree cf (car lst)
      (insert-sort-helper cf (cdr lst)))))

No change (other than using insert-one-tree)…but evaluates to a tree not a list!

(((()) 1 ()) 2 ()) 5 (() 8 ()))

Running time of insert-sort-tree

$n = \text{number of elements in tree}$

$\Theta(\log n)$

$n = \text{number of elements in list}$

$\Theta(n \log n)$

extract-elements

We need to make a list of all the tree elements, from left to right.

(define (extract-elements tree)
  (if (null? tree) null
      (append (extract-elements (get-left tree))
        (cons
          (get-element tree)
          (extract-elements (get-right tree)))))))

Running time of insert-sort-tree

$n = \text{number of elements in tree}$

$\Theta(\log n)$

$n = \text{number of elements in list}$

$\Theta(n \log n)$
Lecture 18: Mutation

What if tree is not well-balanced?

A pathologically unbalanced tree is as bad as a list!

Insert-one worst case requires \( n \) recursive applications, so insert-sort-tree worst case is in \( \Theta(n^2) \)

Comparing sorts

<table>
<thead>
<tr>
<th>Test growth function</th>
<th>Best-first-sort</th>
<th>Insert-sort-halves</th>
<th>Insert-sort-tree</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n = 250 )</td>
<td>time = 110</td>
<td>time = 251</td>
<td>time = 30</td>
</tr>
<tr>
<td>( n = 500 )</td>
<td>time = 371</td>
<td>time = 1262</td>
<td>time = 250</td>
</tr>
<tr>
<td>( n = 1000 )</td>
<td>time = 2363</td>
<td>time = 4025</td>
<td>time = 150</td>
</tr>
<tr>
<td>( n = 2000 )</td>
<td>time = 8162</td>
<td>time = 16454</td>
<td>time = 301</td>
</tr>
<tr>
<td>( n = 4000 )</td>
<td>time = 31757</td>
<td>time = 66137</td>
<td>time = 1001</td>
</tr>
</tbody>
</table>

(4.5 3.17 4.63 4.36)

Can we do better?

• Making all those trees is a lot of work
• Can we divide the problem in two halves, without making trees?

This is the famous "Quicksort" algorithm invented by Sir Tony Hoare. See Chapter 8.

There are lots of ways to do a little bit better, but no way to do asymptotically better. All possible sort procedure have running times in \( \Omega(n \log n) \). (We’ll explain why later in the course...)

Computer Science: CS150 so far

• How to describe information processes by defining procedures
  – Programming with procedures, lists, recursion
  – Chapters 3, 4, 5
• How to predict properties about information processes
  – Predicting running time, \( \Theta \), \( O \), \( \Omega \)
• How to elegantly and efficiently implement information processes
  – Chapter 3 (rules of evaluation)
CS150 upcoming

- How to describe information processes by defining procedures
  - Programming with state, objects, networks
- How to predict properties about information processes
  - What is the fastest process that can solve a given problem?
  - Are there problems which can’t be solved by algorithms?
- How to elegantly and efficiently implement information processes
  - How to implement a Scheme interpreter

The Liberal Arts

Trivium (3 roads)
- Grammar
- Rhetoric
- Logic
Quadrivium (4 roads)
- Arithmetic
- Geometry
- Music
- Astronomy

Liberal Arts Checkup

- Grammar: study of meaning in written expression
- Rhetoric: comprehension of verbal and written discourse
- Logic: argumentative discourse for discovering truth
- Arithmetic: understanding numbers
- Geometry: quantification of space
- Music: number in time
- Astronomy

BNF replacement rules for describing languages, rules of evaluation for meaning
Not much yet... interfaces between components (PS6-9), program and user (PS8-9)
Rules of evaluation, if, recursive definitions
Not much yet... wait until April
Curves as procedures, fractals (PS3)
Yes, listen to “Hey Jude!”

From Lecture 1:

Saturday: read Neil deGrasse Tyson's essay