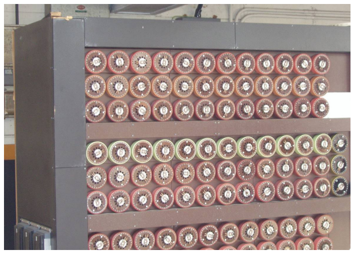
CS216: Program and Data Representation
University of Virginia Computer Science
Spring 2006 David Evans

# Lecture 6: Ordered Data Abstractions



http://www.cs.virginia.edu/cs216

## Why you should actually **read** course syllabi

**Expected Background:** Either (CS101, CS201, and CS202) or (CS150 with a B+ or better) or (Instructor Permission). Students entering CS216 are expected to have background in:

- **Programming:** comfortable creating programs that fill more than one screen, and understanding and modifying programs that involve multiple files. Students should be familiar with control structures commonly found in popular languages including decision and looping structures, and be comfortable with procedures and recursive definitions.
- **Mathematics and Logic:** …



## Schedule Update

- PS3 will be posted before midnight tomorrow
  - Review recursive definitions
  - Preparation for Exam 1
  - Read Chapter 6 (skip skip lists, we are skipping Ch 5 for now)
- Exam 1: out Feb 22, due Feb 27
  - Covers PS1-PS3, Lectures 1-8 (next Weds), book Ch 1-4, 6



## Unordered Data Abstractions

- Our list and tree abstractions have structure (successor, children, etc.) but no notion that structure is associated with values
- What does this mean about the running time of a lookup operation?

Any operation that looks for an element based on element properties must have running time $\in \Omega(N)$ where $N$ is # of elements



## Ordered Data Abstractions

- To do better than $\Omega(N)$ we must be able to know something about where an element can be stored based on its value
  - Can find element without looking at all elements



## Dictionary Data Abstraction

- Set of <key, value> pairs
- Operations:
  - MakeEmptyDictionary ()
    - Returns { }
  - Insert (K, V, S)
    - Add <K, V> to S
  - Lookup (K, S)
    - Return value associated with K in S
      - If $\langle K, I \rangle \in S$, return $I$

Is this enough?

Is this unambiguous?

## Dictionary Operations

- MakeEmptyDictionary ()
  - Returns { }
- Insert $(K, V, S)$
  - If Lookup(K, S) $\neq \Lambda$, $S_{post} = S_{pre} \cup \{<K, V>\}$
  - Otherwise, error
- Lookup $(K, S)$
  - If $<K, I> \in S$, return $I$
  - Otherwise return $\Lambda$



## Python's Dictionary Type

We used it in PS2 code:

memo = {}

memo = MakeEmptyDictionary()

memo[k] = [resU, resV]

Insert (k, [resU, resV], value, memo)

memo.has_key(k)

(Lookup (k, memo) = $\Lambda$)

res = memo[makeKey (U,V)]

res = Lookup (makeKey (U, V), memo)



## Dictionary List Implementation

```
class Record:
    def __init__(self, k, v):
        self.key = k
        self.value = v
    def __str__(self):
        return "<" + str(self.key) + ", " + str(self.value) + ">"

class DictionaryList:
    def __init__(self): self.__node = None

    def lookup (self, key):
        if self.__node == None: return None
        else: return self.__node.lookup (key)

    def insert (self, key, value):
        if self.__node == None:
            self.__node = DictionaryNode (Record (key, value))
        else: self.__node.insert (Record (key, value))
```



## Dictionary Node

```
class DictionaryNode:
    def __init__(self,info):
        self.__info = info
        self.__next = None

    # pre: key must not be a key in self
    # post: self_post = {self[0], ...,self[|self| - 1], value}
    # modifies nothing
    def insert (self, value):
        current = self
        while not current.__next == None:
            current = current.__next
        current.__next = DictionaryNode (value)
```



## Dictionary Lookup

```
def lookup (self, key):
    if self.__info.key == key:
        return self.__info.value
    else:
        if self.__next == None: return None
        else: return self.__next.lookup (key)
```

What is the asymptotic running time?

$\Theta(N)$ where $N$ is the number of dictionary records



## Improving (?) Dictionary

- Order the entries by key
- Stop looking once you get past a key that must be after the lookup key
- Costs: More complex code; insert is more expensive?
- Benefits: Faster lookup?

# Lookup

```
def lookup (self, key):
   if self.__info.key == key:
      return self.__info.value
   elif self.__info.key > key:
      return None
   else:
      if self.__next == None:
         return None
      else:
         res = self.__next.lookup (key)
         return res
```

How does this affect the running time?



# Insert

```
class DictionaryOrderedList:
  def insert (self, key, value):
     rec = DictionaryOrderedNode \
              (Record (key, value))
     if self.__node == None: self.__node = rec
     else:
        if key < self.__node._info.key:
           rec._next = self.__node
           self.__node = rec
        else:
           self.__node.insert (Record (key, value))
```



# Insert Node Code

```
# pre: key must not be a key in self, key must not be before self's
# post: self_post = {self[0], self[1], ...,self[|self| - 1], value}
# modifies nothing
def insert (self, record):
   current = self
   assert (record.key > current._info.key)
   while not current._next == None:
      if current._next._info.key > record.key:
          break
      current = current._next

   r = DictionaryOrderedNode (record)
   r._next = current._next
   current._next = r
```

How does this affect the running time?



# Summary

- Costs:
  - Code size increased by 30%
- Benefits:
  - No growth difference:
    - insert and lookup are still $\Theta(N)$
  - Some absolute difference:
    - Average calls to lookup a non-existent key: $N \rightarrow N/2$



# More Structure

- Current implementation: each comparison eliminates one element
- Ideal comparison implementation: each comparison eliminates half the elements

If our comparison function has Boolean output, can't do better than eliminating half!



# ContinuousTable

```
# invariant: Records in items are sorted on key by <.
def lookup(self, key):
   def lookuprange(items):
      if len(items) == 0: return None
      if len(items) == 1:
         if items[0].key == key: return items[0].value
         else: return None
      middle = len(items) / 2
      if key < items[middle].key:
         return lookuprange (items[:middle])
      else:
         return lookuprange (items[middle:])
   return lookuprange(self.items)
```

## Table Example

```
Lookup VA in: [<CT, 5>, <DE, 1>, <GA, 4>, <MA, 6>, <MD, 7>,
               <NH, 9>, <NJ, 3>, <PA, 2>, <SC, 8>, <VA, 10> ]
Lookup VA in: [<NH, 9>, <NJ, 3>, <PA, 2>, <SC, 8>, <VA, 10> ]
Lookup VA in: [<PA, 2>, <SC, 8>, <VA, 10> ]
Lookup VA in: [<SC, 8>, <VA, 10> ]
Lookup VA in: [<VA, 10> ]
10
```

What is the maximum number of calls to lookuprange?



## Progress

- Ends when len(items) < 2
- Max size of recursive call: len(items)/2 + 1

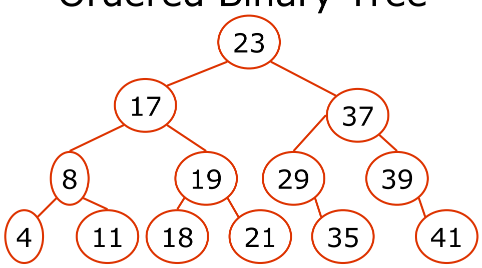
Running time $\in \Theta(\log_2 N)$

```
def lookup(self, key):
    def lookuprange(items):
        if len(items) == 0: return None
        if len(items) == 1:
            if items[0].key == key:
                return items[0].value
            else: return None
        middle = len(items) / 2
        if key < items[middle].key:
            return lookuprange (items[:middle])
        else:
            return lookuprange (items[middle:])
    return lookuprange(self.items)
```

If we double the size of the list, the running time increases by a constant.



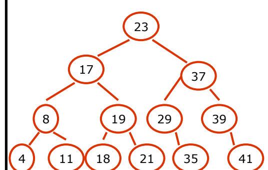
## Ordered Binary Tree




Invariant: node.left.key < node.info.key
node.right.key > node.info.key



## Tree Lookup

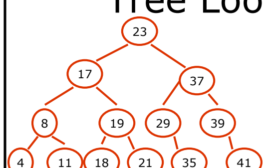



key < current
look left
key > current
look right

```
def lookup(self, k):
    if self.value == k:
        return self
    elif k < self.value:
        if self.left == None:
            return None
        else:
            return self.left.lookup(k)
    else:
        if self.right == None:
            return None
        else:
            return self.right.lookup(k)
```



## Tree Lookup Analysis




Max number of calls: height of tree

***If*** tree is well-balanced:

$N = 2^{h+1}-1$

$h \in \Theta(\log N)$

```
def lookup(self, k):
    if self.value == k:
        return self
    elif k < self.value:
        if self.left == None:
            return None
        else:
            return self.left.lookup(k)
    else:
        if self.right == None:
            return None
        else:
            return self.right.lookup(k)
```



## Tree Lookup Iterative

```
def lookup(self, k):
    if self.value == k:
        return self
    elif k < self.value:
        if self.left == None:
            return None
        else:
            return self.left.lookup(k)
    else:
        if self.right == None:
            return None
        else:
            return self.right.lookup(k)
```

```
def lookup(self, k):
    current = self
    while not current == None:
        if current.value == k:
            return current
        elif k < self.value:
            current = self.left
        else:
            current = self.right
    return None
```

# Comparison

$N$ = number of nodes in self
$h$ = height of self

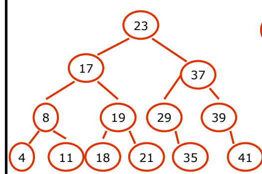
```
def lookup(self, k):
    if self.value == k:
        return self
    elif k < self.value:
        if self.left == None:
            return None
        else:
            return self.left.lookup(k)
    else:
        if self.right == None:
            return None
        else:
            return self.right.lookup(k)
```

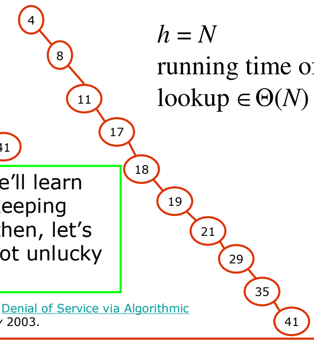
```
def lookup(self, k):
    current = self
    while not current == None:
        if current.value == k:
            return current
        elif k < self.value:
            current = self.left
        else:
                current = self.right
    return None
```

| | | |
|---|---|---|
| Code size | 12 lines | 9 lines |
| Max running time | $\Theta(h)$ | $\Theta(h)$ |
| Max space use | $h$ stack frames | $O(1)$ |



# Worst Running Time




$h = N$
running time of lookup $\in \Theta(N)$

Later in the course, we'll learn some techniques for keeping trees balanced. Until then, let's hope we are usually not unlucky (or being attacked[1]).

1. Scott Crosby and Dan S. Wallach, Denial of Service via Algorithmic Complexity Attacks, *USENIX Security* 2003.



# Charge

- Read Chapter 6
  - You can skip the skip lists section
- PS3 will be posted tomorrow
- Monday:
  - Greedy Algorithms
- Later in the course:
  - More efficient dictionary implementations
  - Python's provides lookup with running time approximately in $O(1)$! (as PS2 #5 asks you to assume)