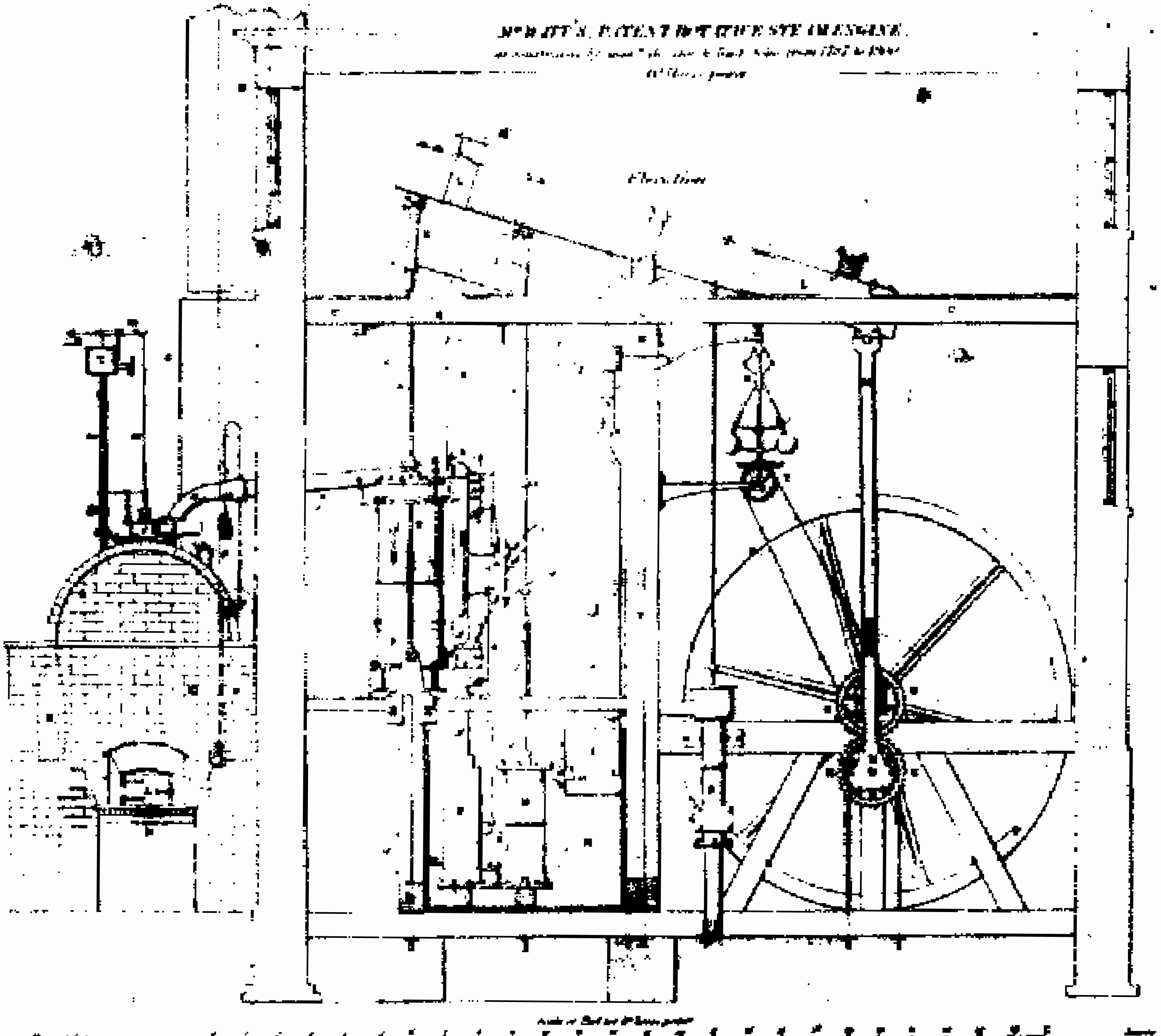
CS302

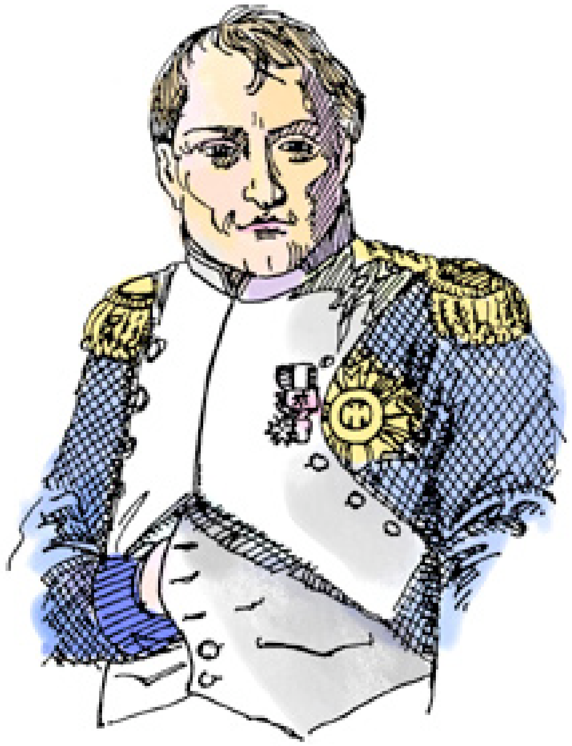
LI

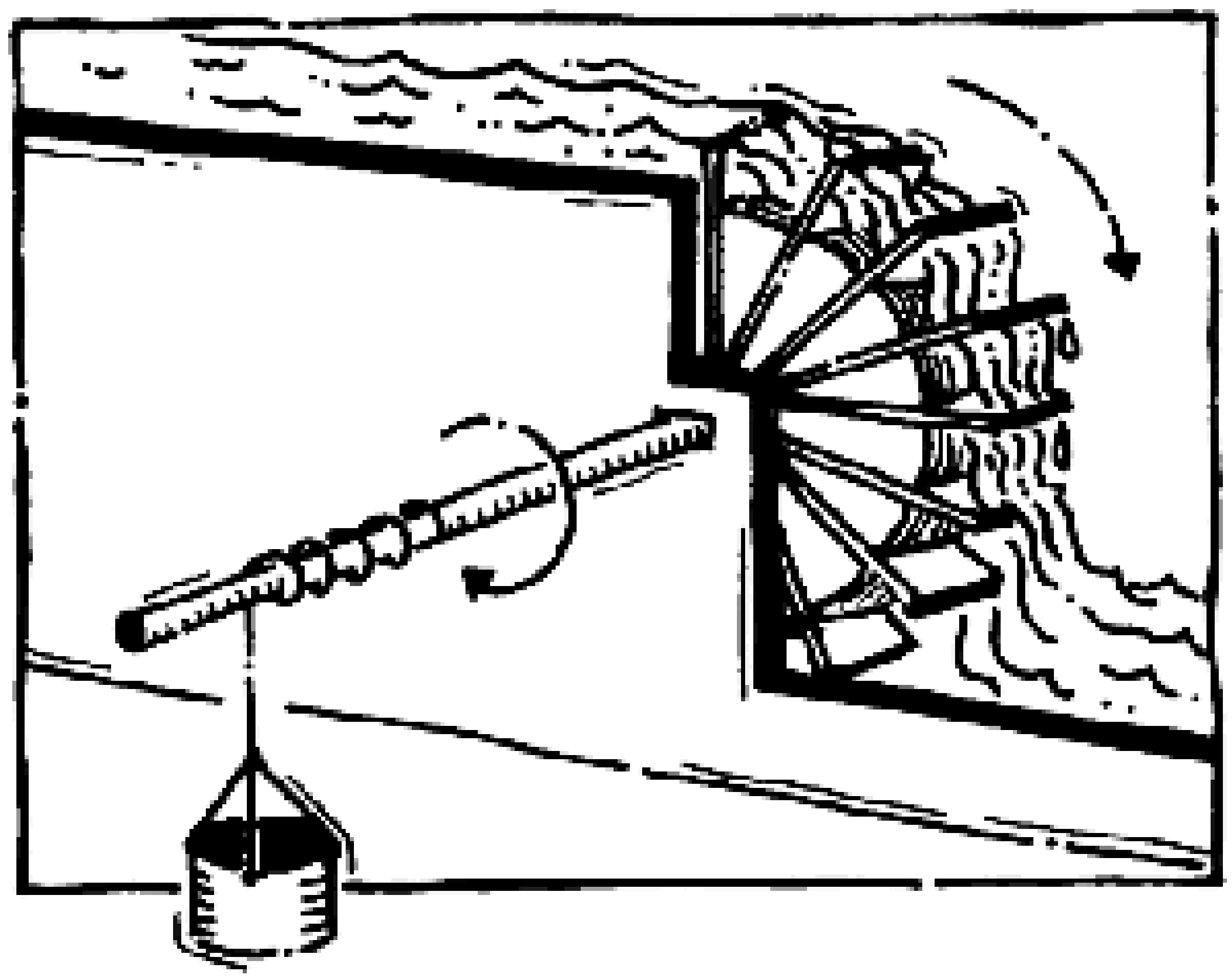
JAN 17 2008

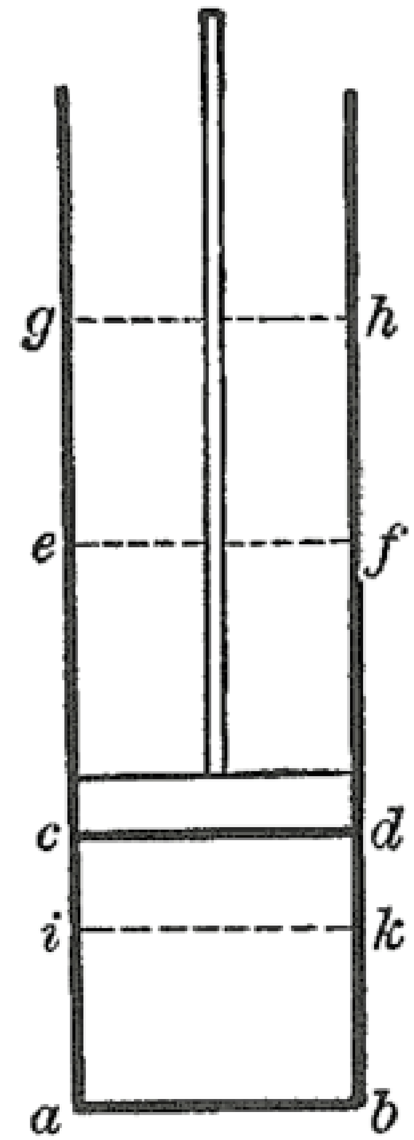
2 STORIES

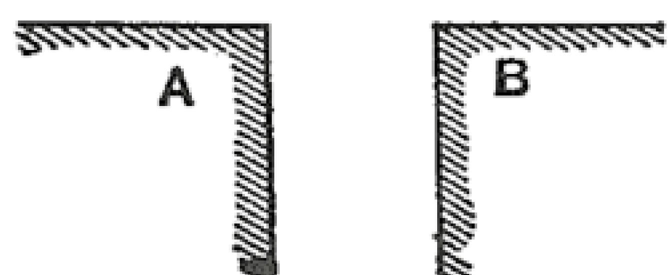
Carnot

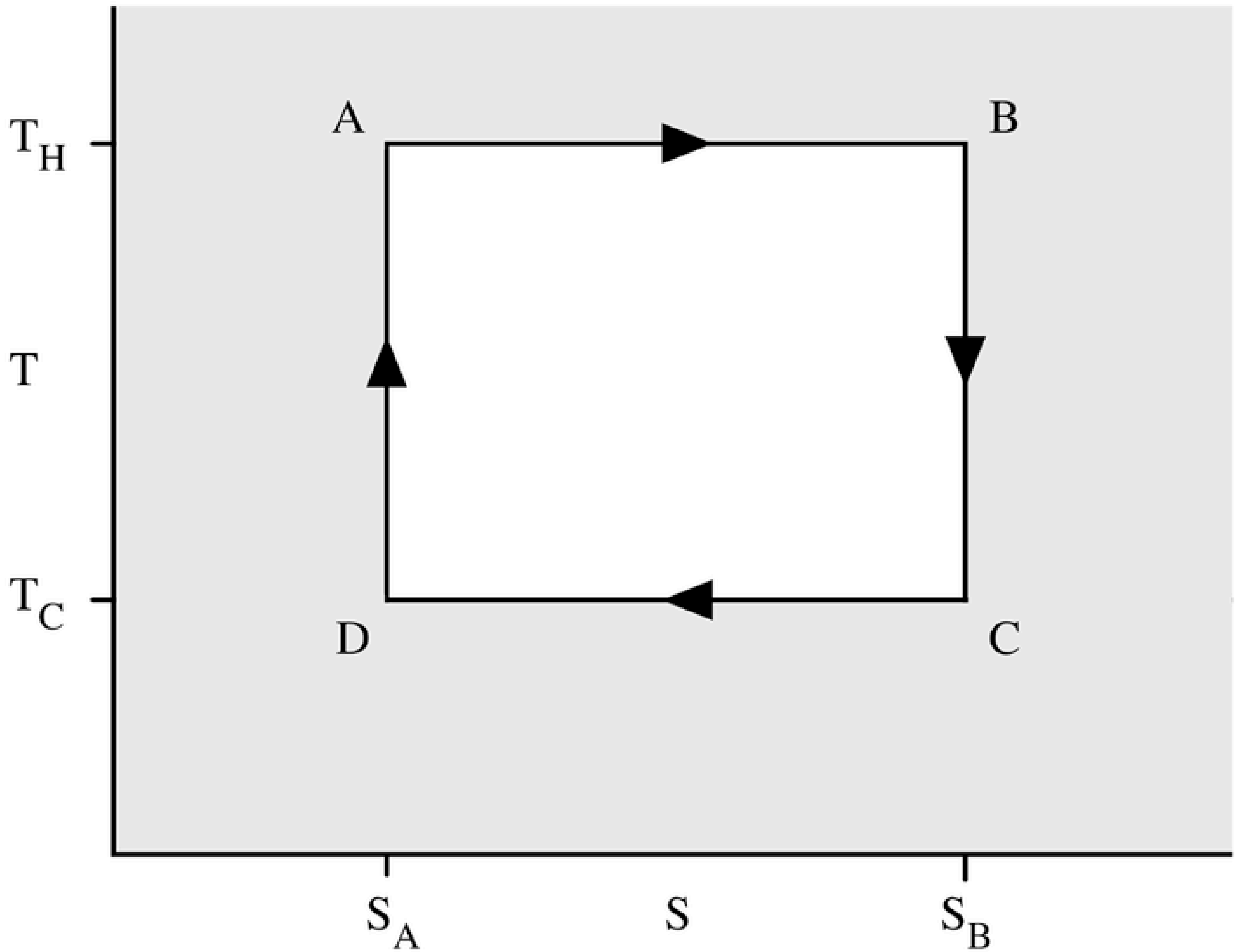

Mr WATT'S PATENT ROTATIVE STEAM ENGINE.






g
h
e
f
c
d
i
k
a
b


A
B



A
B
T_H
T
T_C
D
C
S_A
S
S_B

$$\eta = \frac{\Delta W}{\Delta Q_h} = 1 - \frac{T_c}{T_h}$$

theory
governs
practice
governs
theory

DAVID HILBERT

1900 INT'L CONFERENCE OF MATHEMATICIANS

23

10

ENTSCHEIDUNGSPROBLEM

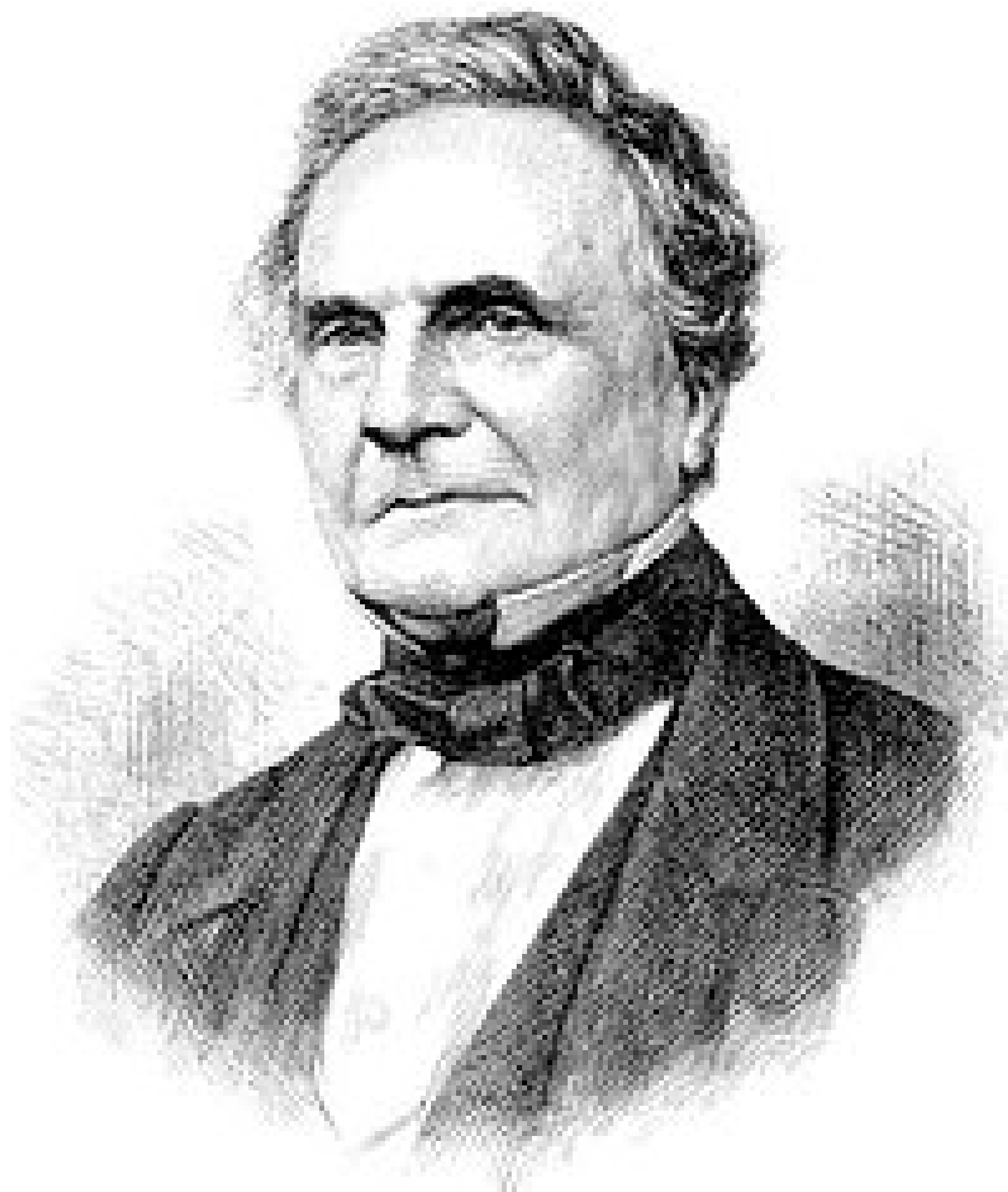
10. Entscheidung der Losbarkeit einer diophantischen Gleichung. Eine diophantische Gleichung mit irgendwelchen Unbekannten und mit ganzen rationalen Zahlkoefficienten sei vorgelegt: man soll ein Verfahren angeben, nach welchen sich mittels einer endlichen Anzahl von Operationen entscheiden lässt, ob die Gleichung in ganzen rationalen Zahlen losbar ist.

"Is there a method to decide whether a given equation with Integer coefficients has an Integer solution?"

1928:

IS THERE A METHOD TO DECIDE WHETHER A MATHEMATICAL STATEMENT IS TRUE OR FALSE?

Leibniz 1670



Babbage 1840

1928:

IS THERE A METHOD TO DECIDE WHETHER A MATHEMATICAL STATEMENT IS TRUE OR FALSE?

Language of Theoretical Computer Science

SET

GROUP OF OBJECTS

SET

GROUP OF OBJECTS

ELEMENTS

MEMBERS

$$\{2, 3, 5, 7\}$$

$$\{2, 3, 5, 7\}$$

$$\in \quad \notin$$

$$2 \in \{2, 3, 5, 7\}$$

$$4 \notin \{2, 3, 5, 7\}$$

A is a subset of B

$$A \subseteq B$$

"every element in A is also in B"

Q: when are 2 sets equal?

$$A \stackrel{?}{=} B$$

Q: WHEN ARE 2 SETS EQUAL?

$$A \stackrel{?}{=} B$$

A: WHENEVER BOTH

$$A \subseteq B$$

$$B \subseteq A$$

SETS CAN CONTAIN

NO ELEMENTS

$$\emptyset$$

SETS CAN CONTAIN INFINITELY MANY ELEMENTS

$$\mathbb{N} = \{1, 2, 3, ...\}$$

$$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$$

SET OPERATIONS

$$A \cup B$$

SET OPERATIONS

$$A \cup B$$

UNION

SET OPERATIONS

$$A \cup B$$

UNION

$$A \cap B$$

SET OPERATIONS

$$A \cup B$$

UNION

$$A \cap B$$

INTERSECTION

SEQUENCE

LIST OF OBJECTS

(ORDER MATTERS)

SEQUENCE

LIST OF OBJECTS

ELEMENTS

MEMBERS

(ORDER MATTERS)

$$(2, 3, 5)$$

$$(2, 3, 5)$$

K ELEMENT SEQUENCE IS CALLED A K-TUPLE

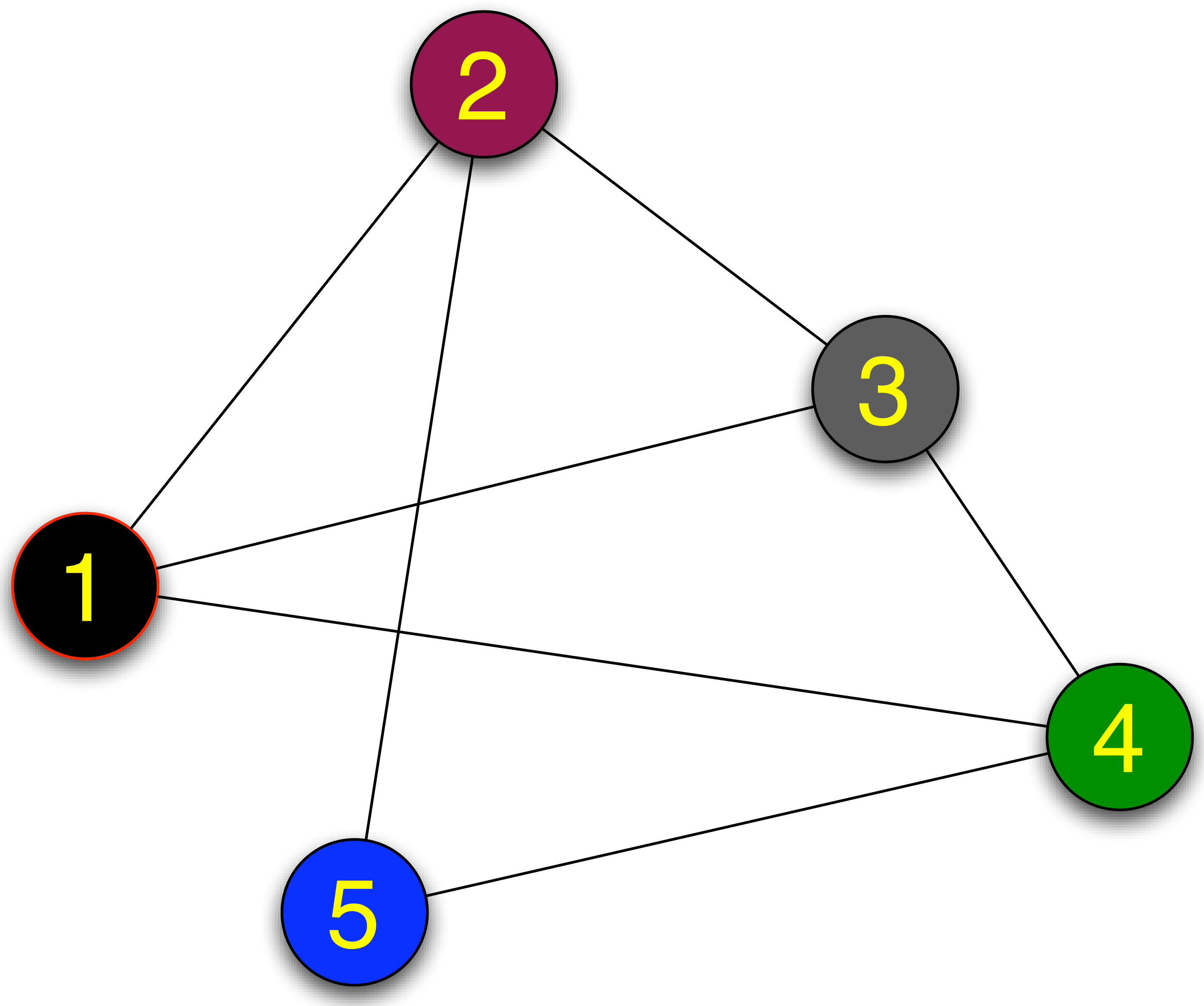
2 ELEMENT SEQUENCE IS CALLED A PAIR

GRAPH

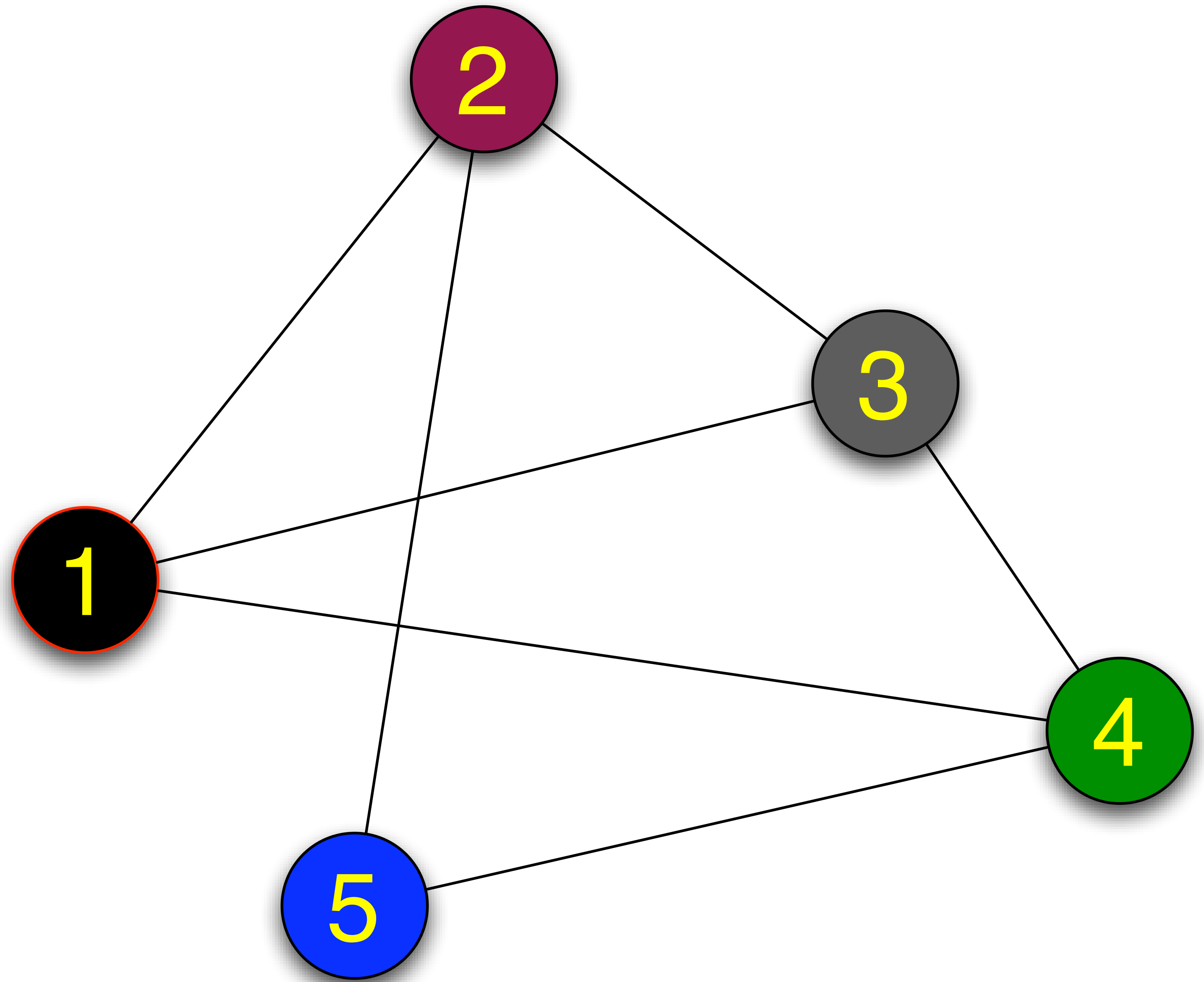
SET OF NODES (VERTICES)

SOME OF WHICH ARE CONNECTED

(EDGES)

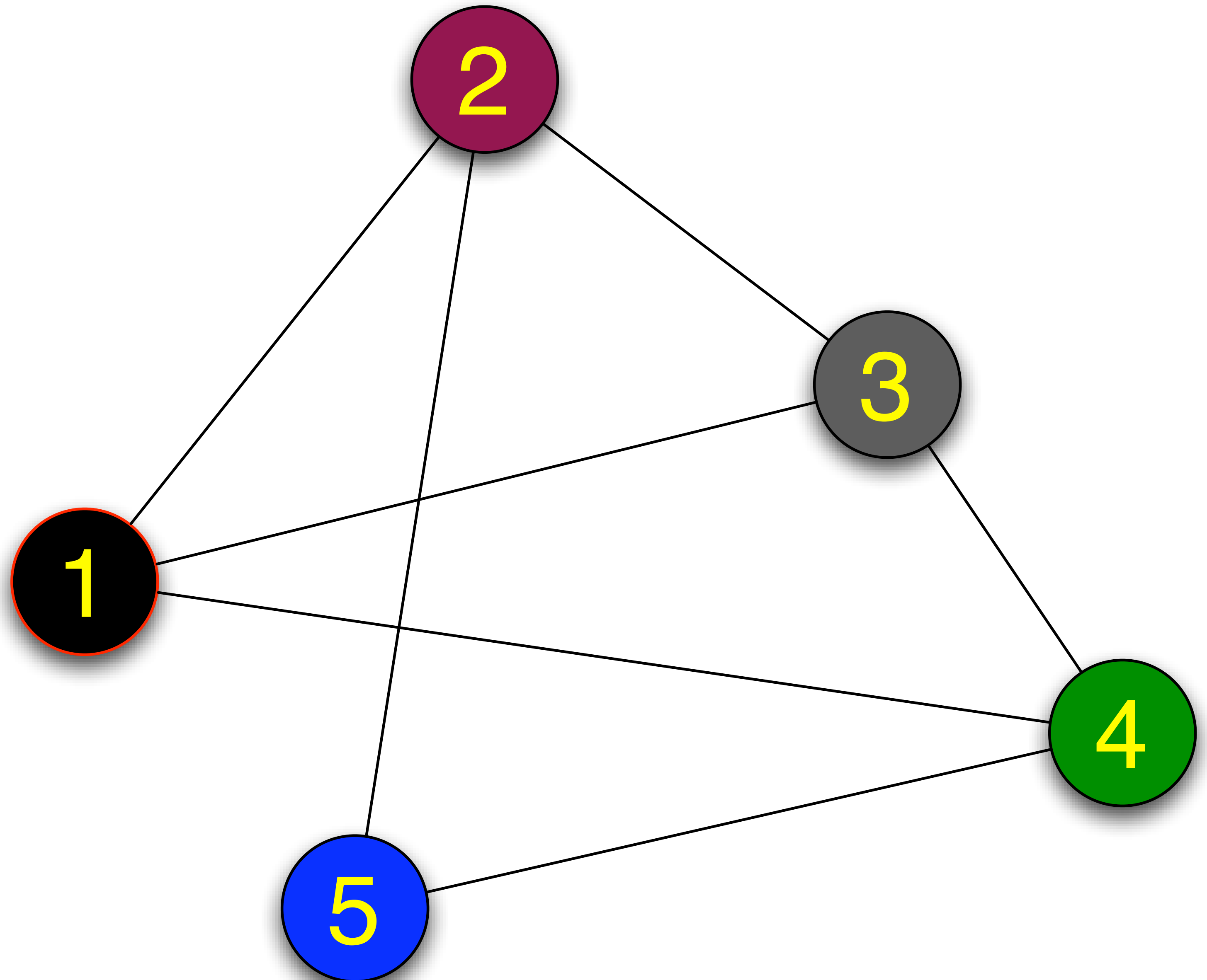


HOW CAN WE FORMALLY REPRESENT A GRAPH?



SET OF NODES

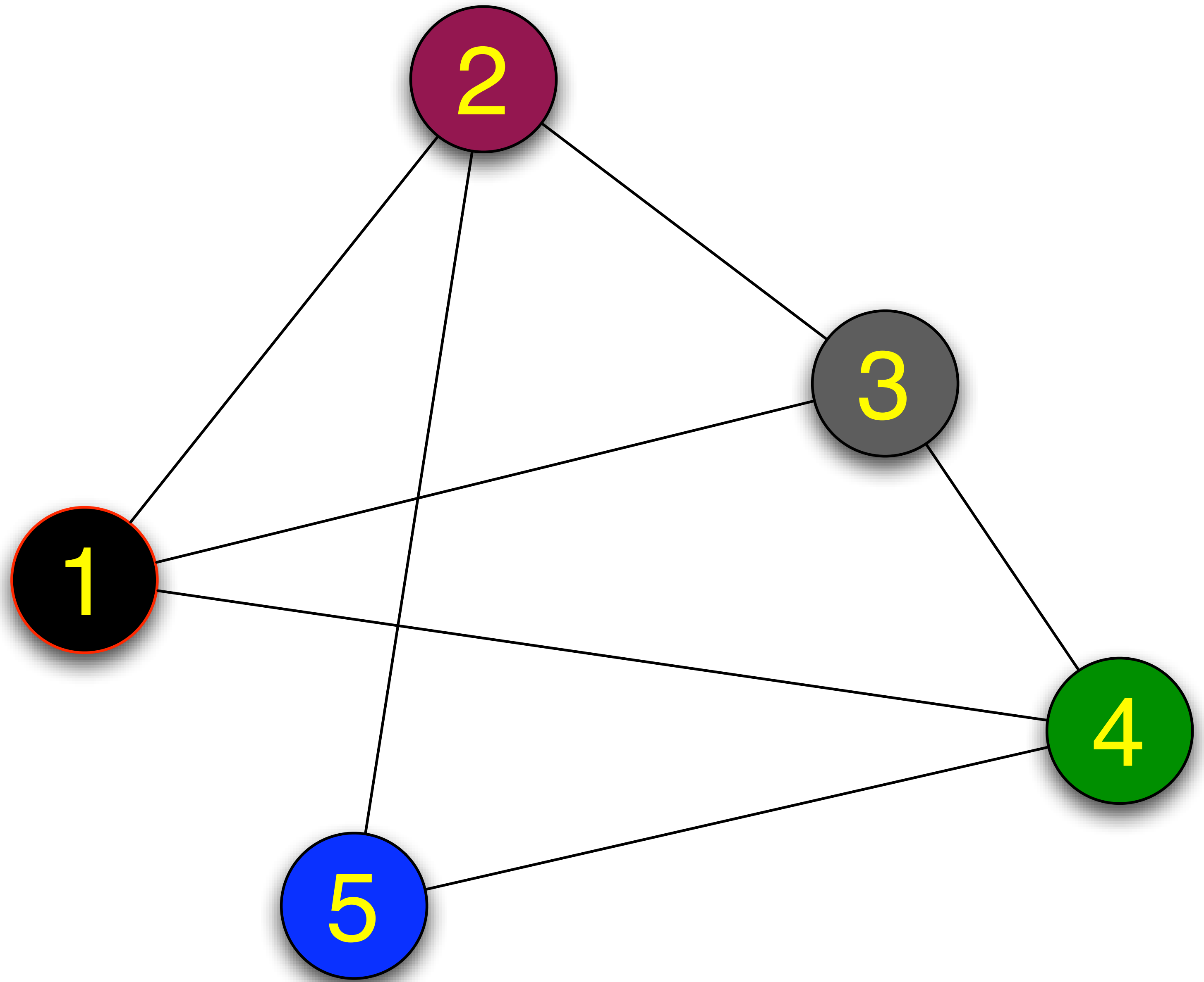
SET OF EDGES



SET OF NODES

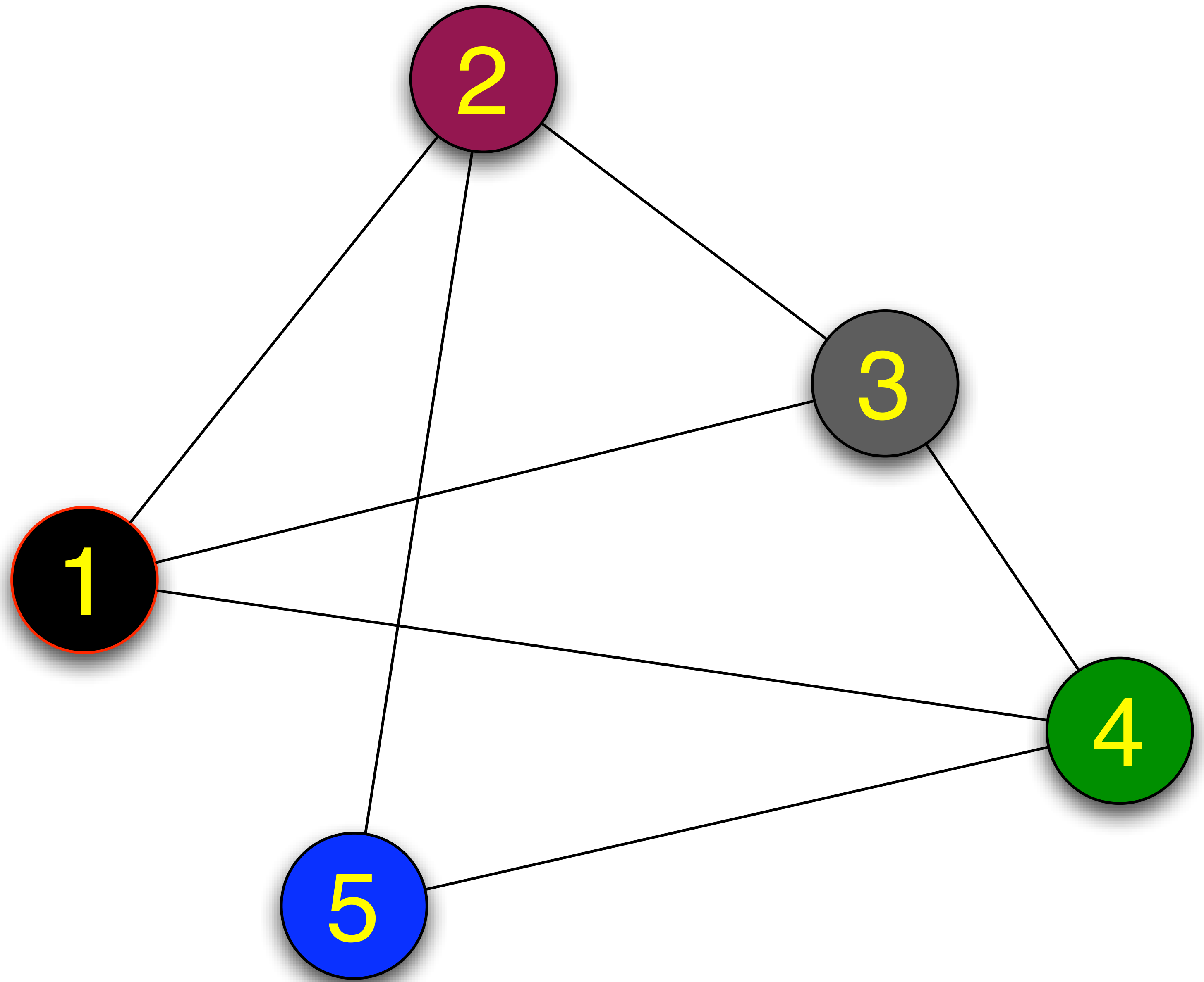
$$G = (V, E)$$

SET OF EDGES



$G = ($ SET OF NODES

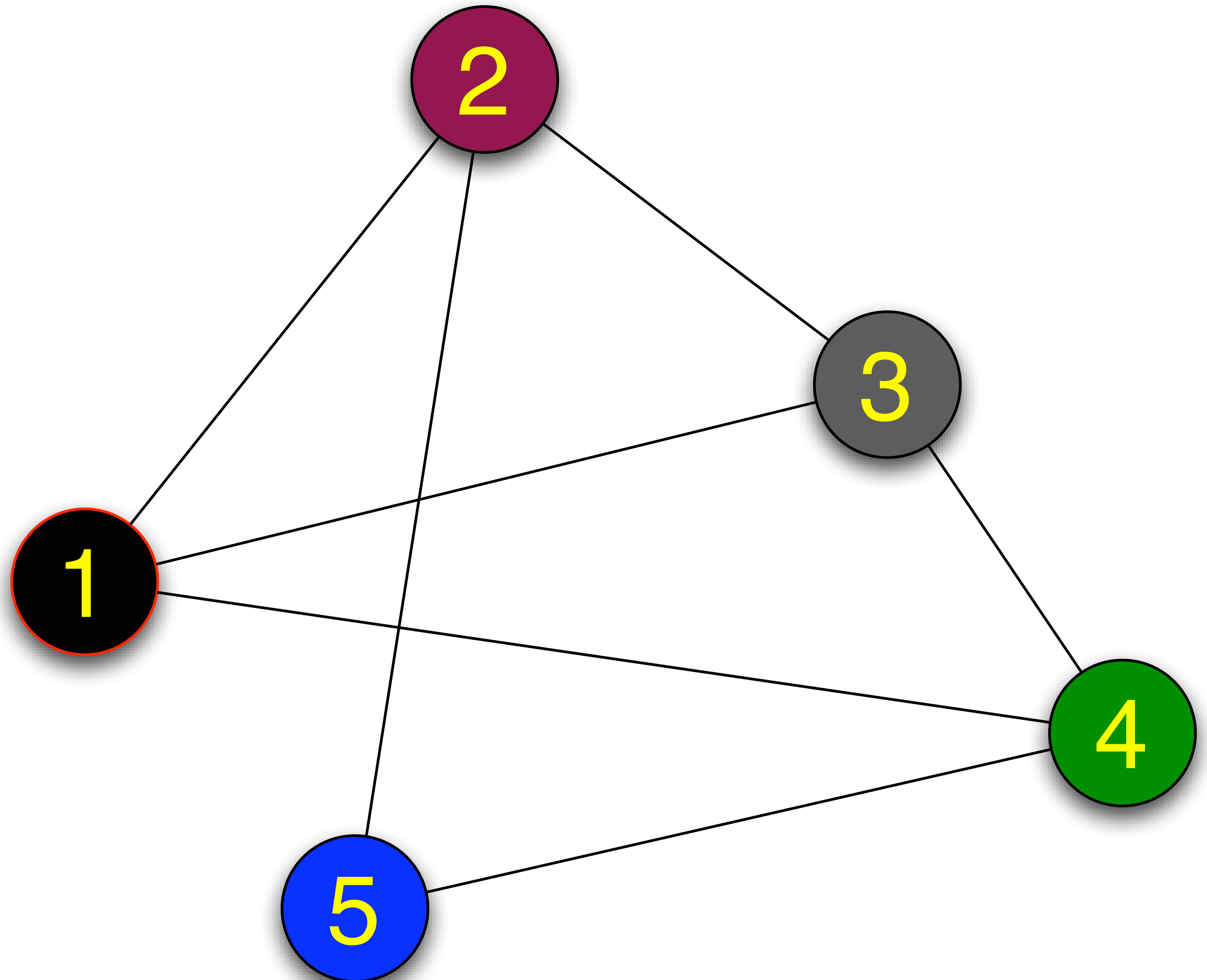
SET OF EDGES $)$



SET OF NODES

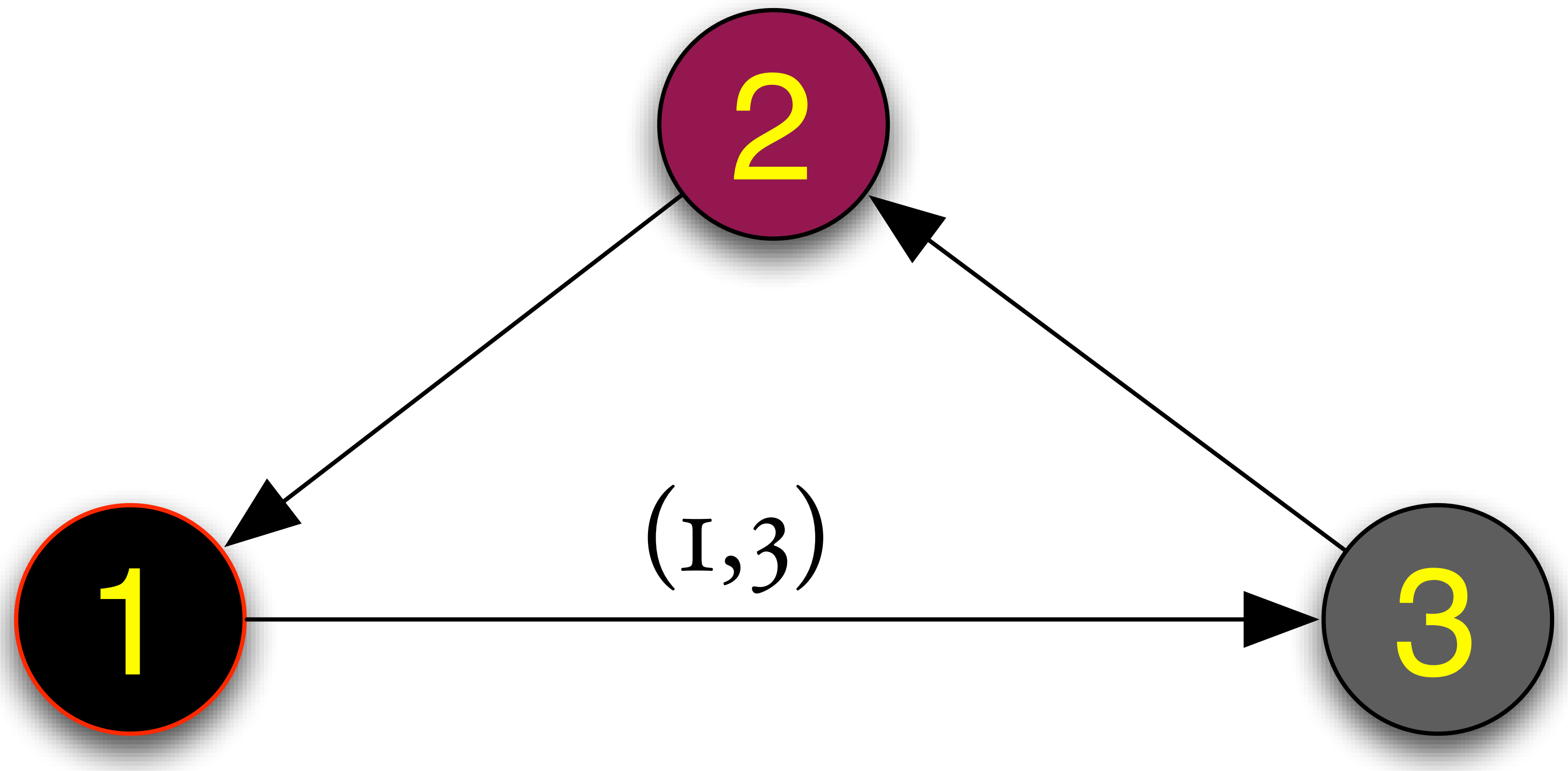
$$G = (\{1,2,3,4,5\}, \quad)$$

SET OF EDGES

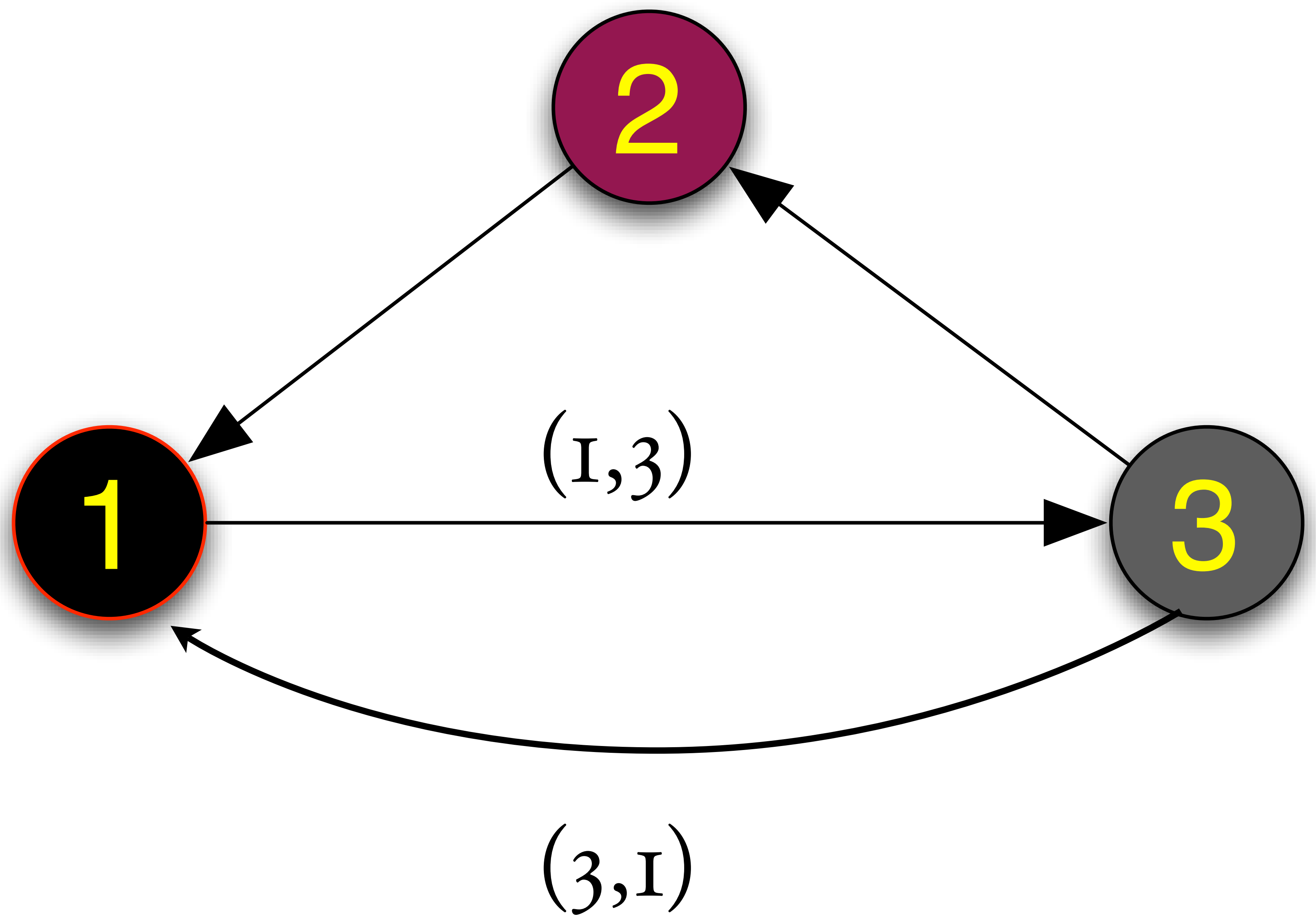


$$G = \left(\overbrace{\{1, 2, 3, 4, 5\}}^{\text{SET OF NODES}}, \overbrace{\{(1,2),(1,3),(1,4),(2,3),(2,5),(3,4),(4,5)\}}^{\text{SET OF EDGES}} \right)$$

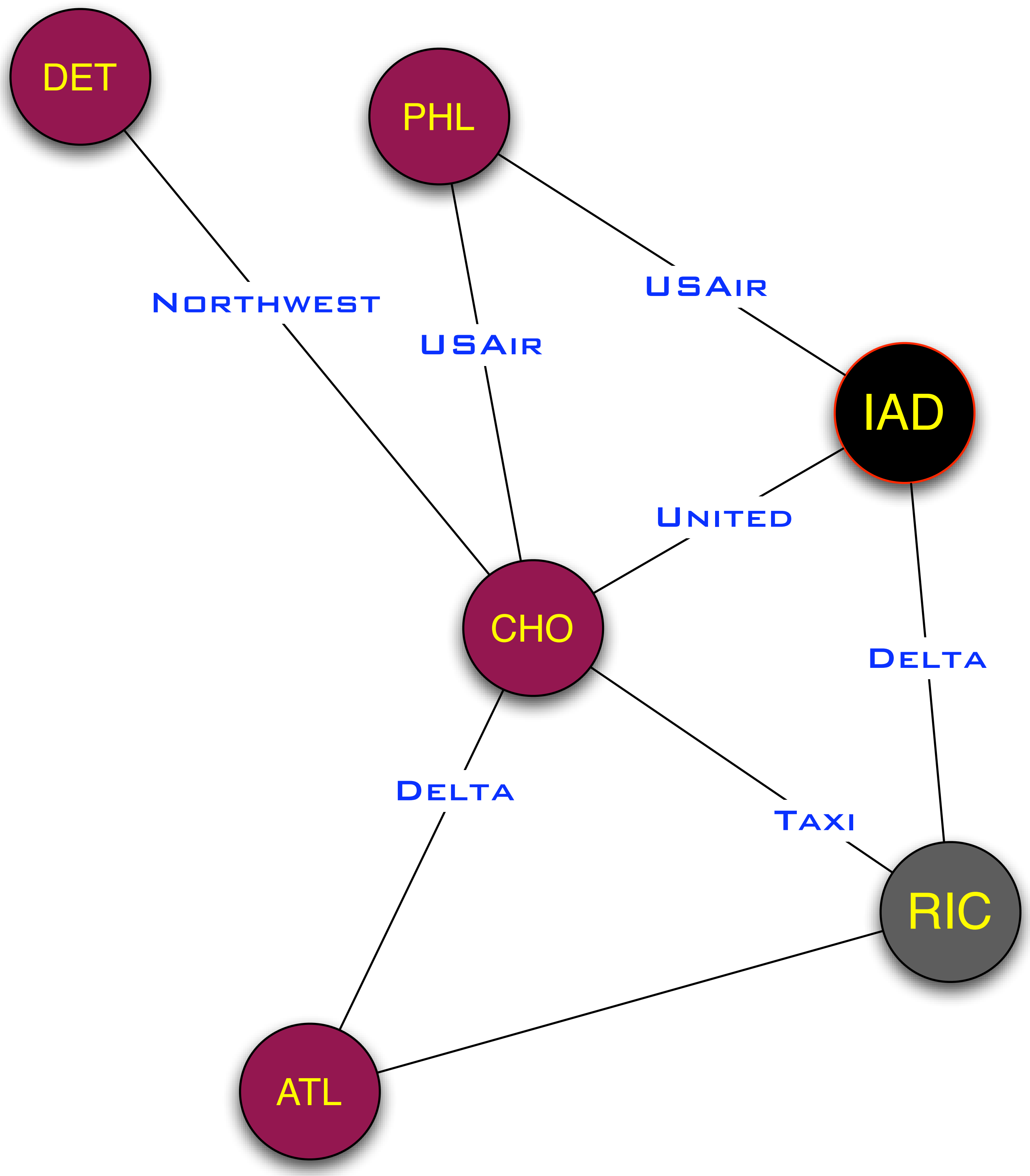
DIRECTED GRAPH



DIRECTED GRAPH



LABELLED GRAPH



ALPHABET

FINITE SET OF SYMBOLS

$$\Sigma_1 = \{0, 1\}$$

BINARY ALPHABET

$$\Sigma_2 = \{a, b, c, d, \ldots, z\}$$

SESAME ST ALPHABET

STRING

FINITE SEQUENCE OF SYMBOLS
FROM AN ALPHABET

STRINGS OVER BINARY ALPHABET

$$\sigma = 010001001$$

ϵ EMPTY STRING (LENGTH 0)

LANGUAGE

SET OF STRINGS

LANGUAGE OF BINARY STRINGS

$$\{\epsilon, 0, 1, 00, 01, 10, 11, ...\}$$

DEFINITIONS

THEOREMS

PROOFS

WE SEEK TO MAKE STATEMENTS ABOUT OUR WORLD.

PREFER TRUE STATEMENTS

PRECISE STATEMENTS

MATHEMATICAL DEFINITIONS OF OBJECTS

PRECISE ARGUMENTS

MATHEMATICAL PROOFS

PROVE: $\overline{A \cup B} = \overline{A} \cap \overline{B}$

WHAT MUST WE SHOW?

PROOF BY CONTRADICTION

"REDUCTIO AD ABSURDUM"

ASSUME THE ABSURD

ASSUME THE ABSURD

DERIVE A FALLACY

ASSUME THE ABSURD

DERIVE A FALLACY

ERGO: ABSURD IS FALSE

PROVE:

THERE ARE INFINITELY MANY PRIMES

PROVE: $\sqrt{2}$ IS AN IRRATIONAL NUMBER