2 STORIES
Carnot
\[ \eta = \frac{\Delta W}{\Delta Q_h} = 1 - \frac{T_c}{T_h} \]
theory
governs
practice
governs
theory
DAVID HILBERT

1900 INT’L CONFERENCE OF MATHEMATICIANS
“Is there a method to decide whether a given equation with Integer coefficients has an Integer solution?”
1928:

IS THERE A METHOD TO DECIDE WHETHER A MATHEMATICAL STATEMENT IS TRUE OR FALSE?
1928:

IS THERE A METHOD TO DECIDE WHETHER A MATHEMATICAL STATEMENT IS TRUE OR FALSE?
Language of Theoretical Computer Science
SET

GROUP OF OBJECTS
SET

GROUP OF OBJECTS

ELEMENTS

MEMBERS
\{2, 3, 5, 7\}
\{2, 3, 5, 7\}

\in \not\in
\[ 2 \in \{2, 3, 5, 7\} \]
\[ 4 \notin \{2, 3, 5, 7\} \]
A is a subset of B

“A ⊆ B”

“Every element in A is also in B”
Q: WHEN ARE 2 SETS EQUAL?

\[ A = B \]
Q: WHEN ARE 2 SETS EQUAL?

\[ A = B \]

A: WHENEVER BOTH

\[ A \subseteq B \]
\[ B \subseteq A \]
SETS CAN CONTAIN NO ELEMENTS

∅
SETS CAN CONTAIN

INFINITELY MANY ELEMENTS

\[ \mathbb{N} = \{1, 2, 3, \ldots\} \]

\[ \mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\} \]
SET OPERATIONS

\[ A \cup B \]
SET OPERATIONS

\[ A \cup B \]

UNION
SET OPERATIONS

\[ A \cup B \]

UNION

\[ A \cap B \]
SET OPERATIONS

A \cup B

UNION

A \cap B

INTERSECTION
SEQUENCE

LIST OF OBJECTS

(ORDER MATTERS)
SEQUENCE

LIST OF OBJECTS
ELEMENTS
MEMBERS

(ORDER MATTERS)
(2, 3, 5)
A $k$-element sequence is called a $k$-tuple.

A 2-element sequence is called a pair.
GRAPH

SET OF NODES (VERTICES)
SOME OF WHICH ARE CONNECTED (EDGES)
HOW CAN WE FORMALLY REPRESENT A GRAPH?
SET OF NODES

SET OF EDGES
$G = (V, E)$

Set of nodes

Set of edges
\[ G = ( \text{SET OF NODES} \quad \text{SET OF EDGES} ) \]
\[ G = (\{1, 2, 3, 4, 5\},\) \\
\text{SET OF NODES} \\
\text{SET OF EDGES} \]
\[ G = \left( \{1, 2, 3, 4, 5\}, \begin{array}{c}
\text{SET OF NODES} \\
\{(1, 2), (1, 3), (1, 4), (2, 3), (2, 5), (3, 4), (4, 5)\}
\end{array} \right) \]
DIRECTED GRAPH

1 → 2 → 3

(1, 3)
DIRECTED GRAPH

1 -> 2
2 -> 3
1 -> 3
3 -> 1
ALPHABET

FINITE SET OF SYMBOLS
$$\Sigma_1 = \{0, 1\}$$

**BINARY ALPHABET**

$$\Sigma_2 = \{a, b, c, d, ..., z\}$$

**SESAME ST ALPHABET**
STRING

FINITE SEQUENCE OF SYMBOLS
FROM AN ALPHABET
STRINGS OVER BINARY ALPHABET

\( \sigma = 010001001 \)

\( \varepsilon \)  \hspace{1cm} \text{EMPTY STRING (LENGTH 0)}
LANGUAGE

SET OF STRINGS
LANGUAGE OF BINARY STRINGS

\{ \epsilon, 0, 1, 00, 01, 10, 11, \ldots \}
DEFINITIONS
THEOREMS
PROOFS
WE SEEK TO MAKE STATEMENTS ABOUT OUR WORLD.
PREFER TRUE
STATEMENTS
PRECISE STATEMENTS
MATHEMATICAL DEFINITIONS OF OBJECTS

PRECISE ARGUMENTS
MATHEMATICAL PROOFS
PROVE: $\overline{A \cup B} = \overline{A} \cap \overline{B}$

WHAT MUST WE SHOW?
PROOF BY

CONTRACTION

“REDUCTIO AD ABSURDUM”
ASSUME THE ABSURD
ASSUME THE ABSURD

DERIVE A FALLACY
ASSUME THE ABSURD

DERIVE A FALLACY

ERGO: ABSURD IS FALSE
PROVE:

THERE ARE INFINITELY MANY PRIMES
PROVE: $\sqrt{2}$ IS AN IRRATIONAL NUMBER