The four pictures above illustrate some solutions of “3D Pie Slicing” problem. (Note: I have spent couple hours but I am not able to decompose the 15 pieces with 4 cuts with nice vision. Here is one exhausting method you could try with scratch paper: choose one cutting-plane and split the pie into two pieces, find the intersecting line of any two planes and then split the pieces again. Keep doing this until you cannot split.)

**Theorem:** The maximum number of pieces that can be produced with n cuts is $(n^3+5n+6)/6$.

**Proof:** Proof by induction on the number of cuts.  
*Basis:* For $n = 0$, $(n^3+5n+6)/6 = 6/6 = 1$ piece.

*Induction:*  
Assume $k$ cuts will give us maximum $(k^3+5k+6)/6$ pieces.  
We can view each cut $C_i$ as a plane. The $k+1$-st cut, $C_{k+1}$, could intersect all the previous $k$ cuts ($C_1$, $C_2$, … , $C_k$). It will form $k$ intersecting lines in the plane created by $C_{k+1}$ (each
two intersecting planes generate an intersecting line).

Note that $k$ lines could cut a plane into maximum $(k^2+k+2)/2$ pieces. $C_{k+1}$ could be divided into a maximum $(k^2+k+2)/2$ pieces. Each piece of $C_{k+1}$ could cut the space that contains it into two parts, increasing the total number of pieces by one.

So $(k^2+k+2)/2$ pieces of the plane will create $(k^2+k+2)/2$ new pie pieces from the $k+1^{st}$ cut. Using the induction hypotheses, the total number of pieces is $(k^3+5k+6)/6 + (k^2+k+2)/2 = ((k+1)^3+5(k+1)+6)/6$. Replacing $n = k + 1$, we get $(n^3 + 5n+6)/6$, thus proving the induction step.