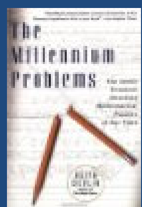
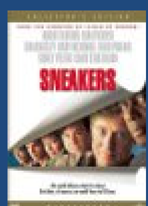
(Teams removed to preserve email address privacy)

Prizes

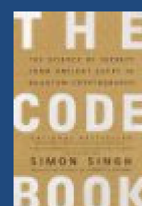


The Millennium Problems: The Seven Greatest Unsolved Mathematical Puzzles Of Our Time
Keith J. Devlin

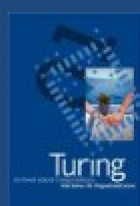
Your Inner Fish: A Journey into the 3.5-Billion-Year History of the Human Body (Vintage)
Neil Shubin



Sneakers



The Code Book: The Science of Secrecy from Ancient Egypt to Quantum Cryptography
Simon Singh



Turing (A Novel about Computation)
Christos H. Papadimitriou

Logicomix: An Epic Search for Truth
Apostolos Doxiadis, Christos Papadimitriou

CS Theory Jeopardy

(All values in $Quadrillions)

Languages	Asymptotic Operators	Undecidable Propositions	Questions too trivial to ask on the final
1	1	1	1
2	2	2	2
4	4	4	4
8	8	8	8

Final Jeopardy

Buffer

Buffer

Languages 1

What is a language?

Languages 1 - Answer

What is a language?

A set of strings.

Return

Buffer

Languages 2

Describe a language that is *context-free* and *finite*.

Languages 2

Describe a language that is *context-free* and *finite*.

Any finite language.

Return

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Languages 4

How many strings are in the language generated by this grammar?

$$S \to 0 \mid S$$

Languages 4 - Answer

1 $S \to 0 \mid S$
generates the language
$\{0\}$

Return

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Languages 8

If A is in **P** and B is in **NP** which of the following *must* be true:

(a) the complement of A is in **P**

(b) the complement of B is in **NP**

(c) $A \cap B$ is non-empty

Languages 8

If A is in **P** and B is in **NP** which of the following *must* be true:

(a) the complement of A is in **P**

True. Simulate M_A and flip result.

(b) the complement of B is in **NP**

Unknown! (False unless P=NP)

(c) $A \cap B$ is non-empty

False. e.g., $A = \{0, 1\}^*$, $B = 0^*$

Return

Buffer

Asymptotic Operators 1

Which of these functions are in $O(2^n)$?

$$a(n) = 2n+3$$

$$b(n) = n!$$

$$c(n) = n2^n$$

$$d(n) = BB(7,2)$$

Asymptotic Operators 1 - Answer

Which of these functions are in $O(2^n)$?

$a(n) = 2n+3$

~~$b(n) = n!$~~

Opps! I was actually wrong on this originally, sorry to the teams that lost points.
n! grows faster than 2*n* so is not in $O(2^n)$

~~$c(n) = n2^n$~~

$d(n) = BB(7,2)$

Return

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Asymptotic Operators 2

What is a tight bound (Θ) of the worst-case asymptotic running time of this Java procedure?

```
int findMatch(int a[], int x) {
   for (int i = 0; i < a.length; i++)
        if (a[i] == x) return i;
   return -1; // not found
}
```

Asymptotic Operators 2 - Answer

What is a tight bound (Θ) of the worst-case asymptotic running time of this Java procedure?

```
int findMatch(int a[], int x) {
   for (int i = 0; i < a.length; i++)
      if (a[i] == x) return i;
   return -1; // not found
}
```

Since Java arrays and integers have bounded size, the running time is in $\Theta(1)$.

Return

Buffer

Asymptotic Operators 4

What is a tight bound (Θ) of the worst-case asymptotic running time of this Python procedure?

```
def findMatch(a, x):
   for i in range(0, len(a)):
       if (a[i] == x): return i
   return -1 # not found
```

Note: do not assume == has constant time!

Asymptotic Operators 4

```
def findMatch(a, x):
    for i in range(0, len(a)):
        if (a[i] == x): return i
    return -1 # not found
```

Worst case:

a is m elements, b bits each

x is b bits

$n = (m+1)b \approx mb$

Loop iterations = $m \approx n/b$

Work/iteration = b (each ==)

The worst case running time is in $\Theta(n)$

Return

Buffer

Asymptotic Operators 8

Order these from **smallest** to **largest**:

A. $BB(6,2)$ (maximum number of steps a 6-state, 2-symbol TM can make before halting)

B. Cost (in dollars) to sequence a human genome today.

C. Cost (in dollars) to sequence a human genome in 2007.

D. 2^{1000}

E. 10000^{20}

F. 20!

G. US Deficit (in dollars)

Write

Asymptotic Operators 8 - Answer

Return

Order these from **smallest** to **largest**:

A. $BB(6,2) > 10^{2879}$

B. Cost (in dollars) to sequence a human genome today ~ \$5000

C. Cost (in dollars) to sequence a human genome in 2007 ~ \$57M

D. $2^{1000} \sim 10^{300}$

E. $10000^{20} = 10^{80}$

F. $20! \sim 10^{18}$

G. US National Debt (in dollars) ~ \$12.9T ~ 10^{13}

B << C << G << F << E << D << A

Buffer

Return

Buffer

Undecidable Propositions 1

What is the meaning and correct pronunciation of "*Entscheidungsproblem*"?

Undecidable Propositions 1 - Answer

What is the meaning and correct pronunciation of "*Entscheidungsproblem*"?

Decision problem.

Input: A mathematical statement.

Output: True if the statement is true, false otherwise.

Return

Buffer

Undecidable Propositions 2

Define an undecidable language that contains only even-length strings.

Undecidable Propositions 2 - Answer

Define an undecidable language that contains only even-length strings.

$$HALTSEVEN_{TM} = \{\langle M, w, z\rangle | M \text{ describes a Turing Machine, } M \text{ halts on input } w, z \in \{\epsilon, 0\} \text{ and } |\langle M, w, z\rangle| \text{ is even } \}$$

Return

Buffer

Undecidable Propositions 4

Which of these languages are ***decidable***?

$A = \{\langle M, w, k\rangle | M$ describes a Turing Machine, $w \in \Sigma^*$, k is a positive integer, and M runs for $\geq k$ steps on input $w\}$

$B = \{\langle M, k\rangle | M$ describes a Turing Machine, k is a positive integer, and there exists a string $w \in \Sigma^*$ on which M runs for $\geq k$ steps $\}$

$C = \{\langle M, w\rangle | M$ describes a Turing Machine, $w \in \Sigma^*$ and there exists some positive integer k where M running on w finishes in $\leq k$ steps $\}$

Undecidable Propositions 4 - Answer

$A = \{\langle M, w, k\rangle | M$ describes a Turing Machine, $w \in \Sigma^*$, k is a positive integer, and M runs for $\geq k$ steps on input $w\}$

Decidable: simulate M on w for up to k steps;
if it halts, accepts; if it hasn't halted, accept.

$B = \{\langle M, k\rangle | M$ describes a Turing Machine, k is a positive integer, and there exists a string $w \in \Sigma^*$ on which M runs for $\geq k$ steps $\}$

Decidable: simulate M for up to k steps on all strings up to length k
(if length > k matters, must take >= k steps on some length k string.)

$C = \{\langle M, w\rangle | M$ describes a Turing Machine, $w \in \Sigma^*$ and there exists some positive integer k where M running on w finishes in $\leq k$ steps $\}$

Undecidable: C is the same as HALTS

Return

Buffer

Undecidable Propositions 8

Prove that there exist languages that cannot be *recognized* by any Turing Machine.

Write

Undecidable Propositions 8 - Answers

The number of languages is uncountable: proof by diagonalization.

The number of TMs is countable.

Return

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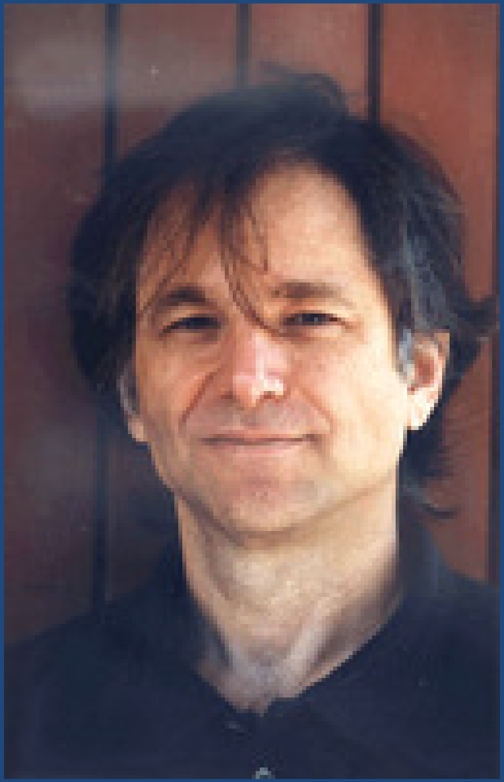
Too Trivial 1

Who was credited as the Mathematical Consultant for *Sneakers*?

Too Trivial 1

Leonard Adleman



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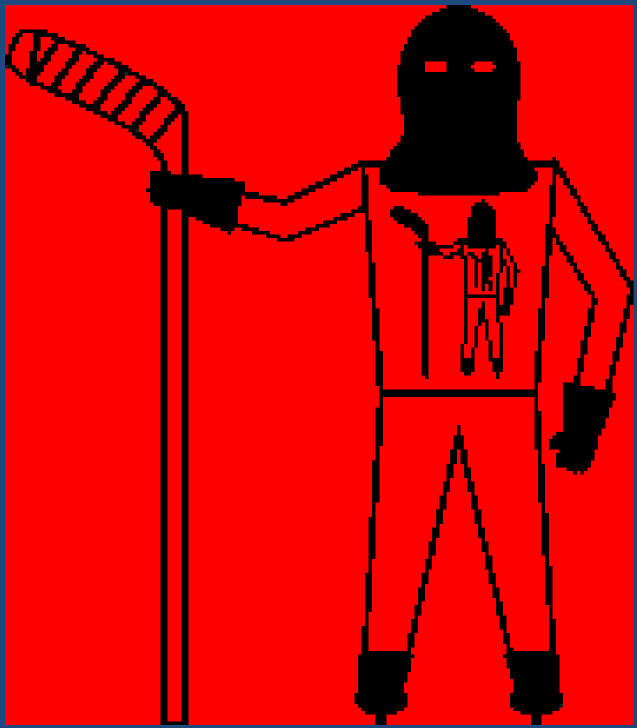
Too Trivial 2

What is the name of MIT's Lab for Computer Science D-league intramural hockey team?



Too Trivial 2

What is the name of MIT's Lab for Computer Science D-league intramural hockey team?

Halting Problem

Return

Buffer

Too Trivial 4

How many Turing Award winners have I had a meal with? (Tiebreaker: How many have I taken a class from?)

Too Trivial 4

How many Turing Award winners have I had a meal with?
7-8: John Backus, Ed Clarke, Sir Tony Hoare, John Hopcroft, Butler Lampson, Barbara Liskov, Alan Kay, Ron Rivest

(Tiebreaker: How many have I taken a class from?)
3: Barbara Liskov, Marvin Minsky, Ron Rivest

Return

Buffer

Too Trivial 8

Identify the source (author, paper, year) of each of these quotes:

A. “Namely, it would obviously mean that inspite of the undecidability of the Entscheidungsproblem, the mental work of a mathematician concerning Yes-or-No questions could be completely replaced by a machine.”
B. “It is my contention that these operations include all those which are used in the computation of a number. The defence of this contention will be easier when the theory of the machines is familiar to the reader.”
C. “It is shown that any recognition problem solved by a polynomial time-bounded nondeterministic Turing machine can be “reduced” to the problem of determining whether a given propositional formula is a tautology.”

Write

Return

Too Trivial 8

Identify the source (author, paper, year) of each of these quotes:

A. "Namely, it would obviously mean that inspite of the undecidability of the Entscheidungsproblem, the mental work of a mathematician concerning Yes-or-No questions could be completely replaced by a machine." Gödel's 1956 Letter to von Neumann

B. "It is my contention that these operations include all those which are used in the computation of a number. The defence of this contention will be easier when the theory of the machines is familiar to the reader." Alan Turing, *On Computable Numbers …,* 1936

C. "It is shown that any recognition problem solved by a polynomial time-bounded nondeterministic Turing machine can be "reduced" to the problem of determining whether a given propositional formula is a tautology."

Stephen Cook, *The Complexity of Theorem-Proving Procedures,* 1971

Buffer

Final Jeopardy:

Proving NP-Completeness

Final Jeopardy

Prove the language *4-SAT* is **NP-Complete**.

$$4SAT = \{\phi | \phi \text{ is a satisfiable formula in 4-cnf }\}$$

4-cnf = strings of the form

$$(v_{0,0} \vee v_{0,1} \vee v_{0,2} \vee v_{0,3}) \wedge \ldots \wedge (v_{i_n,0} \vee v_{i_n,1} \vee v_{i_n,2} \vee v_{i_n,3})$$

where $v_{i,j} \in \{x_i | i \geq 0\} \cup \{\overline{x_i} | i \geq 0\}$

Final Charge

- Don't forget to show up for the final:

  Thursday, May 13, 9am-noon

  (studying for it would be a good idea too!)
- Do the course evaluations
  - Official University evaluation
  - Course-specific evaluation (will be posted on website)
- **Work on big, important problems**
  - Don't spend your career making improvements that are hidden inside Θ notation!

How will P=NP be resolved?

Richard Karp: (Berkeley, unsure, P $\subset$ NP)
My intuitive belief is that P is unequal to NP, but the only supporting arguments I can offer are the failure of all efforts to place specific NP-complete problems in P by constructing polynomial-time algorithms. I believe that the traditional proof techniques will not suffice. Something entirely novel will be required. **My hunch is that the problem will be solved by a young researcher who is not encumbered by too much conventional wisdom about how to attack the problem.**

The P=?NP Poll, William Gasarch

Answer

The easiest proof is a reduction from $3SAT$ to $4SAT$. Suppose we have a polynomial-time decider for $4SAT$, M_{4SAT}. We can use it to make a polynomial-time decider for $3SAT$ by adding a new variable to each clause and forcing it to be *false*:

$$3SAT(\phi) = \text{result of simulating } M_{4SAT}(\phi') \text{ where}$$

$$\begin{aligned}
\phi' \quad = \quad & (v_{0,0} \vee v_{0,1} \vee v_{0,2} \vee x_z) \\
& \wedge \ldots \\
& \wedge (v_{i_n,0} \vee v_{i_n,1} \vee v_{i_n,2} \vee x_z) \\
& \wedge (\overline{x_z} \vee x_0 \vee x_0 \vee x_0) \\
& \wedge (\overline{x_z} \vee \overline{x_0} \vee \overline{x_0} \vee \overline{x_0})
\end{aligned}$$

Thank you!

Handouts before you leave today:

- Your PS6
- PS6 Comments
- Final Exam Preview