(Teams removed to preserve email address privacy)
Prizes

The Millennium Problems: The Seven Greatest Unsolved Mathematical Puzzles Of Our Time
Keith J. Devlin

Your Inner Fish: A Journey into the 3.5-Billion-Year History of the Human Body (Vintage)
Neil Shubin

Sneakers

The Code Book: The Science of Secrecy from Ancient Egypt to Quantum Cryptography
Simon Singh

Turing (A Novel about Computation)
Christos H. Papadimitriou

Logicomix: An Epic Search for Truth
Apostolos Doxiadis, Christos Papadimitriou
## CS Theory Jeopardy

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(All values in $Quadrillions$)

**Final Jeopardy**
Buffer
Buffer
What is a language?
Languages 1 - Answer

What is a language?

A set of strings.
Buffer
Describe a language that is context-free and finite.
Describe a language that is context-free and finite.

Any finite language.
Buffer
Languages 4

How many strings are in the language generated by this grammar?

\[ S \rightarrow 0 \mid S \]
Languages 4 - Answer

1. \[ S \rightarrow 0 \mid S \]
generates the language \[ \{0\} \]
Buffer
Languages 8

If $A$ is in $\mathbf{P}$ and $B$ is in $\mathbf{NP}$ which of the following must be true:

(a) the complement of $A$ is in $\mathbf{P}$
(b) the complement of $B$ is in $\mathbf{NP}$
(c) $A \cap B$ is non-empty
Languages 8

If $A$ is in $\mathbf{P}$ and $B$ is in $\mathbf{NP}$ which of the following *must* be true:

(a) the complement of $A$ is in $\mathbf{P}$
   
   **True.** Simulate $M_A$ and flip result.

(b) the complement of $B$ is in $\mathbf{NP}$
   
   **Unknown! (False unless P=NP)**

(c) $A \cap B$ is non-empty
   
   **False.** e.g., $A = \{0, 1\}^*$, $B = 0^*$
Buffer
Asymptotic Operators 1

Which of these functions are in $O(2^n)$?

- $a(n) = 2n + 3$
- $b(n) = n!$
- $c(n) = n2^n$
- $d(n) = BB(7,2)$
Asymptotic Operators 1 - Answer

Which of these functions are in $O(2^n)$?

\[ a(n) = 2n+3 \]

\[ b(n) = n! \] \(\text{Opps! I was actually wrong on this originally, sorry to the teams that lost points.} \)

\[ c(n) = n2^n \] \(n! \text{ grows faster than } 2n \text{ so is not in } O(2^n) \)

\[ d(n) = BB(7,2) \]
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Asymptotic Operators 2

What is a tight bound ($\Theta$) of the worst-case asymptotic running time of this Java procedure?

```java
int findMatch(int a[], int x) {
    for (int i = 0; i < a.length; i++)
        if (a[i] == x) return i;
    return -1; // not found
}
```
Asymptotic Operators 2 - Answer

What is a tight bound (Θ) of the worst-case asymptotic running time of this Java procedure?

```java
int findMatch(int a[], int x) {
    for (int i = 0; i < a.length; i++)
        if (a[i] == x) return i;
    return -1; // not found
}
```

Since Java arrays and integers have bounded size, the running time is in Θ(1).
Buffer
What is a tight bound (Θ) of the worst-case asymptotic running time of this Python procedure?

```python
def findMatch(a, x):
    for i in range(0, len(a)):
        if (a[i] == x): return i
    return -1 # not found
```

Note: do not assume == has constant time!
Asymptotic Operators 4

```python
def findMatch(a, x):
    for i in range(0, len(a)):
        if a[i] == x:
            return i
    return -1 # not found
```

Worst case:
- a is $m$ elements, $b$ bits each
- x is $b$ bits
- $n = (m+1)b \approx mb$
- Loop iterations = $m \approx n/b$
- Work/iteration = $b$ (each ==)
- The worst case running time is in $\Theta(n)$
Buffer
Asymptotic Operators 8

Order these from smallest to largest:
A. $BB(6,2)$ (maximum number of steps a 6-state, 2-symbol TM can make before halting)
B. Cost (in dollars) to sequence a human genome today.
C. Cost (in dollars) to sequence a human genome in 2007.
D. $2^{1000}$
E. $10000^{20}$
F. $20!$
G. US Deficit (in dollars)
Asymptotic Operators 8 - Answer

Order these from **smallest** to **largest**:

A. \( BB(6,2) > 10^{2879} \)
B. Cost (in dollars) to sequence a human genome today \( \sim \$5000 \)
C. Cost (in dollars) to sequence a human genome in 2007 \( \sim \$57M \)
D. \( 2^{1000} \sim 10^{300} \)
E. \( 10000^{20} = 10^{80} \)
F. \( 20! \sim 10^{18} \)
G. US National Debt (in dollars) \( \sim \$12.9T \sim 10^{13} \)

\[ B << C << G << F << E << D << A \]
Buffer
Buffer
Undecidable Propositions 1

What is the meaning and correct pronunciation of "Entscheidungsproblem"?
What is the meaning and correct pronunciation of “Entscheidungsproblem”?

Decision problem.

**Input:** A mathematical statement.

**Output:** True if the statement is true, false otherwise.
Buffer
Define an undecidable language that contains only even-length strings.
Undecidable Propositions 2 - Answer

Define an undecidable language that contains only even-length strings.

\[ \text{HALTSEVEN}_{TM} = \{ \langle M, w, z \rangle | M \text{ describes a Turing Machine, } M \text{ halts on input } w, z \in \{ \epsilon, 0 \} \text{ and } |\langle M, w, z \rangle| \text{ is even} \} \]
Buffer
Which of these languages are **decidable**?

\[ A = \{ \langle M, w, k \rangle \mid M \text{ describes a Turing Machine, } w \in \Sigma^*, k \text{ is a positive integer, and } M \text{ runs for } \geq k \text{ steps on input } w \} \]

\[ B = \{ \langle M, k \rangle \mid M \text{ describes a Turing Machine, } k \text{ is a positive integer, and there exists a string } w \in \Sigma^* \text{ on which } M \text{ runs for } \geq k \text{ steps} \} \]

\[ C = \{ \langle M, w \rangle \mid M \text{ describes a Turing Machine, } w \in \Sigma^* \text{ and there exists some positive integer } k \text{ where } M \text{ running on } w \text{ finishes in } \leq k \text{ steps} \} \]
Undecidable Propositions 4 - Answer

\[ A = \{ \langle M, w, k \rangle \mid M \text{ describes a Turing Machine, } w \in \Sigma^*, k \text{ is a positive integer, and } M \text{ runs for } \geq k \text{ steps on input } w \} \]

Decidable: simulate \( M \) on \( w \) for up to \( k \) steps; if it halts, accept; if it hasn’t halted, accept.

\[ B = \{ \langle M, k \rangle \mid M \text{ describes a Turing Machine, } k \text{ is a positive integer, and there exists a string } w \in \Sigma^* \text{ on which } M \text{ runs for } \geq k \text{ steps} \} \]

Decidable: simulate \( M \) for up to \( k \) steps on all strings up to length \( k \) (if length > \( k \) matters, must take \( \geq k \) steps on some length \( k \) string.)

\[ C = \{ \langle M, w \rangle \mid M \text{ describes a Turing Machine, } w \in \Sigma^* \text{ and there exists some positive integer } k \text{ where } M \text{ running on } w \text{ finishes in } \leq k \text{ steps} \} \]

Undecidable: \( C \) is the same as HALTS
Buffer
Undecidable Propositions 8

Prove that there exist languages that cannot be recognized by any Turing Machine.
Undecidable Propositions 8 - Answers

The number of languages is uncountable: proof by diagonalization.

The number of TMs is countable.
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Who was credited as the Mathematical Consultant for *Sneakers*?
Too Trivial 1

Leonard Adleman
Buffer
What is the name of MIT’s Lab for Computer Science D-league intramural hockey team?
Too Trivial 2

What is the name of MIT’s Lab for Computer Science D-league intramural hockey team?

Halting Problem
Buffer
Too Trivial 4

How many Turing Award winners have I had a meal with? (Tiebreaker: How many have I taken a class from?)
How many Turing Award winners have I had a meal with?

7-8: John Backus, Ed Clarke, Sir Tony Hoare, John Hopcroft, Butler Lampson, Barbara Liskov, Alan Kay, Ron Rivest

(Tiebreaker: How many have I taken a class from?)

3: Barbara Liskov, Marvin Minsky, Ron Rivest
Buffer
Too Trivial 8

Identify the source (author, paper, year) of each of these quotes:

A. “Namely, it would obviously mean that inspite of the undecidability of the Entscheidungsproblem, the mental work of a mathematician concerning Yes-or-No questions could be completely replaced by a machine.”

B. “It is my contention that these operations include all those which are used in the computation of a number. The defence of this contention will be easier when the theory of the machines is familiar to the reader.”

C. “It is shown that any recognition problem solved by a polynomial time-bounded nondeterministic Turing machine can be “reduced” to the problem of determining whether a given propositional formula is a tautology.”
Too Trivial 8

Identify the source (author, paper, year) of each of these quotes:

A. “Namely, it would obviously mean that inspite of the undecidability of the Entscheidungsproblem, the mental work of a mathematician concerning Yes-or-No questions could be completely replaced by a machine.”

Gödel’s 1956 Letter to von Neumann

B. “It is my contention that these operations include all those which are used in the computation of a number. The defence of this contention will be easier when the theory of the machines is familiar to the reader.”

Alan Turing, On Computable Numbers ..., 1936

C. “It is shown that any recognition problem solved by a polynomial time-bounded nondeterministic Turing machine can be “reduced” to the problem of determining whether a given propositional formula is a tautology.”

Stephen Cook, The Complexity of Theorem-Proving Procedures, 1971
Final Jeopardy:
Proving NP-Completeness
Final Jeopardy

Prove the language 4-SAT is NP-Complete.

$$4SAT = \{ \phi | \phi \text{ is a satisfiable formula in } 4\text{-cnf} \}$$

4-cnf = strings of the form

$$(\nu_{0,0} \lor \nu_{0,1} \lor \nu_{0,2} \lor \nu_{0,3}) \land \ldots \land (\nu_{n,0} \lor \nu_{n,1} \lor \nu_{n,2} \lor \nu_{n,3})$$

where $\nu_{i,j} \in \{ x_i | i \geq 0 \} \cup \{ \overline{x_i} | i \geq 0 \}$
Final Charge

• Don’t forget to show up for the final:
  Thursday, May 13, 9am-noon
  (studying for it would be a good idea too!)
• Do the course evaluations
  – Official University evaluation
  – Course-specific evaluation (will be posted on website)
• Work on big, important problems
  – Don’t spend your career making improvements that are hidden inside $\Theta$ notation!
How will P=NP be resolved?

Richard Karp: (Berkeley, unsure, P ⊂ NP)
My intuitive belief is that P is unequal to NP, but the only supporting arguments I can offer are the failure of all efforts to place specific NP-complete problems in P by constructing polynomial-time algorithms. I believe that the traditional proof techniques will not suffice. Something entirely novel will be required. **My hunch is that the problem will be solved by a young researcher who is not encumbered by too much conventional wisdom about how to attack the problem.**

The P=?NP Poll, William Gasarch
Answer

The easiest proof is a reduction from 3SAT to 4SAT. Suppose we have a polynomial-time decider for 4SAT, $M_{4SAT}$. We can use it to make a polynomial-time decider for 3SAT by adding a new variable to each clause and forcing it to be false:

$$3SAT(\phi) = \text{result of simulating } M_{4SAT}(\phi')$$

where

$$\phi' = (v_{0,0} \lor v_{0,1} \lor v_{0,2} \lor x_z)$$

$$\ldots$$

$$\land (v_{i_n,0} \lor v_{i_n,1} \lor v_{i_n,2} \lor x_z)$$

$$\land (\overline{x_z} \lor x_0 \lor x_0 \lor x_0)$$

$$\land (\overline{x_z} \lor \overline{x_0} \lor \overline{x_0} \lor \overline{x_0})$$
Thank you!

Handouts before you leave today:
Your PS6
PS6 Comments
Final Exam Preview