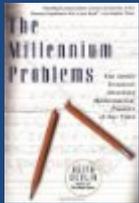
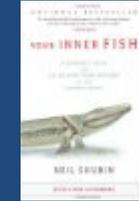


(Teams removed to preserve email address privacy)

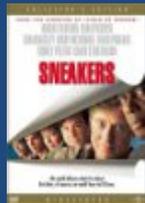
Prizes



The Millennium Problems: The
Seven Greatest Unsolved
Mathematical Puzzles Of Our
Time
Keith J. Devlin



Your Inner Fish: A Journey into the
3.5-Billion-Year History of the
Human Body (Vintage)
Neil Shubin



Sneakers



The Code Book: The Science of Secrecy from
Ancient Egypt to Quantum Cryptography
Simon Singh



Turing (A Novel about Computation)
Christos H. Papadimitriou



Logicomix: An Epic Search for Truth
Apostolos Doxiadis, Christos Papadimitriou

CS Theory Jeopardy

(All values in
\$Quadrillions)

Asymptotic Languages Operators Undecidable Propositions Questions too trivial to ask on the final

1	1	1	1
2	2	2	2
4	4	4	4
8	8	8	8

Final Jeopardy

Buffer

Buffer

Languages 1

What is a language?

Languages 1 - Answer

What is a language?

A set of strings.

Return

Buffer

Languages 2

Describe a language that is
context-free and *finite*.

Languages 2

Describe a language that is
context-free and *finite*.

Any finite language.

Return

Buffer

Languages 4

How many strings are in the language generated by this grammar?

$$S \rightarrow 0 \mid S$$

Languages 4 - Answer

1 $S \rightarrow 0 \mid S$

generates the language
 $\{0\}$

Return

Buffer

Languages 8

If A is in \mathbf{P} and B is in \mathbf{NP} which of the following *must* be true:

- (a) the complement of A is in \mathbf{P}
- (b) the complement of B is in \mathbf{NP}
- (c) $A \cap B$ is non-empty

Languages 8

If A is in \mathbf{P} and B is in \mathbf{NP} which of the following *must* be true:

(a) the complement of A is in \mathbf{P}

True. Simulate M_A and flip result.

(b) the complement of B is in \mathbf{NP}

Unknown! (False unless $\mathbf{P}=\mathbf{NP}$)

(c) $A \cap B$ is non-empty

False. e.g., $A = \{0, 1\}^*$, $B = 0^*$

Return

Buffer

Asymptotic Operators 1

Which of these functions are in $O(2^n)$?

$$a(n) = 2n+3$$

$$b(n) = n!$$

$$c(n) = n2^n$$

$$d(n) = BB(7,2)$$

Asymptotic Operators 1 - Answer

Which of these functions are in $O(2^n)$?

$$a(n) = 2n+3$$

~~$$b(n) = n!$$~~

Oops! I was actually wrong on this originally, sorry to the teams that lost points.

$n!$ grows faster than $2n$ so is not in $O(2^n)$

~~$$c(n) = n2^n$$~~

$$d(n) = BB(7,2)$$

Return

Buffer

Asymptotic Operators 2

What is a tight bound (Θ) of the worst-case asymptotic running time of this Java procedure?

```
int findMatch(int a[], int x) {  
    for (int i = 0; i < a.length; i++)  
        if (a[i] == x) return i;  
    return -1; // not found  
}
```

Asymptotic Operators 2 - Answer

What is a tight bound (Θ) of the worst-case asymptotic running time of this Java procedure?

```
int findMatch(int a[], int x) {  
    for (int i = 0; i < a.length; i++)  
        if (a[i] == x) return i;  
    return -1; // not found  
}
```

Since Java arrays and integers have bounded size, the running time is in $\Theta(1)$.

Return

Buffer

Asymptotic Operators 4

What is a tight bound (Θ) of the worst-case asymptotic running time of this Python procedure?

```
def findMatch(a, x):  
    for i in range(0, len(a)):  
        if (a[i] == x): return i  
    return -1 # not found
```

Note: do not assume == has constant time!

Asymptotic Operators 4

```
def findMatch(a, x):  
    for i in range(0, len(a)):  
        if (a[i] == x): return i  
    return -1 # not found
```

Worst case:

a is m elements, b bits each

x is b bits

$$n = (m+1)b \approx mb$$

$$\text{Loop iterations} = m \approx n/b$$

$$\text{Work/iteration} = b \text{ (each ==)}$$

The worst case running time is in $\Theta(n)$

Return

Buffer

Asymptotic Operators 8

Order these from **smallest** to **largest**:

- A. $BB(6,2)$ (maximum number of steps a 6-state, 2-symbol TM can make before halting)
- B. Cost (in dollars) to sequence a human genome today.
- C. Cost (in dollars) to sequence a human genome in 2007.
- D. 2^{1000}
- E. 10000^{20}
- F. $20!$
- G. US Deficit (in dollars)



Write

Asymptotic Operators 8 - Answer

Return

Order these from **smallest** to **largest**:

- A. $BB(6,2) > 10^{2879}$
- B. Cost (in dollars) to sequence a human genome today \sim \$5000
- C. Cost (in dollars) to sequence a human genome in 2007 \sim \$57M
- D. $2^{1000} \sim 10^{300}$
- E. $10000^{20} = 10^{80}$
- F. $20! \sim 10^{18}$
- G. US National Debt (in dollars) \sim \$12.9T $\sim 10^{13}$

B << C << G << F << E << D << A

Buffer

Return

Buffer

Undecidable Propositions 1

What is the meaning and correct
pronunciation of
“Entscheidungsproblem”?

Undecidable Propositions 1 - Answer

What is the meaning and correct pronunciation of “*Entscheidungsproblem*”?

Decision problem.

Input: A mathematical statement.

Output: True if the statement is true, false otherwise.

Return

Buffer

Undecidable Propositions 2

Define an undecidable language that contains only even-length strings.

Undecidable Propositions 2 - Answer

Define an undecidable language that contains only even-length strings.

$$\text{HALTSEVEN}_{TM} = \{ \langle M, w, z \rangle \mid M \text{ describes a Turing Machine, } M \text{ halts on input } w, z \in \{\epsilon, 0\} \text{ and } |\langle M, w, z \rangle| \text{ is even} \}$$

Return

Buffer

Undecidable Propositions 4

Which of these languages are *decidable*?

$A = \{ \langle M, w, k \rangle \mid M \text{ describes a Turing Machine, } w \in \Sigma^*, k \text{ is a positive integer, and } M \text{ runs for } \geq k \text{ steps on input } w \}$

$B = \{ \langle M, k \rangle \mid M \text{ describes a Turing Machine, } k \text{ is a positive integer, and there exists a string } w \in \Sigma^* \text{ on which } M \text{ runs for } \geq k \text{ steps } \}$

$C = \{ \langle M, w \rangle \mid M \text{ describes a Turing Machine, } w \in \Sigma^* \text{ and there exists some positive integer } k \text{ where } M \text{ running on } w \text{ finishes in } \leq k \text{ steps } \}$

Undecidable Propositions 4 - Answer

$A = \{ \langle M, w, k \rangle \mid M \text{ describes a Turing Machine, } w \in \Sigma^*, k \text{ is a positive integer, and } M \text{ runs for } \geq k \text{ steps on input } w \}$

Decidable: simulate M on w for up to k steps;
if it halts, accepts; if it hasn't halted, accept.

$B = \{ \langle M, k \rangle \mid M \text{ describes a Turing Machine, } k \text{ is a positive integer, and there exists a string } w \in \Sigma^* \text{ on which } M \text{ runs for } \geq k \text{ steps} \}$

Decidable: simulate M for up to k steps on all strings up to length k
(if length $> k$ matters, must take $\geq k$ steps on some length k string.)

$C = \{ \langle M, w \rangle \mid M \text{ describes a Turing Machine, } w \in \Sigma^* \text{ and there exists some positive integer } k \text{ where } M \text{ running on } w \text{ finishes in } \leq k \text{ steps} \}$

Undecidable: C is the same as HALTS

Return

Buffer

Undecidable Propositions 8

Prove that there exist languages that cannot be *recognized* by any Turing Machine.



Write

Undecidable Propositions 8 - Answers

The number of languages is uncountable:
proof by diagonalization.

The number of TMs is countable.

[Return](#)

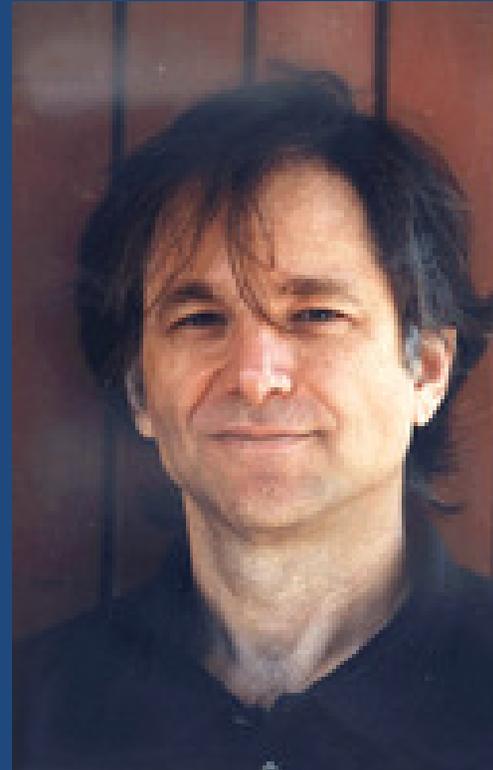
Buffer

Too Trivial 1

Who was credited as
the Mathematical
Consultant for
Sneakers?

Too Trivial 1

Leonard
Adleman

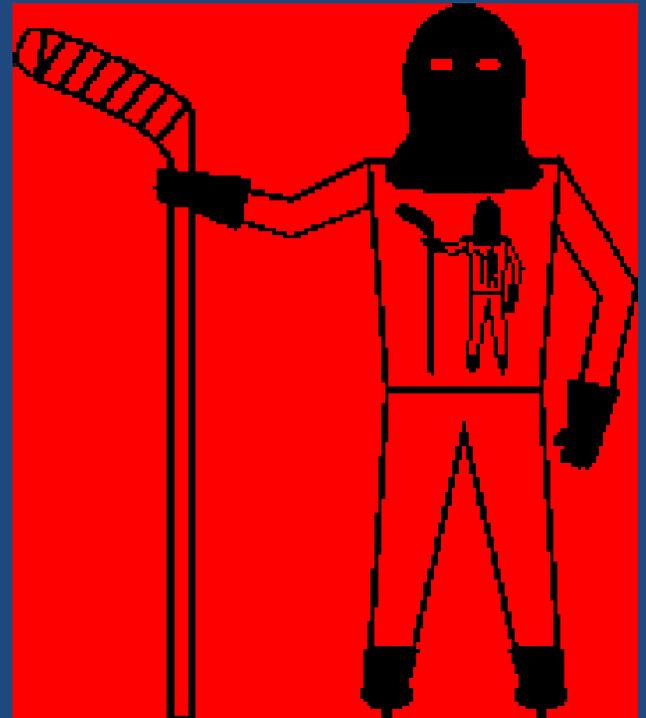


[Return](#)

Buffer

Too Trivial 2

What is the name of MIT's Lab for Computer Science D-league intramural hockey team?



Too Trivial 2

What is the name of MIT's
Lab for Computer Science
D-league intramural
hockey team?

Halting Problem

Return

Buffer

Too Trivial 4

How many Turing Award winners have I had a meal with?
(Tiebreaker: How many have I taken a class from?)

Too Trivial 4

How many Turing Award winners have I had a meal with?

7-8: John Backus, Ed Clarke, Sir Tony Hoare, John Hopcroft, Butler Lampson, Barbara Liskov, Alan Kay, Ron Rivest

(Tiebreaker: How many have I taken a class from?)

3: Barbara Liskov, Marvin Minsky, Ron Rivest

[Return](#)

Buffer

Too Trivial 8

Identify the source (author, paper, year) of each of these quotes:

- A. “Namely, it would obviously mean that inspite of the undecidability of the Entscheidungsproblem, the mental work of a mathematician concerning Yes-or-No questions could be completely replaced by a machine.”
- B. “It is my contention that these operations include all those which are used in the computation of a number. The defence of this contention will be easier when the theory of the machines is familiar to the reader.”
- C. “It is shown that any recognition problem solved by a polynomial time-bounded nondeterministic Turing machine can be “reduced” to the problem of determining whether a given propositional formula is a tautology.”



Write

Too Trivial 8

Identify the source (author, paper, year) of each of these quotes:

- A. “Namely, it would obviously mean that inspite of the undecidability of the Entscheidungsproblem, the mental work of a mathematician concerning Yes-or-No questions could be completely replaced by a machine.” [Gödel’s 1956 Letter to von Neumann](#)
- B. “It is my contention that these operations include all those which are used in the computation of a number. The defence of this contention will be easier when the theory of the machines is familiar to the reader.” [Alan Turing, *On Computable Numbers ...*, 1936](#)
- C. “It is shown that any recognition problem solved by a polynomial time-bounded nondeterministic Turing machine can be “reduced” to the problem of determining whether a given propositional formula is a tautology.”

[Stephen Cook, *The Complexity of Theorem-Proving Procedures*, 1971](#)

Buffer

Final Jeopardy:
Proving NP-Completeness

Final Jeopardy

Prove the language 4-SAT is NP-Complete.

$4SAT = \{ \phi \mid \phi \text{ is a satisfiable formula in 4-cnf} \}$

4-cnf = strings of the form

$$(v_{0,0} \vee v_{0,1} \vee v_{0,2} \vee v_{0,3}) \wedge \dots \wedge (v_{i_n,0} \vee v_{i_n,1} \vee v_{i_n,2} \vee v_{i_n,3})$$

where $v_{i,j} \in \{x_i \mid i \geq 0\} \cup \{\bar{x}_i \mid i \geq 0\}$

Final Charge

- Don't forget to show up for the final:
Thursday, May 13, 9am-noon
(studying for it would be a good idea too!)
- Do the course evaluations
 - Official University evaluation
 - Course-specific evaluation (will be posted on website)
- **Work on big, important problems**
 - Don't spend your career making improvements that are hidden inside Θ notation!

How will $P=NP$ be resolved?

Richard Karp: (Berkeley, unsure, $P \subset NP$)

My intuitive belief is that P is unequal to NP , but the only supporting arguments I can offer are the failure of all efforts to place specific NP -complete problems in P by constructing polynomial-time algorithms. I believe that the traditional proof techniques will not suffice. Something entirely novel will be required. **My hunch is that the problem will be solved by a young researcher who is not encumbered by too much conventional wisdom about how to attack the problem.**

The $P=?NP$ Poll, William Gasarch

Answer

The easiest proof is a reduction from 3SAT to 4SAT. Suppose we have a polynomial-time decider for 4SAT, M_{4SAT} . We can use it to make a polynomial-time decider for 3SAT by adding a new variable to each clause and forcing it to be *false*:

$3SAT(\phi) = \text{result of simulating } M_{4SAT}(\phi')$ where

$$\begin{aligned}\phi' &= (v_{0,0} \vee v_{0,1} \vee v_{0,2} \vee x_z) \\ &\quad \wedge \dots \\ &\quad \wedge (v_{i_n,0} \vee v_{i_n,1} \vee v_{i_n,2} \vee x_z) \\ &\quad \wedge (\overline{x_z} \vee x_0 \vee x_0 \vee x_0) \\ &\quad \wedge (\overline{x_z} \vee \overline{x_0} \vee \overline{x_0} \vee \overline{x_0})\end{aligned}$$

Thank you!

Handouts before you leave today:

Your PS6

PS6 Comments

Final Exam Preview