CS6501: Great Works in Computer Science

Presented by Longze Chen
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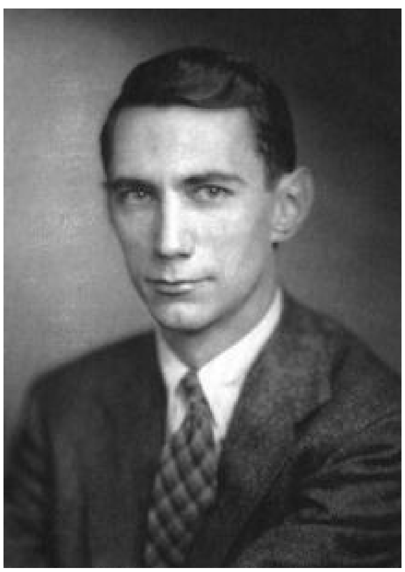
Communication Theory of Secrecy Systems (C. E. Shannon, 1949)

A Mathematical Theory of Cryptography (C. E. Shannon, 1946)

Claude Elwood Shannon (1916 - 2001)

The Father of Information Theory

*

Boolean Theory

- A Symbolic Analysis of Relay and Switching Circuits (1937)
- An Algebra for Theoretical Genetics (1940)

Cryptography

- A Mathematical Theory of Cryptography (1946)
- Communication Theory of Secrecy Systems (1949)

Information Thoery

- A Mathematical Theory of Communication (1948)

Secrecy Systems

- Schematic of A General Secrecy System
  - $E = f(M, K)$
  - $E = T_i M$

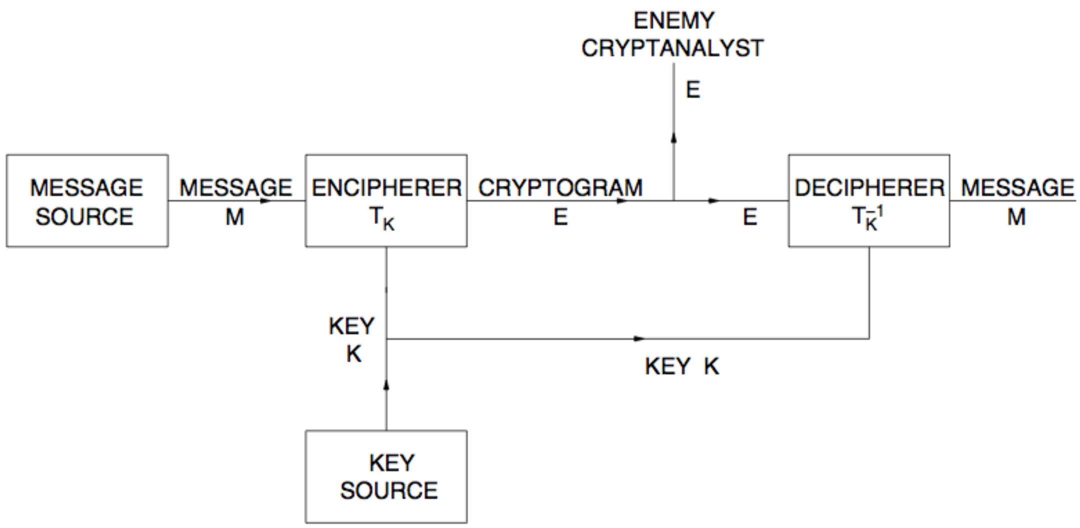


Fig. 1. Schematic of a general secrecy system

- Definition of Secrecy Systems
  - A Secrecy System is a family of uniquely reversible transformations T_i of a set of possible messages into a set of cryptograms, the transformation T_i having an associated probability p_i.
  - A set of transformations with associated probabilities
  - Domain and Range
  - More on the definition

- Threat Model
    - The enemy knows the system being used. (Shannon' Maxim)
        - Objection

- Deciphering vs Cryptanalysis

- Representation of Secrecy Systems
    - Line diagram

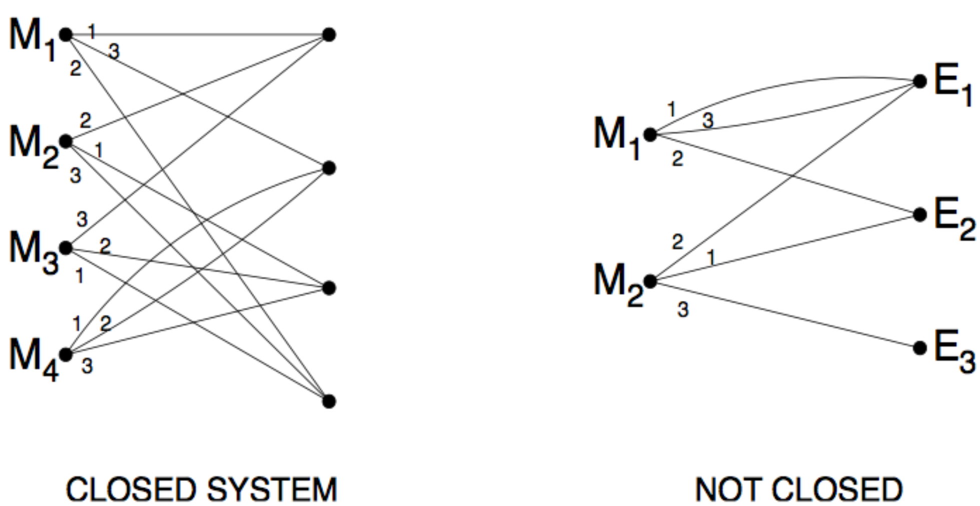


Fig. 2. Line drawings for simple systems

- Closed system

Examples of Secrecy Systems

- Substitution
  - Simple Substitution
    - Key
    - wklv phvvdjh lv qrw wrr kdug wr euhdn
  - Vigenère
    - Degree
    - $e_i = m_i + k_i \ (mod\ 26)$
- Transposition
  - Columnar Transposition
- Combination
- One-time Pads
  - Unbreakable if used correctly / Information-theoretically secure
    - Perfect Secrecy
  - Problems
    - True randomness
    - Key size
    - Synchronization
  - Vernam Cipher

Characteristics of a Good Cryptosystem

- Shannon's Criteria
  - Amount of Secrecy
    - Perfect
    - Not Perfect but never yield unique solution
    - Not Perfect and yield unique solution, but the amount of effort varies
  - Size of Key
  - Complexity of Enciphering and Deciphering Operations
  - Propagation of Errors
  - Expansion of Messages

- Are these criteria still reasonable?

- Anything else?

Mathematical Structure of Secrecy Systems

- Secrecy System

- Combination
  - Weighted Sum
  - Product

- Properties
  - Associative?
  - Distributive?
  - Commutative?
  - Endomorphic?

Pure Cipher

- Homogenenity
    - Group property

- Unrefined Defination
    - T forms a group
    - Endomorphic

- Proper Definination
    - A cipher T is pure if for every T_i, T_j, T_k there is a T_s such that $T_i T_j^{-1} T_k = T_s$, and every key is equally likely. Otherwise the cipher is mixed.

- Property
    - **Theorem 1**
    - **Theorem 2**
    - **Theorem 3**
    - **Theorem 4**

Perfect Secrecy

- Questions:
  - How immune a system is when the cryptanalyst has unlimited time and manpower available for the analysis of cryptograms?

- Natural Definition of Perfect Secrecy
  - It is natural to define perfect secrecy by the condition that, for all E the *a posteriori* probabilities are equal to the *a priori* probabilities independent of the value of these.

- **Theorem 6**
  - *A necessary and sufficient condition for perfect secrecy is that* $P_M(E) = P(E)$ *for all* M *and* E*. That is,* $P_M(E)$ *must be independent of* M*.*

- Important relationship between keys and messages

General Idea of Ideal Secrecy

- Problem with Perfect Secrecy
  - Key size

- Entropy and Equivocation
  - $H(M)$ and $H(K)$
  - $H_E(M)$ and $H_E(K)$

- Properties of Equivocation

- Definition of Ideal Secrecy
  - Ideal secrecy
    - $H_E(M)$ and $H_E(K)$ do not approach zero as $N \to \infty$.
  - Strongly ideal secrecy
    - $H_E(K)$ remains constant at $H_E(M)$.