Mathematics in Poker

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Mathematics in Poker

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“The Theory of Poker”
by David Sklansky (1987)

“Practical Poker Math”
by Pat Dittmar (2008)

- www.pokerjunkie.com
Pot Odds

• How much you can win to the cost for playing

• Example:

  $30 : in pot
  $10 : to call
  * Pot odds are “4 to 1”
Pot Odds

- Compare Pot Odds to your Winning Odds

- Example:

  If pot odds are “4 to 1”,
  - Call if your winning odds are better than “4 to 1” (20%)
  - Fold if your winning odds are worse than that.
Pot Odds with more cards to come

- One more card to come

Example:

* Pot odds: 5 to 1
- If you’re sure you can win with flush,
- The odds of making the flush are “4.22 to 1”
  (9 of the 47 unseen cards)
Pot Odds with more cards to come

• One more card to come

Example:
* If pot odds are “3 to 1”
  = FOLD
Effective Odds

- More than one card to come

Example:
In hold’em, you have four-flush after the flop.
* Pot odds: 3 to 1 ($20 in pot, $10 bet)

- Odds of making the flush (with two more cards): 1.8 to 1

= Call??
Effective Odds

- More than one card to come

Example:
In hold’em, you have four-flush after the flop.
- Odds of making the flush (with two more cards) : 1.8 to 1

If you miss your hand: Lose $10(this) + $20(next)
If you make your hand: Win $30 + $20(next)
* Pot odds: 5 to 3 = 1.67 to 1
Effective Odds

• More than one card to come

Example:
In hold’em, you have four-flush after the flop.
- Odds of making the flush (with two more cards):
  : 1.8 to 1

If you miss your hand: Lose $10(this) + $20(next)
If you make your hand: Win $50 + $20(opponent)
* Pot odds: 7 to 3 = 2.33 to 1
Implied Odds

- Based on the possibility of winning money in later betting rounds.
- Sometimes it’s okay to make a call even if the pot is too small to give you the correct pot odds.
- Cannot be calculated with certainty.
Bluffing Frequency

• If you never bluff = Never call your bet
• If you always bluff = Always call

• Find Optimum Bluffing Frequency.
Bluffing Frequency

- T = Pot size
- B = Bet size
- p = The probability that you have the best hand
- b = Your bluff frequency

When opponent calls your bet,
- Wins (T+B) if you’re on a bluff.
- Loses (B) if you have the best hand.

\[ EV(\text{opponent}) = b(1-p)(T+B) - pB \]
Bluffing Frequency

- $T =$ Pot size
- $B =$ Bet size
- $p =$ The probability that you have the best hand
- $b =$ Your bluff frequency

Optimal strategy:

$$\text{EV(opponent)} = 0$$

$$b(1-p)(T+B) - pB = 0$$

$$b = \left(\frac{p}{1-p}\right) * \left(\frac{B}{T+B}\right)$$
Bluffing Frequency

• Example:

- Bluffing frequencies for pot sized bets (B=T)

\[ b = \frac{1}{2} \times \frac{p}{1-p} \]

* if \( p = \frac{2}{3} \), then \( b = 1 \)
**Bluffing Frequency**

- **Example:**

- Bluffing frequencies for pot sized bets (B=T)

<table>
<thead>
<tr>
<th>p(%)</th>
<th>b(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>20</td>
<td>12</td>
</tr>
<tr>
<td>30</td>
<td>21</td>
</tr>
<tr>
<td>40</td>
<td>33</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>60</td>
<td>75</td>
</tr>
</tbody>
</table>
Bluffing Frequency

- David Sklansky (Theory of Poker):

  “Optimal bluffing strategy is to bluff in such a way that the chances against your bluffing equals his pot odds.”
Bluffing Frequency

• David Sklansky:

“Optimal bluffing strategy is to bluff in such a way that the chances against your bluffing equals his pot odds.”

* If you make pot-sized bet,

= Bluff half as much as you bet.
Bluffing Frequency

- Example:

- Bluffing frequencies for pot sized bets (B=T)

<table>
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<th>p(%)</th>
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Bet: 20 times / 100
Bluff:
*Sklansky: half of 20
  = 10 times
*table: 12% of 80 times
  = about 10 times
Bluff Equity

• Before folding, you should always consider bluffing.

• If you bluff equity is positive, you need to carry out the bluff.
Bluff Equity

- $P = \text{Pot size}$
- $B = \text{Bet size (your bluff bet)}$
- $f = \text{The probability that the opponent folds if you bet}$

When you bluff
- If opponent folds, you win $P$.
- If opponent calls, you lose your bet $B$.

So, expected value (EV) of your bluff is...

$$EV = fP - (1-f)B$$
Bluff Equity

- P = Pot size
- B = Bet size (your bluff bet)
- f = The probability that the opponent folds if you bet

- We want EV to be positive.

\[ fP - (1 - f)B > 0 \]

\[ f > B/(P+B) \]
Bluff Equity

- Example 1 - Pot sized bet
- \( P = B \)
- \( f > 1/2 \)

* If your opponent tends to fold more than 50% when you bet, a pot sized bluff earns you money.
Bluff Equity

• Example 2 - Smaller Bet

• $P = 3B$

• $f > 1/4$

* If your opponent tends to fold more than 25% when you bet (usually calls your bet), a small bluff will earn you money.
Example 3 - Bluff bet size limit and the opponent folding probability

\[ B < P \frac{f}{1 - f} \]

\[ B_{\text{max}} = P \frac{f}{1 - f} = P \frac{1}{\frac{1}{f} - 1} \]

So bluff bet size increases as folding probability increases
Bluff Equity

• Limitations

• Too simplified.

• $f$ is actually a function of $B$ and $P$. And the function is hard to know.

• Typically, bigger bets are harder to call.