



# Scaling Secure Computation

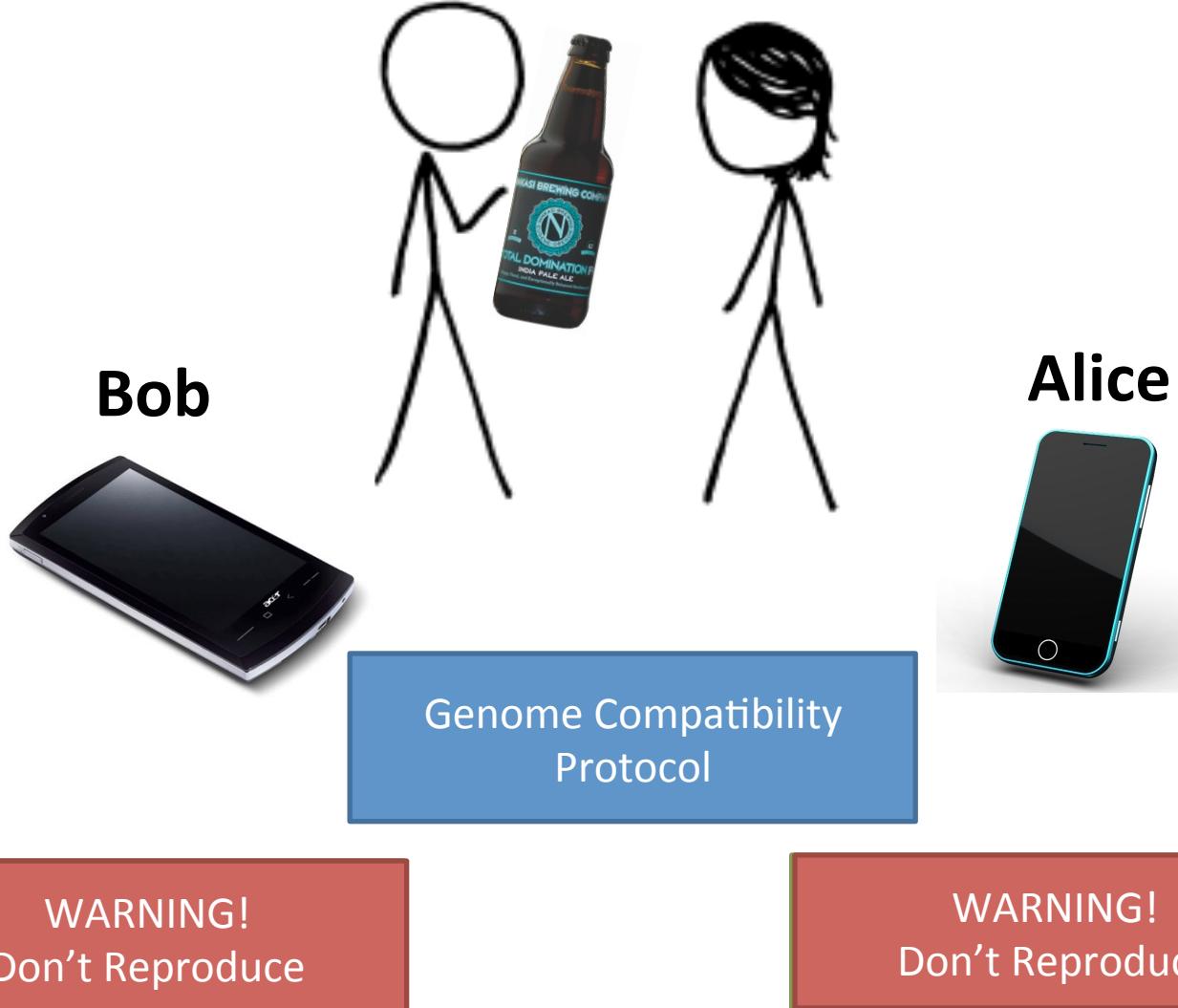
Oregon Security Day  
5 April 2013

David Evans

University of Virginia

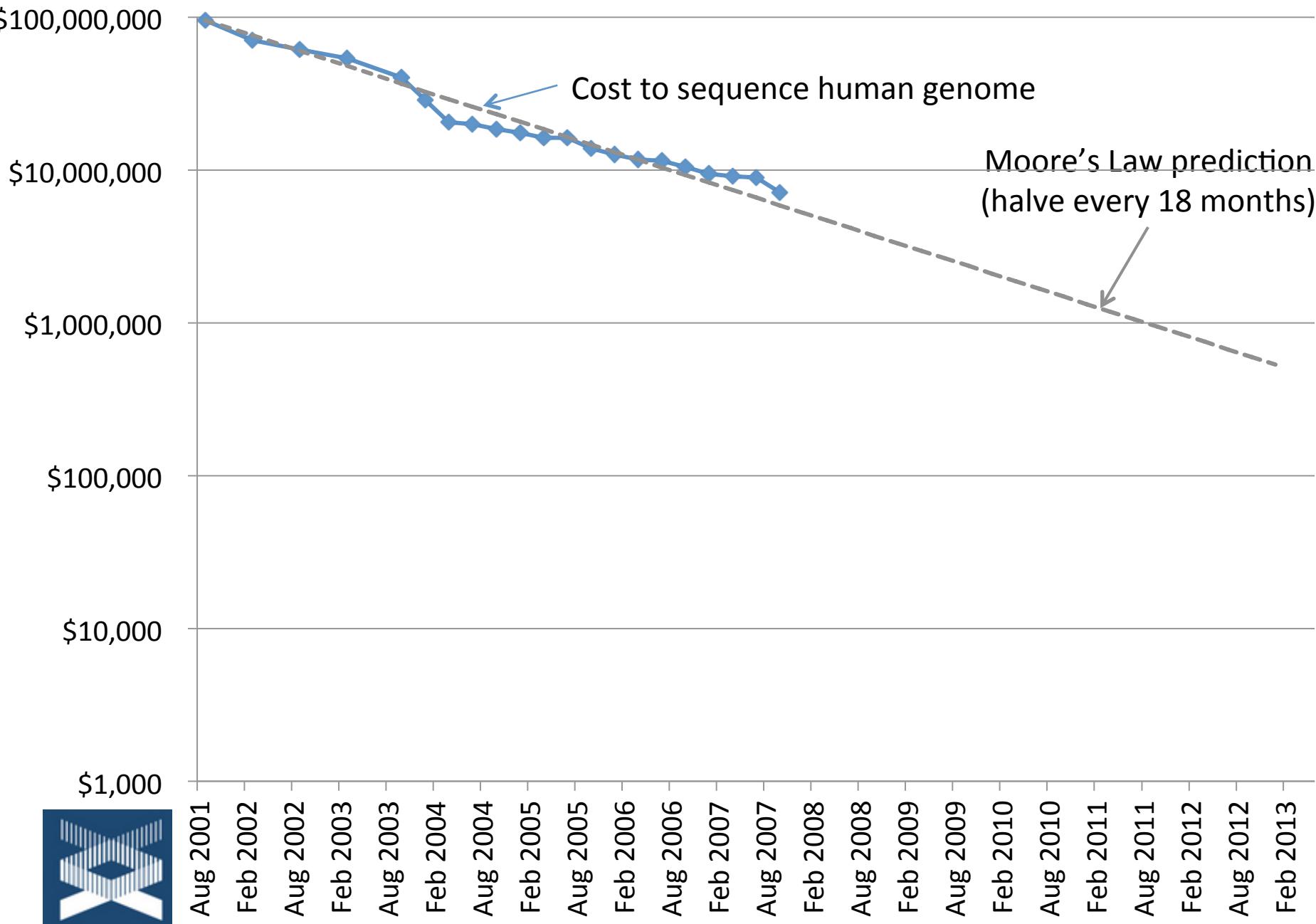
<http://www.cs.virginia.edu/evans>  
<http://MightBeEvil.com>

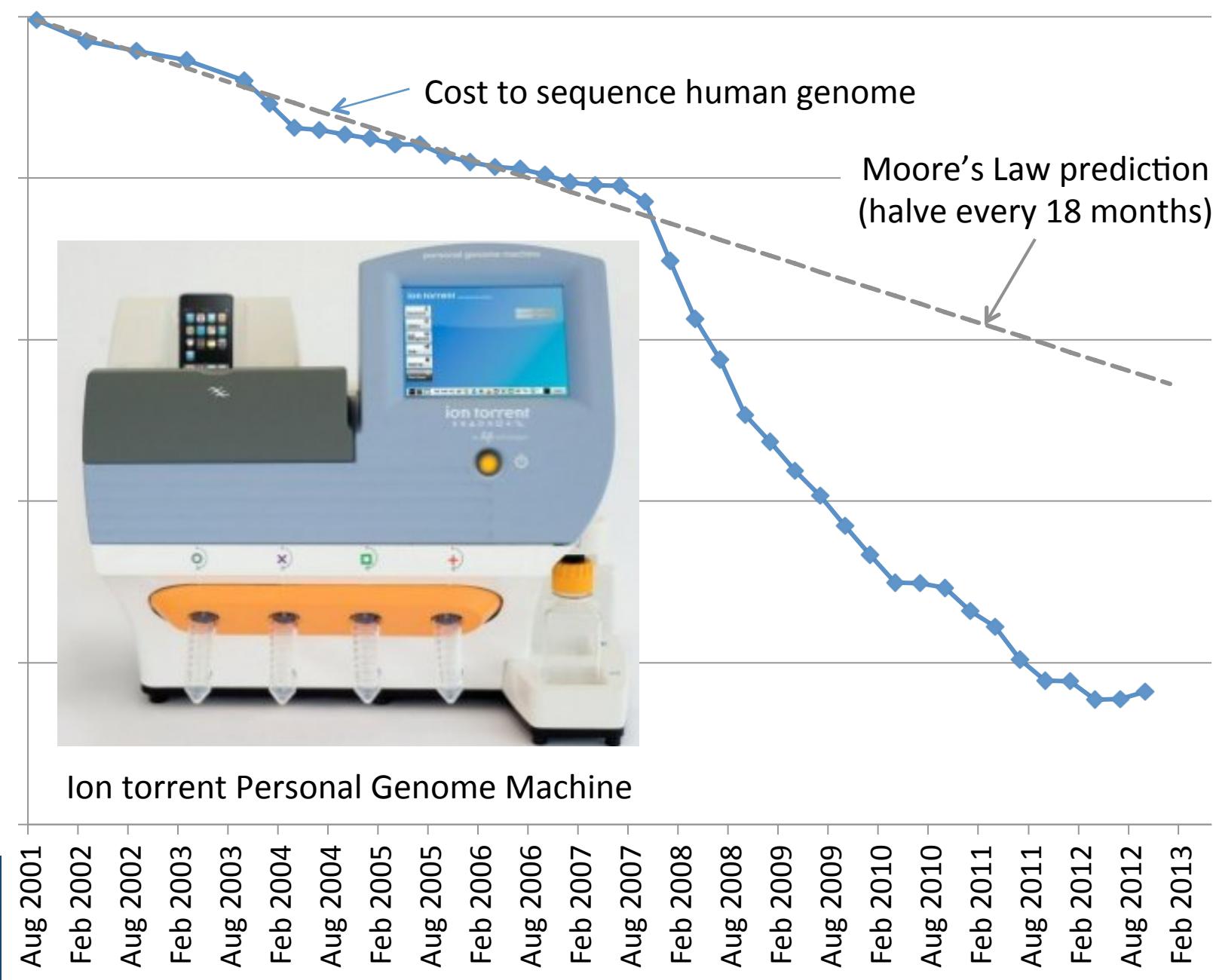
# (De)Motivating Application: “Genetic Dating”





genome.gov/sequencingcosts

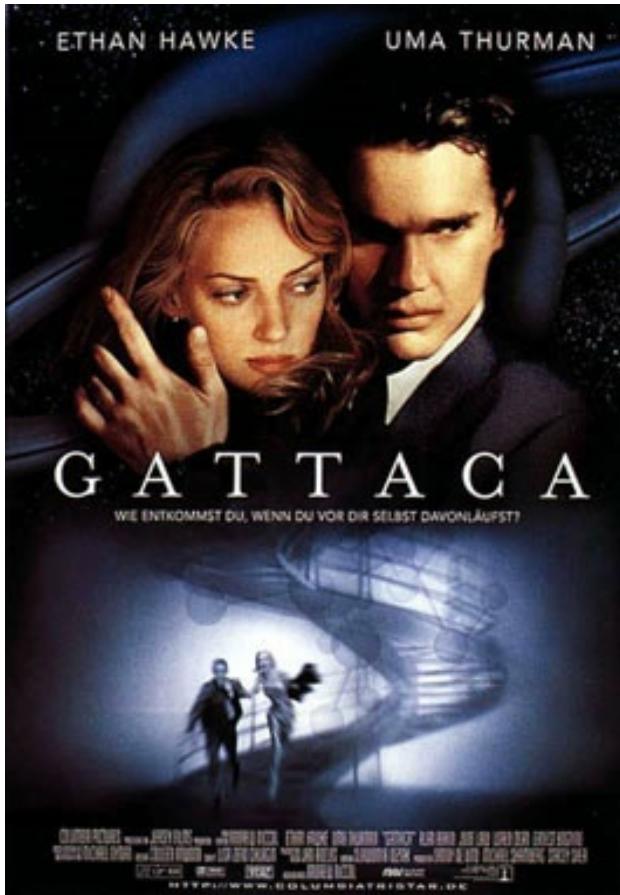




Year	reference	Technology	Sample	Average Reported Coverage depth (fold)	Reported sequencing consumables cost	Estimated cost per 40-fold coverage
	S4	Sanger (ABI)	JCV	7	\$10,000,000	\$57,000,000
	S5	Roche(454)	JDW	7	\$1,000,000	\$5,700,000
	S6	Illumina	NA18507	30	\$250,000	\$330,000
	S7	Helicos	SRQ	28	\$48,000	\$69,000
2009	this work	this work	NA07022	87	\$8,005	\$3,700
2009	this work	this work	NA19240	63	\$3,451	\$2,200
2009	this work	this work	NA20431	45	\$1,726	\$1,500

**Human Genome Sequencing Using Unchained Base Reads on Self-Assembling DNA Nanoarrays.** Radoje Drmanac, Andrew B. Sparks, Matthew J. Callow, Aaron L. Halpern, Norman L. Burns, Bahram G. Kermani, Paolo Carnevali, Igor Nazarenko, Geoffrey B. Nilsen, George Yeung, Fredrik Dahl, Andres Fernandez, Bryan Staker, Krishna P. Pant, Jonathan Baccash, Adam P. Borcherding, Anushka Brownley, Ryan Cedeno, Linsu Chen, Dan Chernikoff, Alex Cheung, Razvan Chirita, Benjamin Curson, Jessica C. Ebert, Coleen R. Hacker, Robert Hartlage, Brian Hauser, Steve Huang, Yuan Jiang, Vitali Karpinchyk, Mark Koenig, Calvin Kong, Tom Landers, Catherine Le, Jia Liu, Celeste E. McBride, Matt Morenzoni, Robert E. Morey, Karl Mutch, Helena Perazich, Kimberly Perry, Brock A. Peters, Joe Peterson, Charit L. Pethiyagoda, Kaliprasad Pothuraju, Claudia Richter, Abraham M. Rosenbaum, Shaunak Roy, Jay Shafto, Uladzislau Sharahovich, Karen W. Shannon, Conrad G. Sheppy, Michel Sun, Joseph V. Thakuria, Anne Tran, Dylan Vu, Alexander Wait Zaranek, Xiaodi Wu, Snezana Drmanac, Arnold R. Oliphant, William C. Banyai, Bruce Martin, Dennis G. Ballinger, George M. Church, Clifford A. Reid. ***Science***, January 2010.

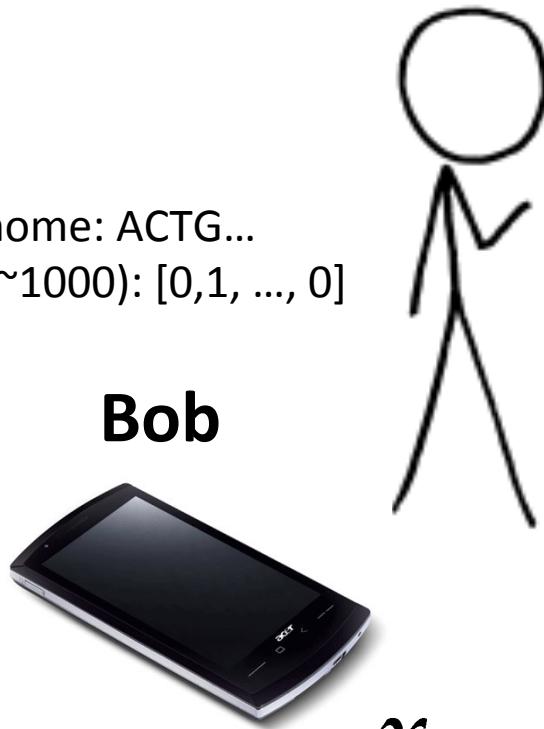
# Dystopia



Personalized Medicine

# Secure Two-Party Computation

Bob's Genome: ACTG...  
Markers (~1000): [0,1, ..., 0]



Alice's Genome: ACTG...  
Markers (~1000): [0, 0, ..., 1]



$$x = f(g_A, g_B)$$

Can Alice and Bob compute a function of their private data,  
without exposing anything about their data besides the result?

# Secure Function Evaluation

Alice (circuit generator)

Picks  $a \in \{0, 1\}^s$

Bob (circuit evaluator)

Picks  $b \in \{0, 1\}^t$

Agree on

$$f(a, b) \rightarrow x$$

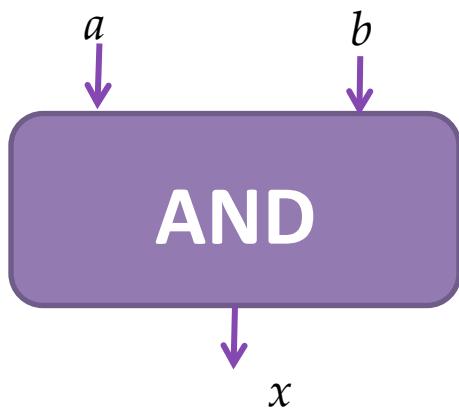
Garbled Circuit Protocol

Outputs  $x = f(a, b)$   
without revealing  $a$   
to Bob or  $b$  to Alice.

Andrew Yao, 1982/1986

# Regular Logic

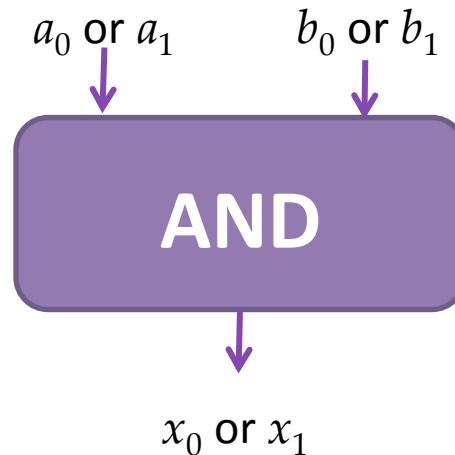
Inputs		Output
$a$	$b$	$x$
0	0	0
0	1	0
1	0	0
1	1	1



# Computing with Meaningless Values?

Inputs		Output
$a$	$b$	$x$
$a_0$	$b_0$	$x_0$
$a_0$	$b_1$	$x_0$
$a_1$	$b_0$	$x_0$
$a_1$	$b_1$	$x_1$

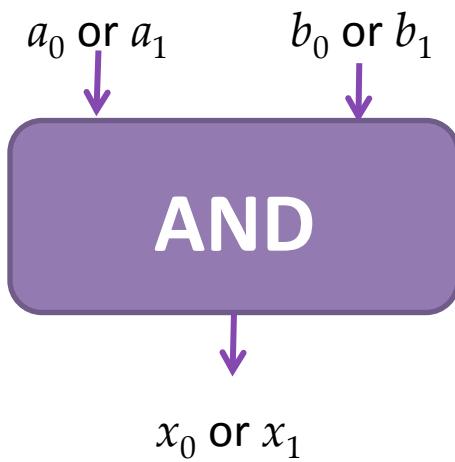
$a_i, b_i, x_i$  are **random** values, chosen by the **circuit generator** but **meaningless** to the **circuit evaluator**.



# Computing with Garbled Tables

Inputs		Output
$a$	$b$	$x$
$a_0$	$b_0$	$Enc_{a_0,b_0}(x_0)$
$a_0$	$b_1$	$Enc_{a_0,b_1}(x_0)$
$a_1$	$b_0$	$Enc_{a_1,b_0}(x_0)$
$a_1$	$b_1$	$Enc_{a_1,b_1}(x_1)$

Bob can only decrypt  
one of these!



Garbled And Gate
$Enc_{a_0,b_1}(x_0)$
$Enc_{a_1,b_1}(x_1)$
$Enc_{a_1,b_0}(x_0)$
$Enc_{a_0,b_0}(x_0)$



Random  
Permutation

# Garbled Circuit Protocol

**Alice (circuit generator)**

Creates random keys:  $a_0, a_1, b_0, b_1, x_0, x_1$

Garbled Gate
$Enc_{a_0, b_1}(x_0)$
$Enc_{a_1, b_1}(x_1)$
$Enc_{a_1, b_0}(x_0)$
$Enc_{a_0, b_0}(x_0)$

**Bob (circuit evaluator)**

Sends  $a_i$  to Bob  
based on her input  
value

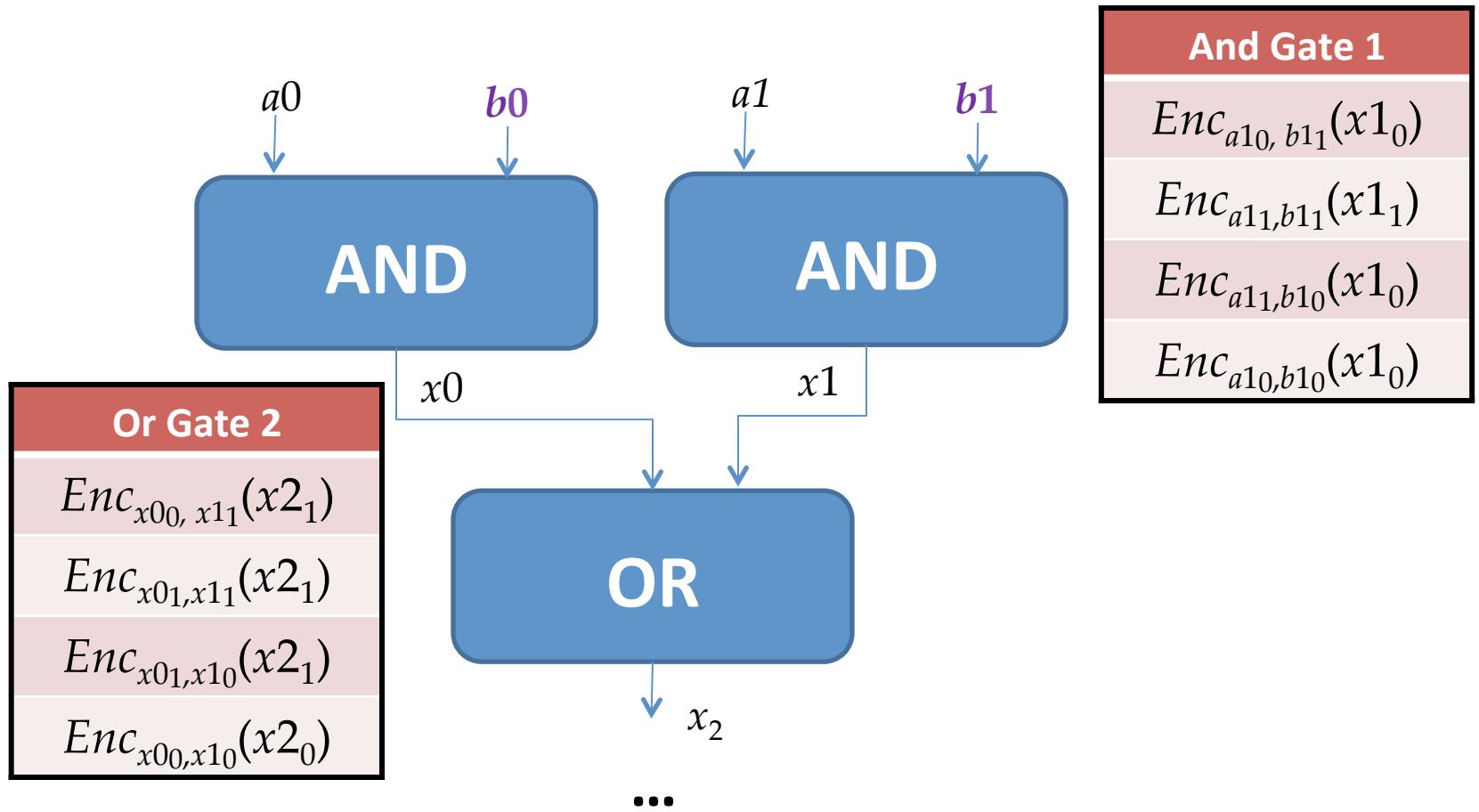
How does the Bob learn his own input wires?

# Primitive: Oblivious Transfer



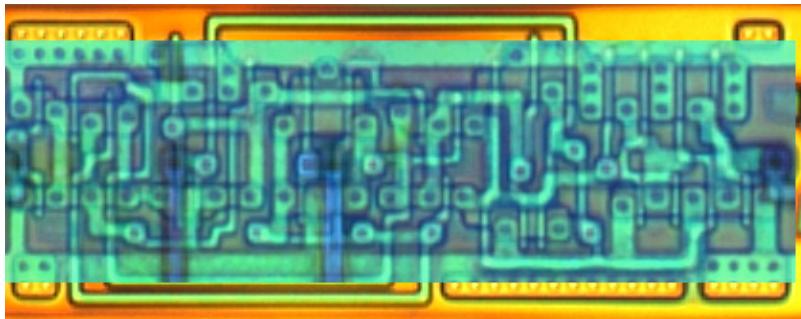
**Oblivious:** Alice doesn't learn which secret Bob obtains  
**Transfer:** Bob learns one of Alice's secrets

# Chaining Garbled Circuits



We can do *any* computation privately this way!

# Building Computing Systems


$$Enc_{x00, x11}(x2_1)$$
$$Enc_{x01, x11}(x2_1)$$
$$Enc_{x01, x10}(x2_1)$$
$$Enc_{x00, x10}(x2_0)$$

Digital Electronic Circuits	Garbled Circuits
Operate on <b>known data</b>	Operate on <b>encrypted wire labels</b>
One-bit logical operation requires moving a few electrons a few nanometers (hundreds of Billions per second)	One-bit logical operation requires performing (up to) 4 encryption operations: <b>very slow execution</b>
Reuse is great!	Reuse is not allowed for privacy: <b>huge circuits needed</b>

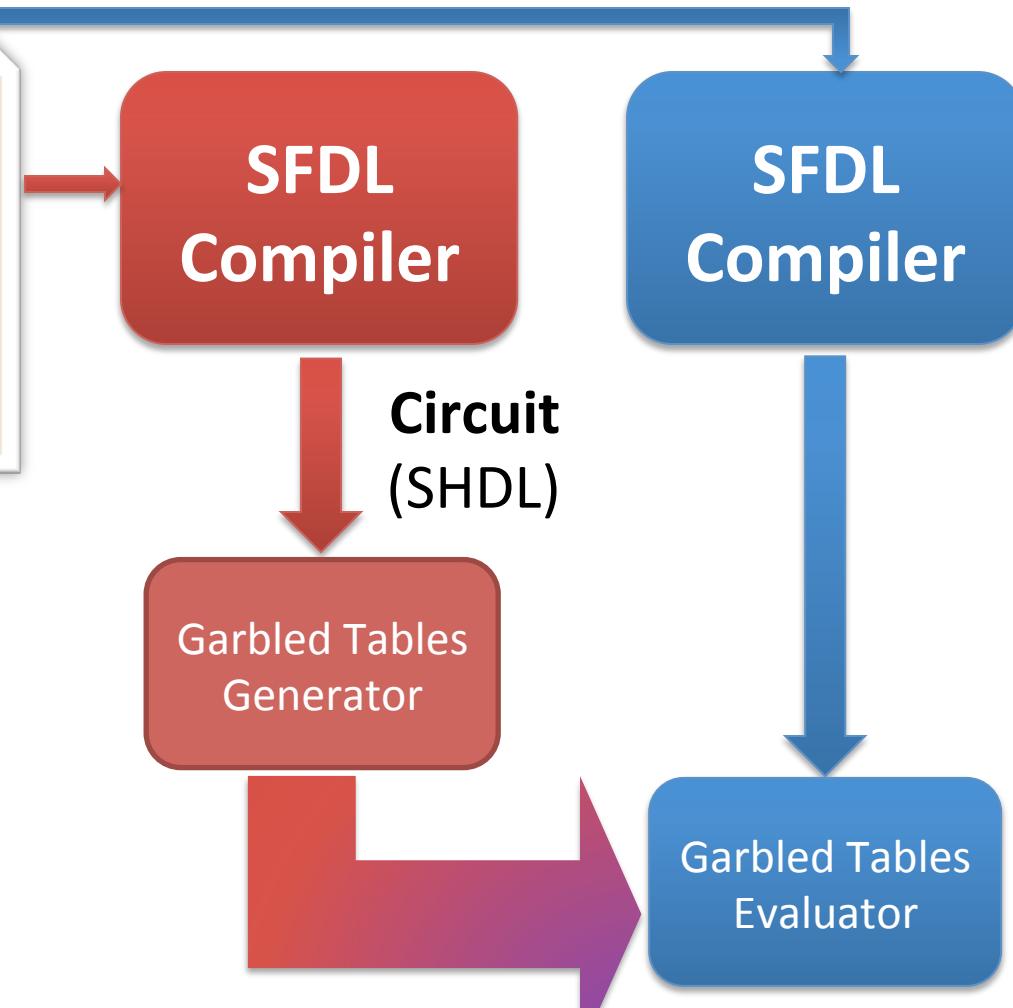
# Fairplay

## Alice

Bob

```
program Millionaires {  
    type int = Int<4>; // 4-bit integer  
    type AliceInput = int;  
    type BobInput = int;  
    type AliceOutput = Boolean;  
    type BobOutput = Boolean;  
    type Output = struct {  
        AliceOutput alice, BobOutput bob};  
    type Input = struct {  
        AliceInput alice, BobInput bob};  
  
    function Output out(Input inp) {  
        out.alice = inp.alice > inp.bob;  
        out.bob = inp.bob > inp.alice;  
    }  
}
```

SFDL Program



Dahlia Malkhi, Noam Nisan,  
Benny Pinkas and Yaron Sella  
[USENIX Sec 2004]

# Faster Circuit Execution



**Yan Huang**  
(UVa PhD Student)  
Graduate

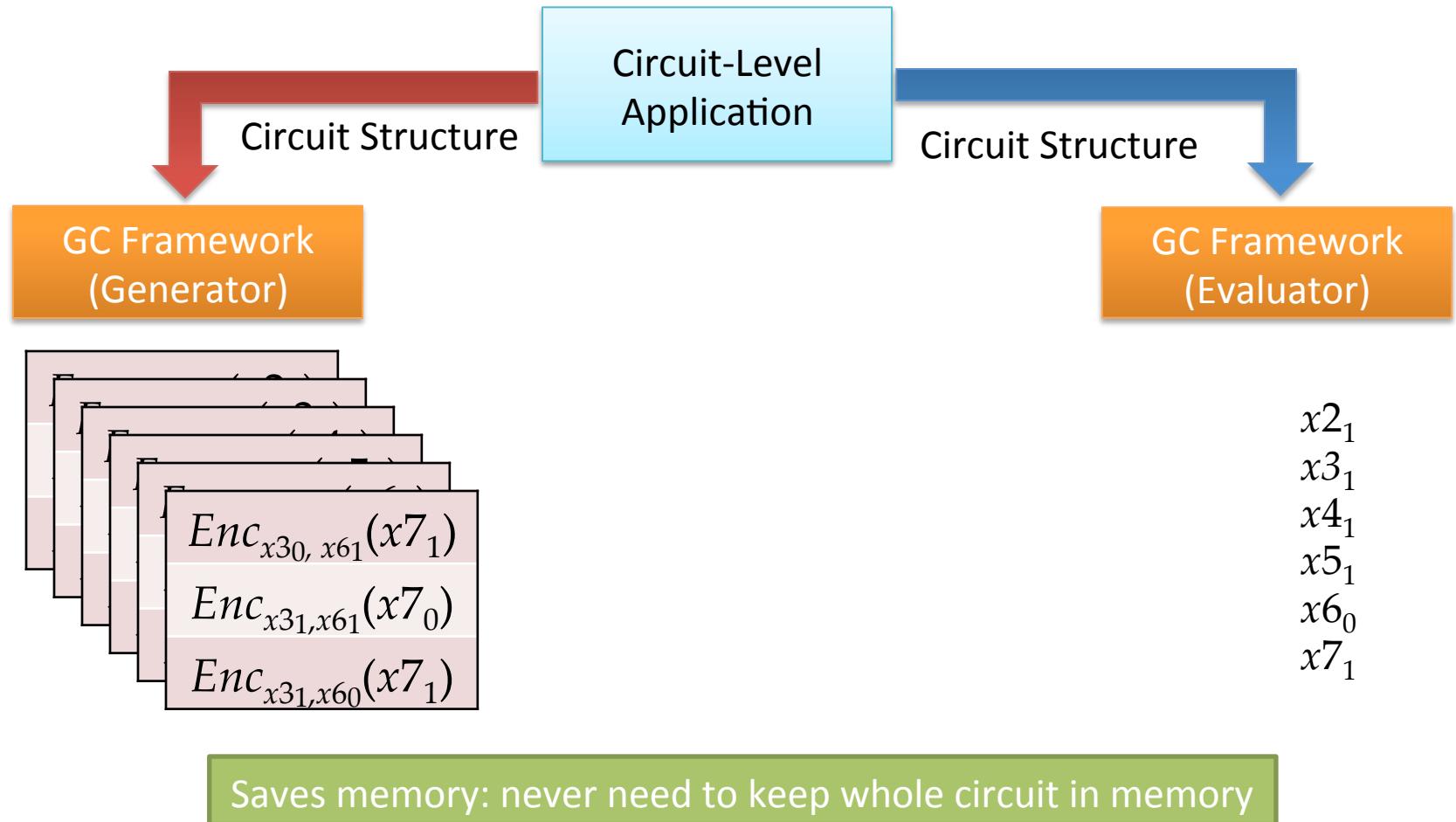
**Pipelined Execution**

**Optimized Circuit Library**

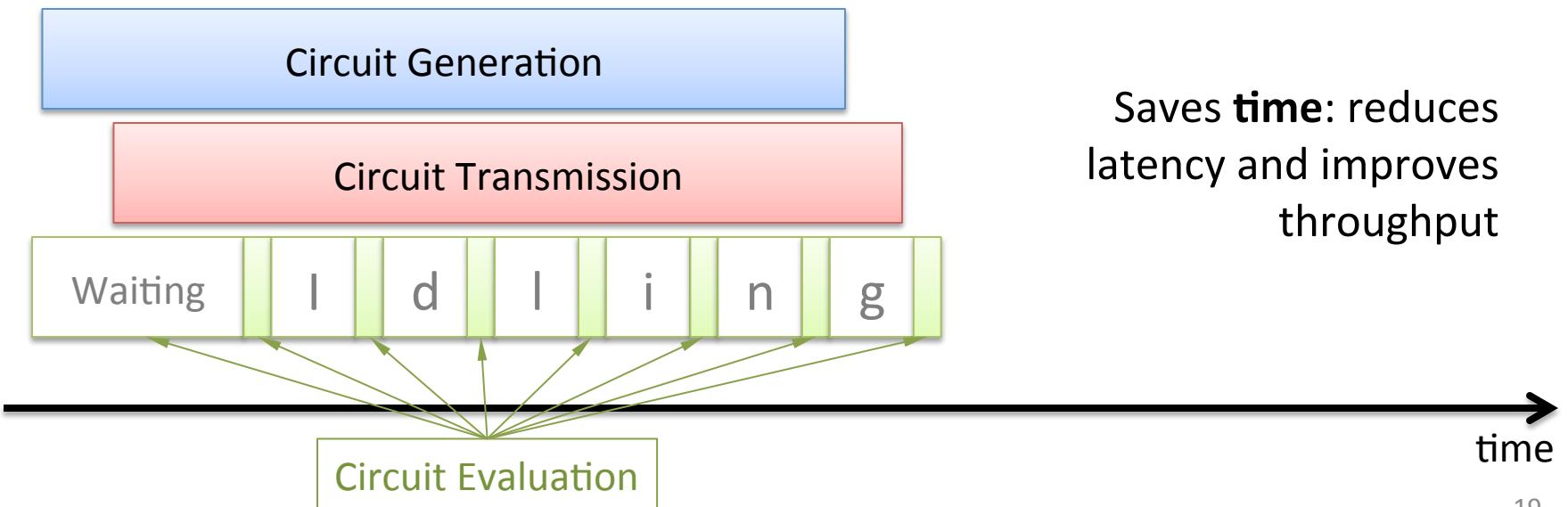
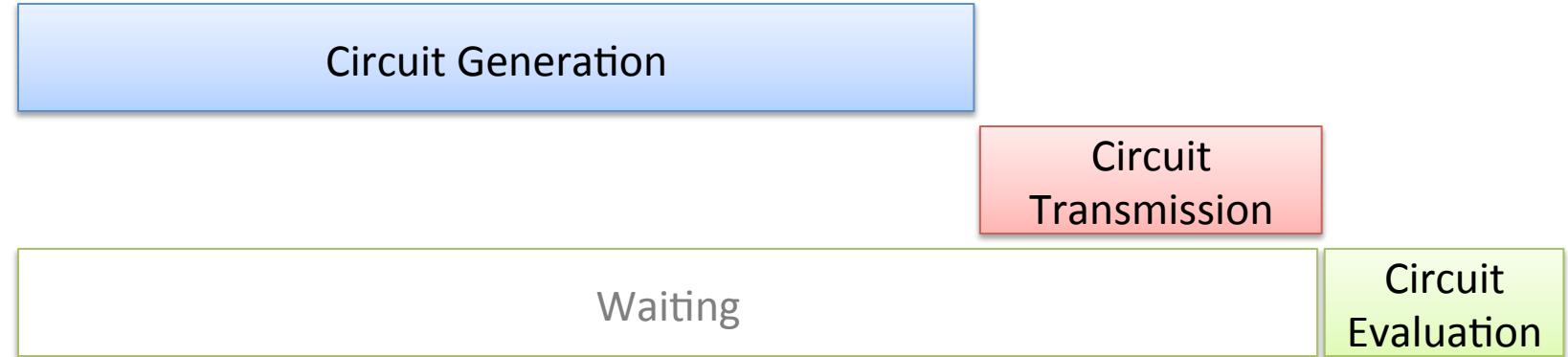
**Partial Evaluation**

Yan Huang, David Evans, Jonathan Katz, and Lior Malka. *Faster Secure Two-Party Computation Using Garbled Circuits*. USENIX Security 2011.

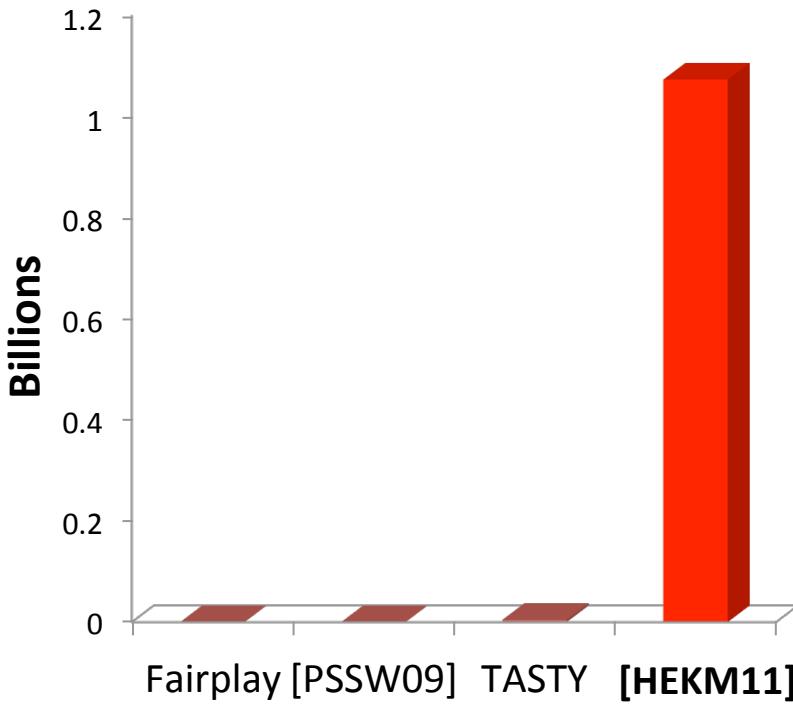
# Pipelined Execution



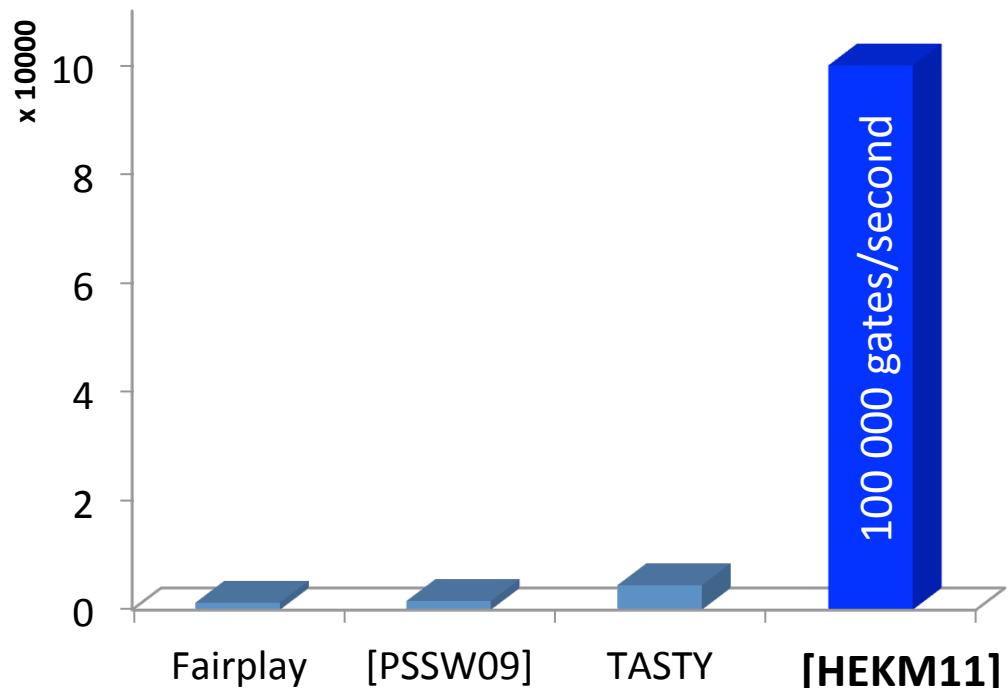
# Pipelining



# Results



**Scalability**  
(billions of gates)



**Performance**  
(10,000x non-free gates per second)

# Semi-Honest is Half-Way There

## Privacy

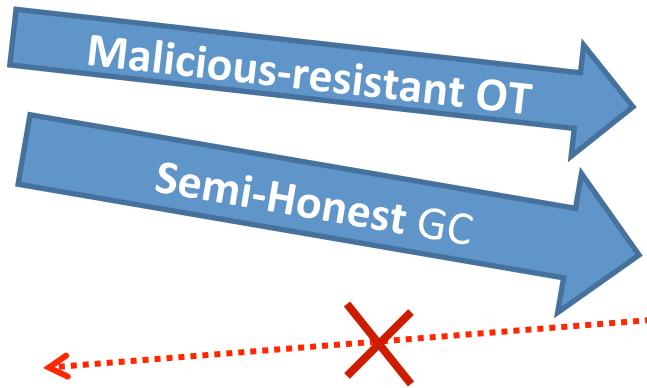
Nothing is revealed  
other than the output

## Correctness

The output of the  
protocol is indeed  $f(x,y)$

Generator

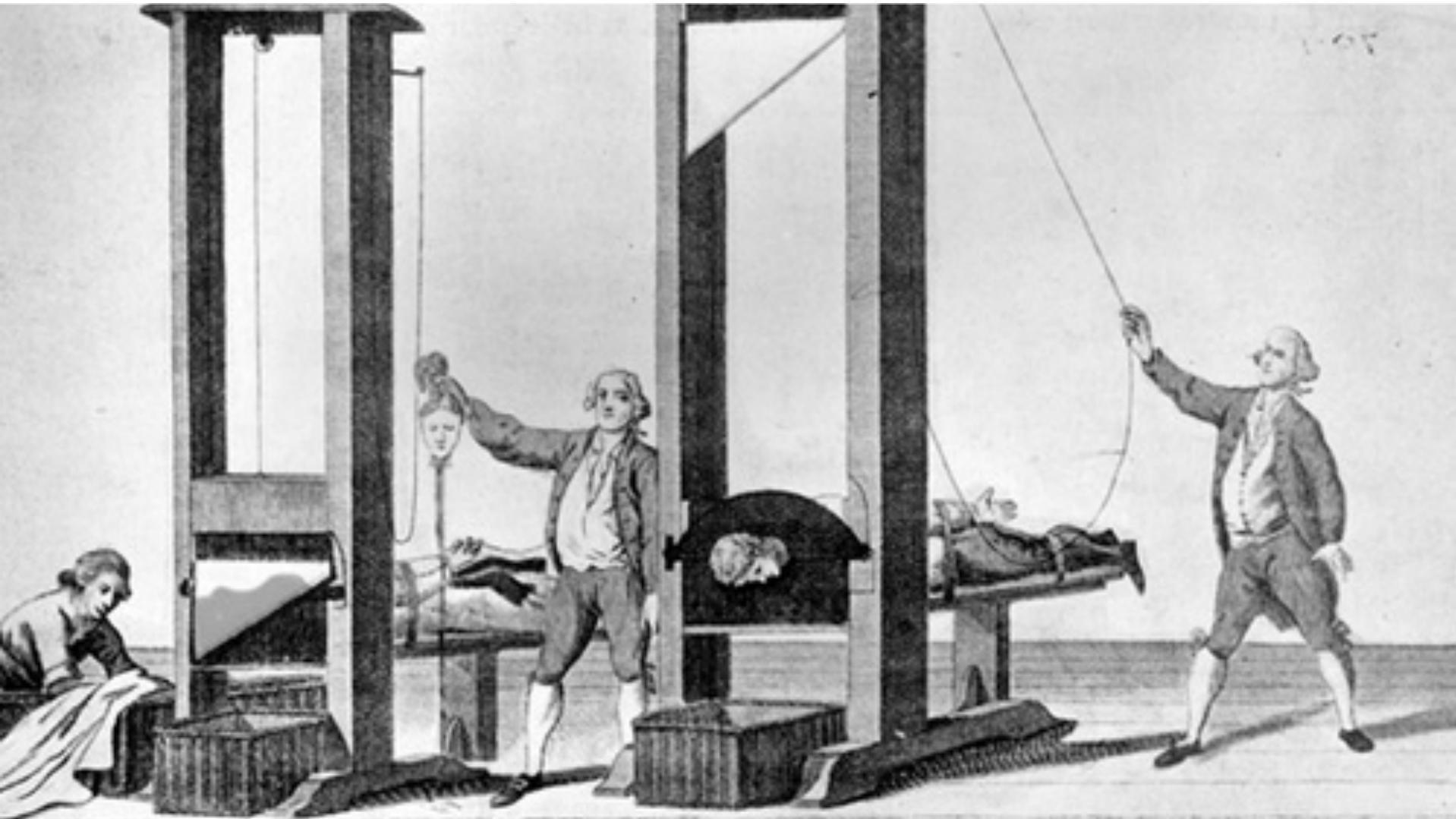
Evaluator



As long as evaluator doesn't send result back, and a malicious-resistant OT is used, **privacy** for evaluator is guaranteed.

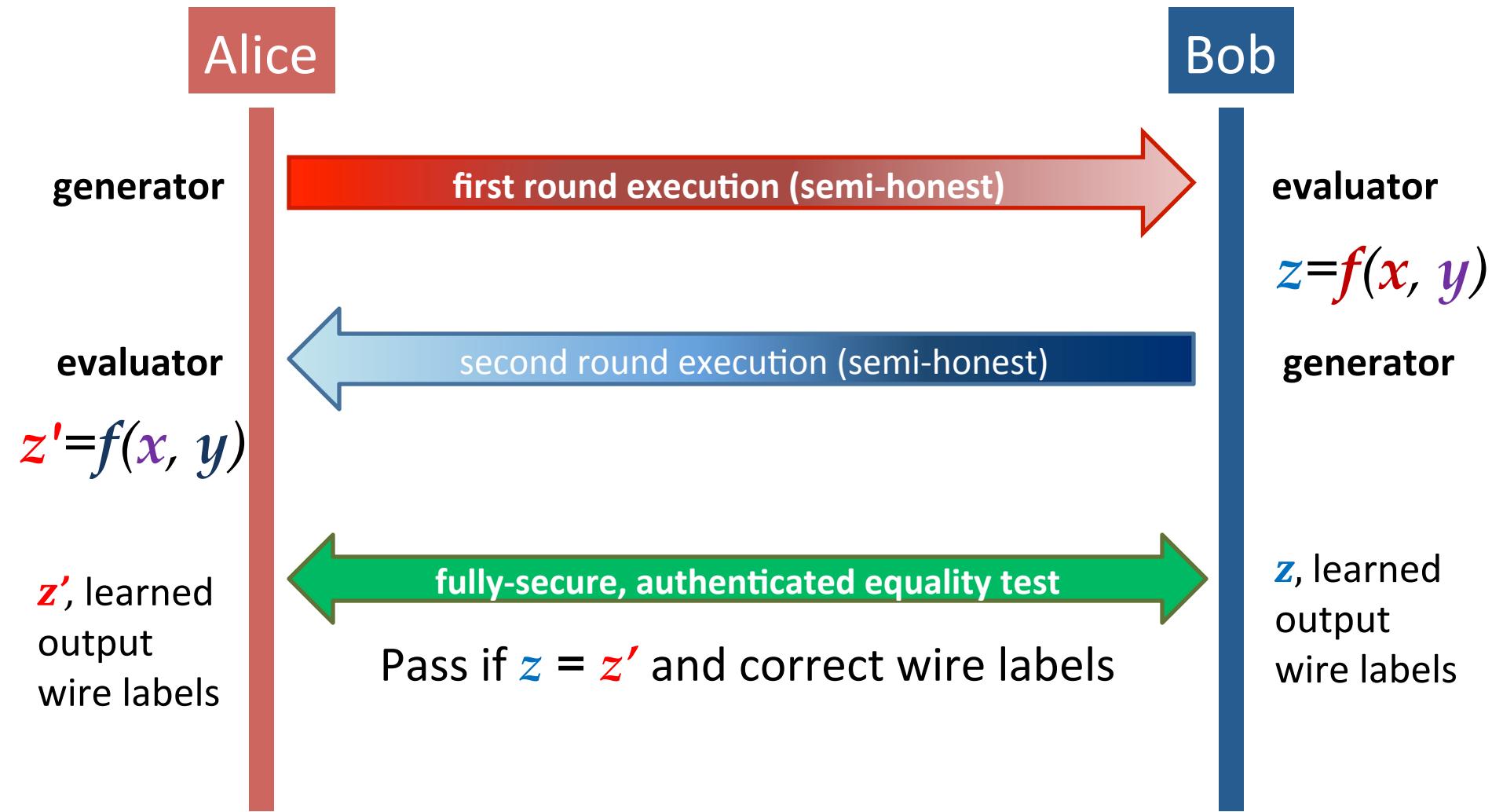
How can we get both correctness, and maintain privacy while giving both parties result?

# Dual Execution Protocols



Yan Huang, Jonathan Katz, and David Evans. *Quid-Pro-Quo-tocols: Strengthening Semi-Honest Protocols with Dual Execution*. IEEE Security and Privacy (Oakland) 2012.

# Dual Execution Protocol



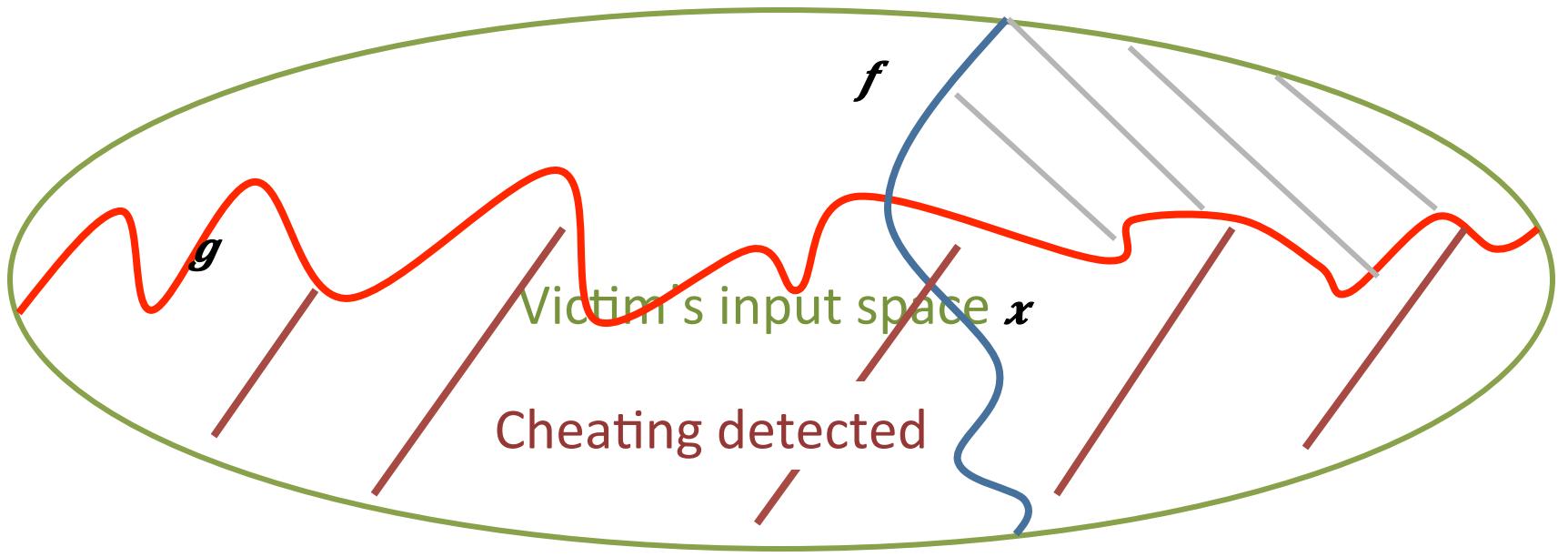
# Security Properties

**Correctness:** guaranteed by authenticated, secure equality test

**Privacy:** Leaks **one (extra) bit** on average  
adversarial circuit generator provides a circuit that fails on  $\frac{1}{2}$  of inputs

Malicious generator can **decrease likelihood of being caught**, and **increase information leaked when caught** (but decreases average information leaked): at extreme, circuit fails on just one input

# 1-bit Leak

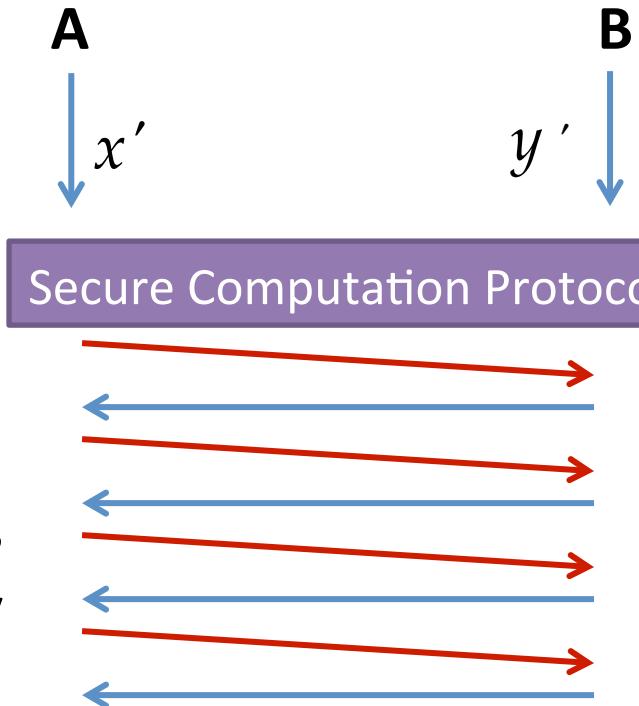
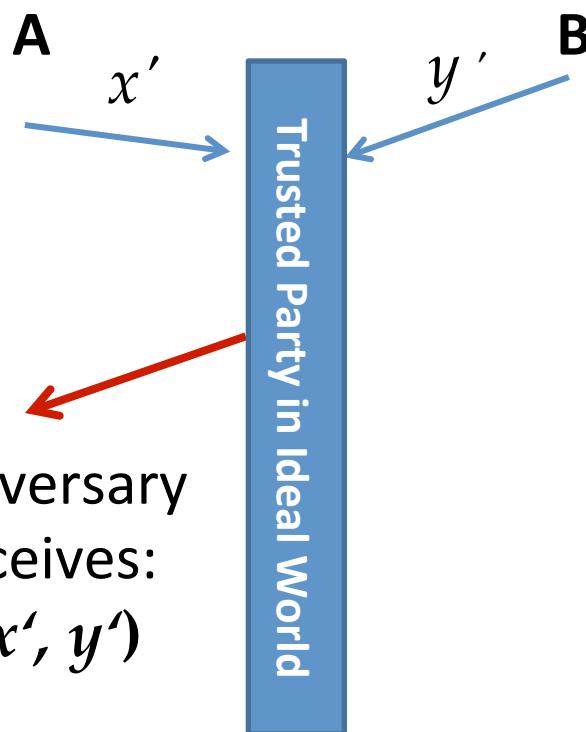


# Proving Security: Malicious

Ideal World

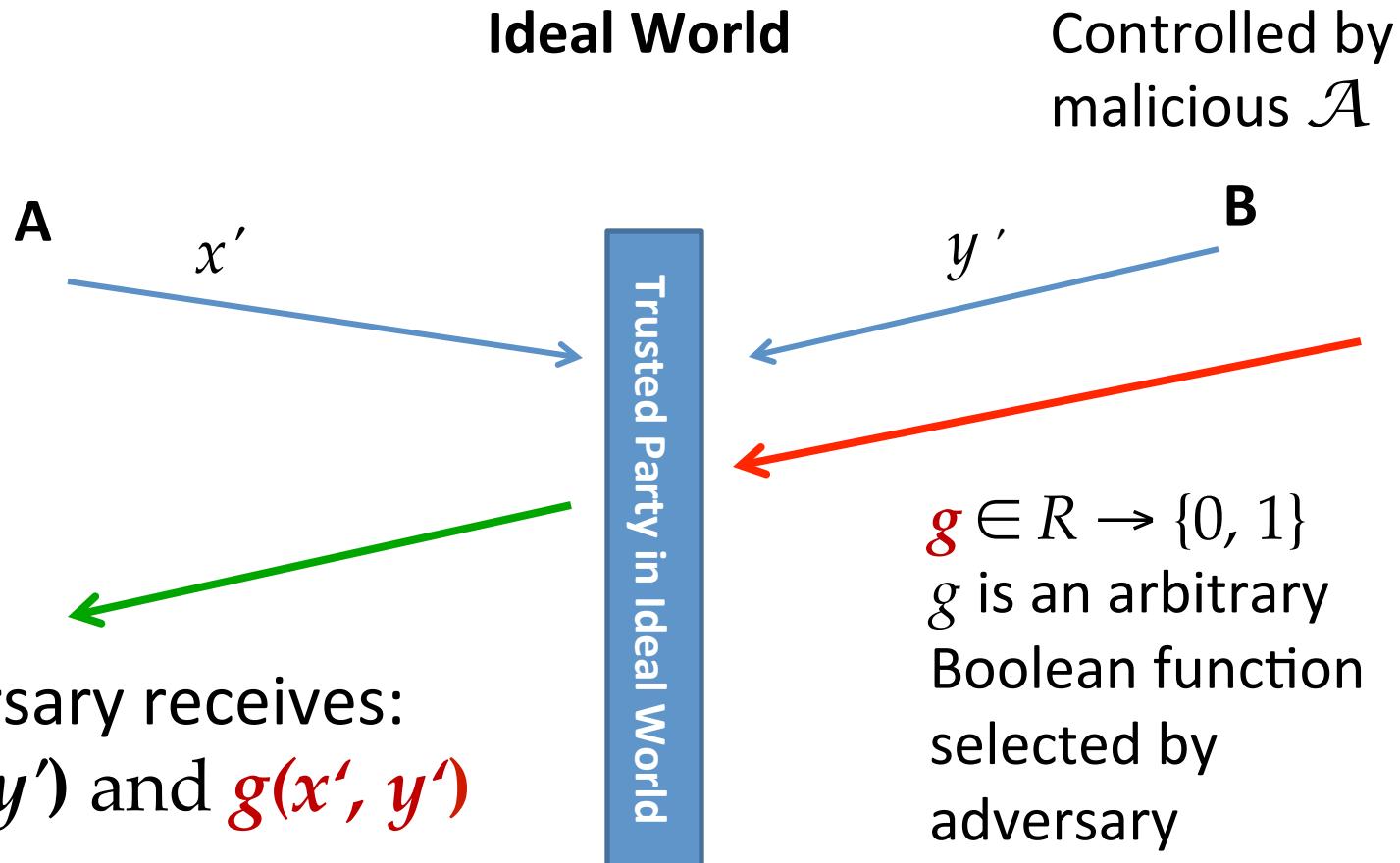
Show equivalence

Real World



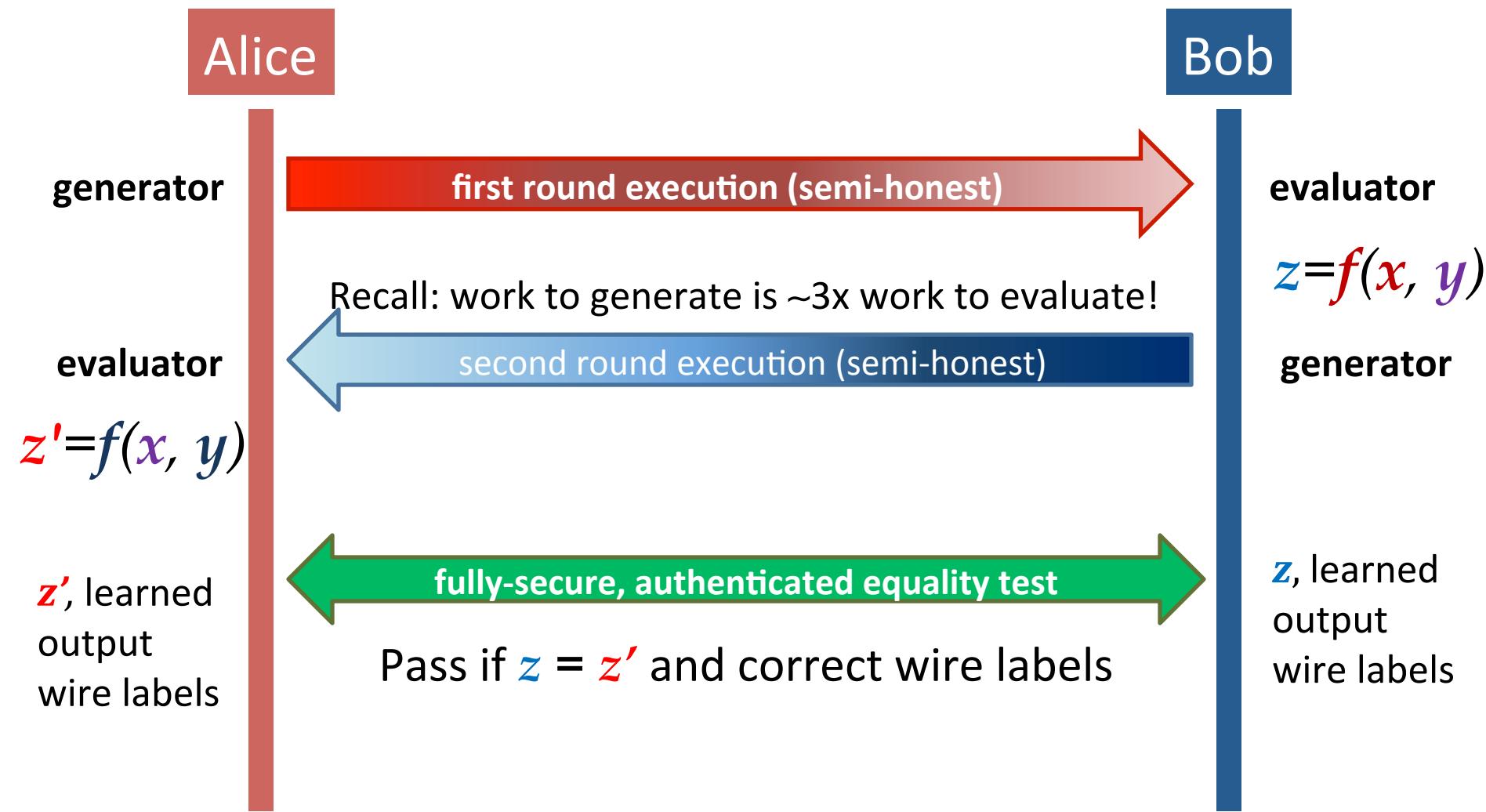
Standard Malicious Model: can't prove this for Dual Execution

# Proof of Security: One-Bit Leakage

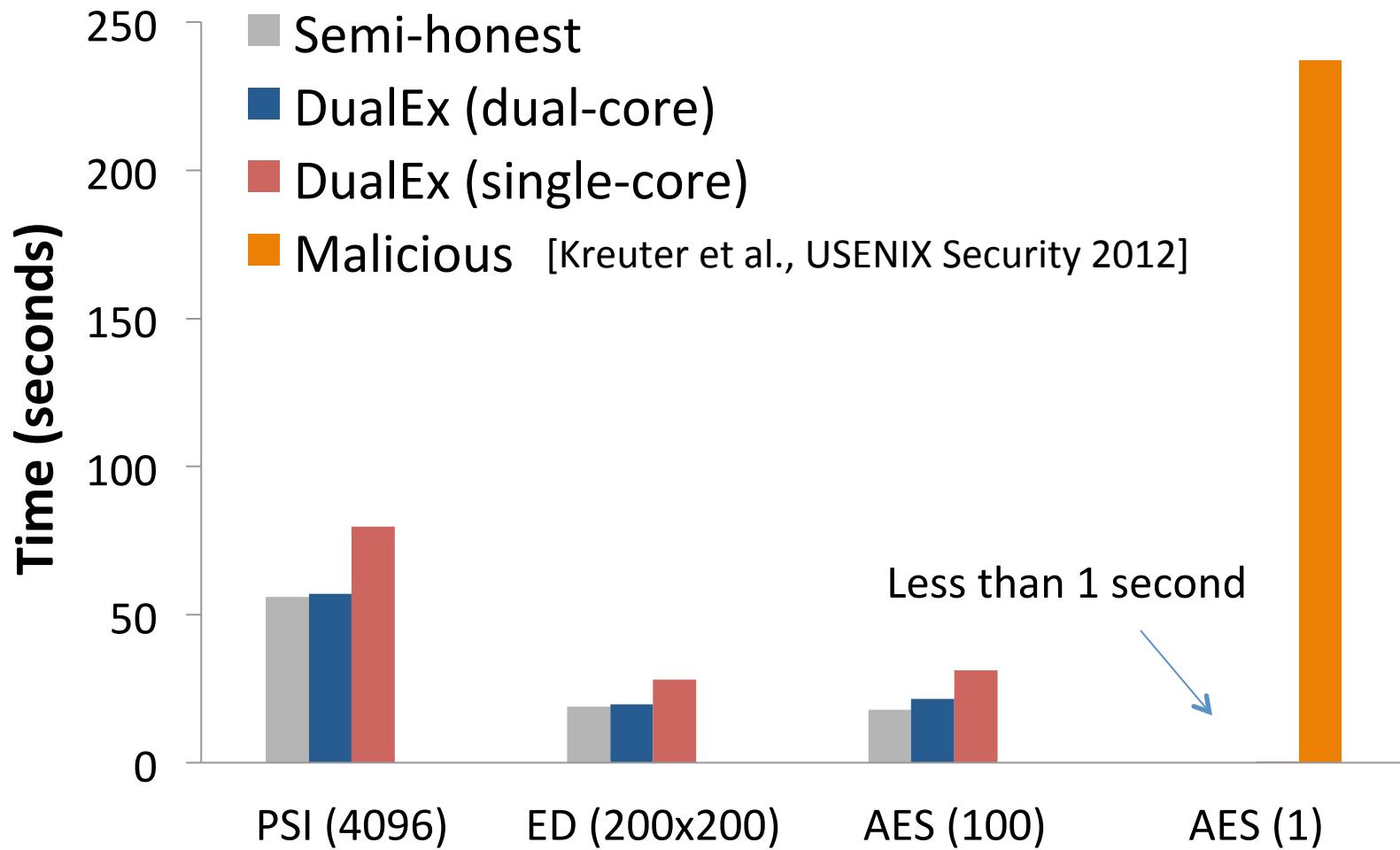


Can prove equivalence to this for Dual Execution protocols

# Implementation



# Performance



Circuits of arbitrary size can be done this way



Privacy-Preserving  
Biometric Matching

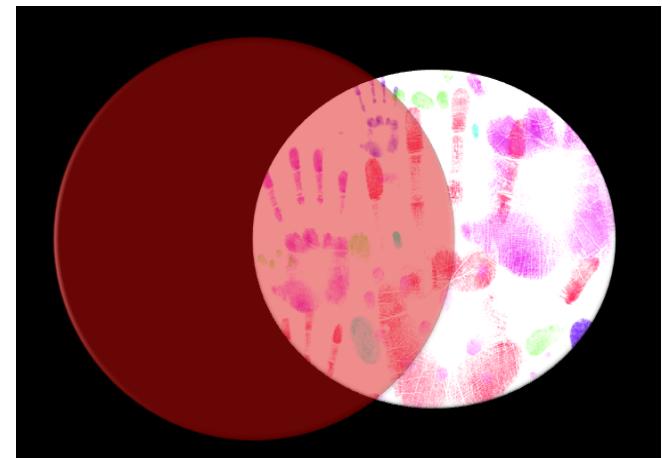


Private AES  
Encryption

Private  
Personal  
Genomics



# Applications



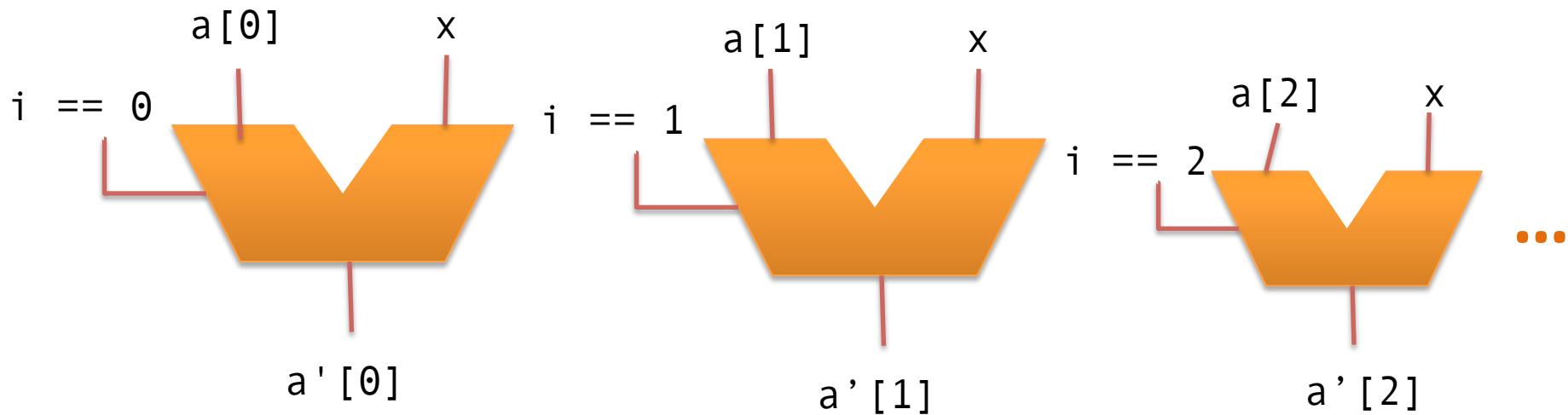
Private Set Intersection

	Problem	Best Previous Result	Our Result	Speedup
NDSS 2012	<b>Private Set Intersection</b> (contact matching, common disease carrier)	Competitive with best custom protocols, scales to millions of 32-bit elements		
USENIX Security 2011	<b>Hamming Distance</b> (Face Recognition)	213s [SCiFI, 2010]	<b>0.051s</b>	<b>4176</b>
	<b>Levenshtein Distance</b> (genome, text comparison) – two 200-character inputs	534s [Jha+, 2008]	<b>18.4s</b>	<b>29</b>
	<b>Smith-Waterman</b> (genome alignment) – two 60-nucleotide sequences	[Not Implementable]	<b>447s</b>	-
	<b>AES Encryption</b>	3.3s [Henecka, 2010]	<b>0.2s</b>	<b>16.5</b>
NDSS 2011	<b>Fingerprint Matching</b> (1024-entry database, 640x8bit vectors)	~83s [Barni, 2010]	<b>18s</b>	<b>4.6</b>

# Crazy Things in Typical Code

```
a[ i ] = x
```

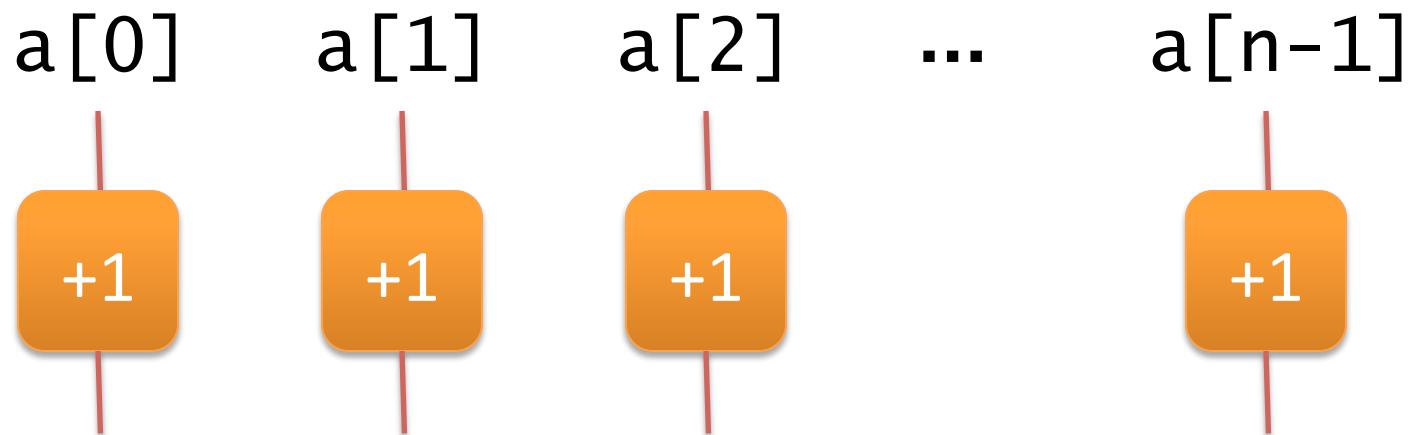
# Circuit for Array Update



$a[i] = x$

# Easy (and Common) Case

```
for (i = 0; i < n; i++)  
    a[i] += 1
```



# Circuit Structures

Design circuits to support typical  
**data structures** efficiently

Non-trivial access patterns, but  
**patterns nonetheless**

Main opportunities:

**Locality and Batching**



**Samee Zahur**  
(UVa PhD Student)

Samee Zahur and David Evans. *Circuit Structures for Improving Efficiency of Security & Privacy Tools*.  
**IEEE Security and Privacy (Oakland) 2013.**

# Locality: Stacks and Queues

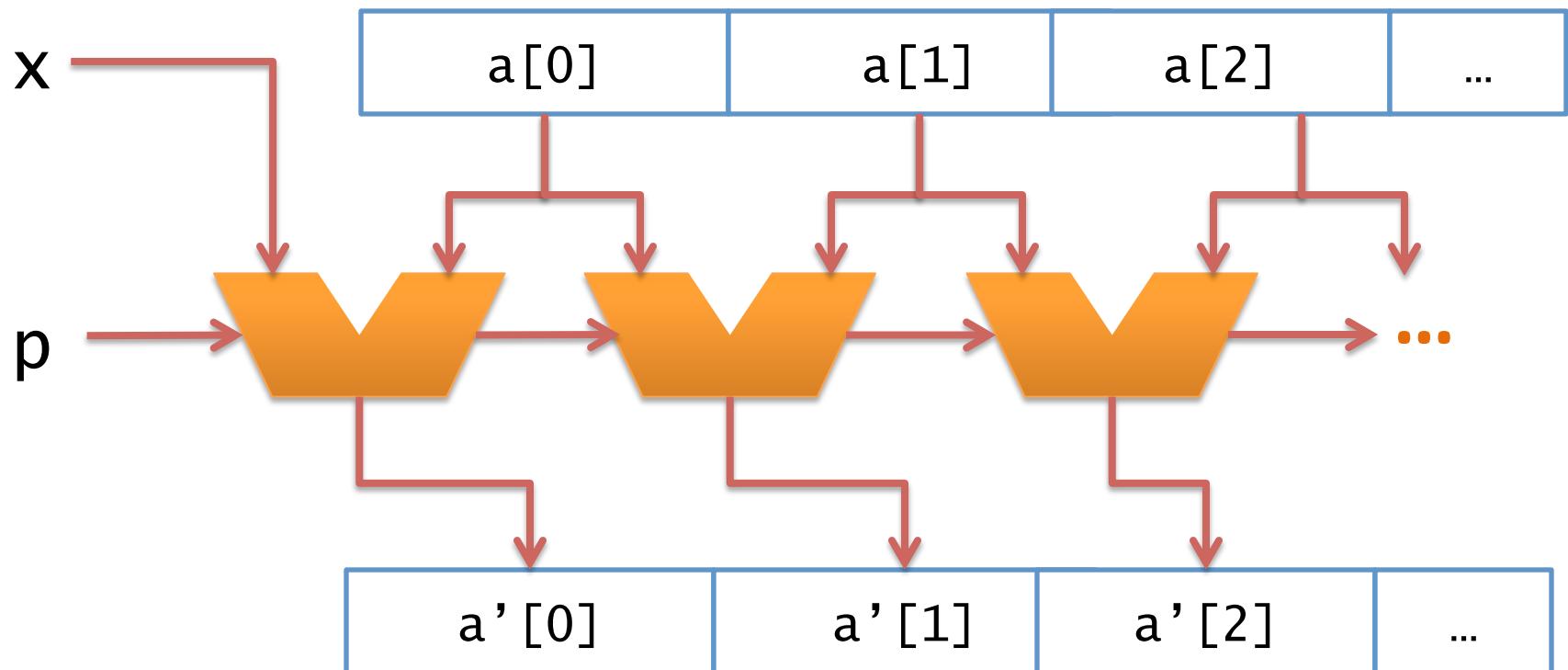
```
if (x != 0)
    a[i] += 1
    if (a[i] > 10)
        i += 1
    a[i] = 5
```

Data-oblivious code  
No branching allowed

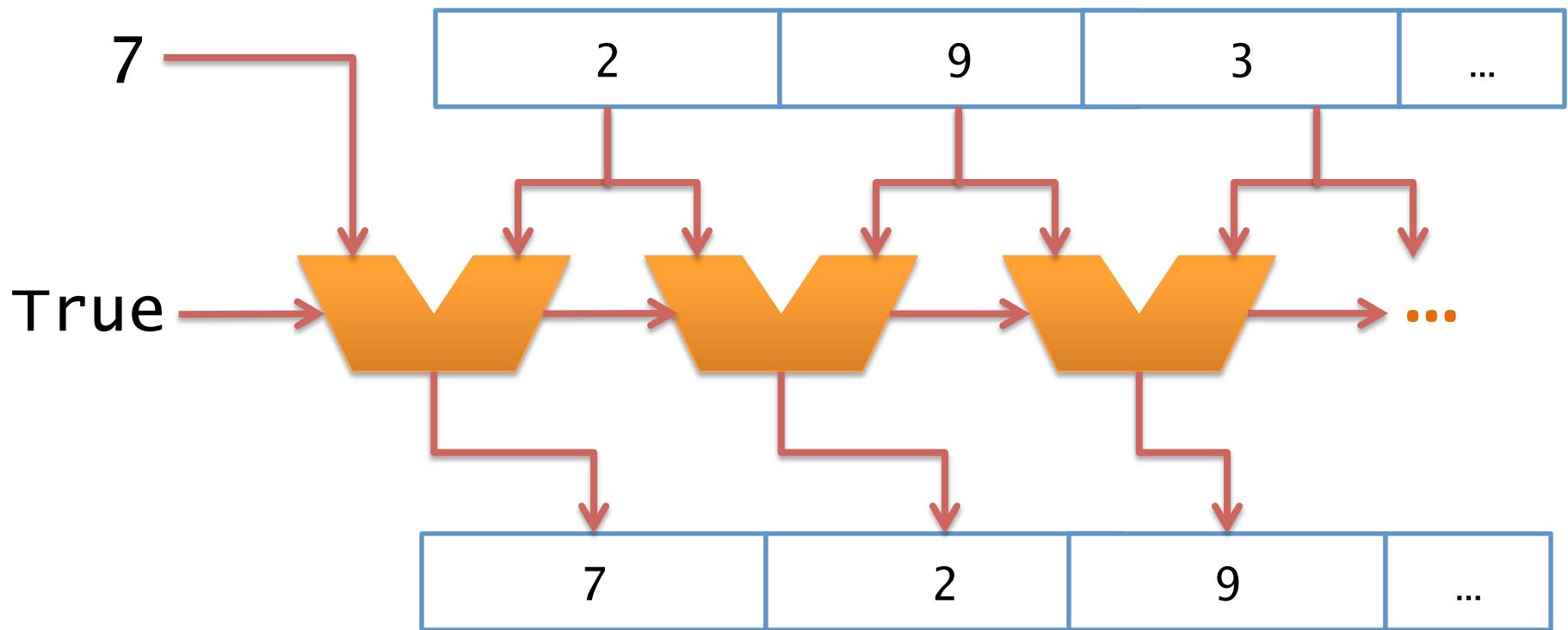


```
t := a.top() + 1
a.cond_update(x != 0, t)
a.cond_push(x != 0 && t > 10, *)
a.cond_update(x != 0, 5)
```

# Naïve Conditional Push



# Naïve Conditional Push



# More Efficient Stack

Level 0: 

$t = 3$

Level 1: 

$t = 2$

Level 2: 

$t = 3$

...

Block size =  $2^{level}$

Each level has 5 blocks, at least 2 full and 2 empty

Level 0



t = 3

Level 1



t = 2

Level 2



4 7

5 4

t = 3

Conditional push (True, 7)



t = 4



t = 2



4 7

5 4

t = 3

Conditional push (True, 8)



t = 5



t = 2



4 7

5 4

t = 3

Shift



t = 3



t = 3



9 3



4 7

5 4

t = 3

Level 0



t = 3



Level 1



t = 2

Level



t = 3

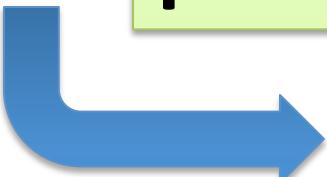


t = 4



t = 5

Amortized  
 $\Theta(\log n)$  gates  
per operation



t = 3



t = 3



t = 3

# Arbitrary Array Accesses (*Associative Maps*)

	0	2	7	9
$m[0] = 'A'$	'A'			
$m[2] = 'U'$		'U'		
$m[9] = 'M'$				'M'
$m[7] = 'R'$			'R'	
$m[0] = 'D'$	'D'			
$m[9] = 'Y'$				'Y'
$m[9] = 'K'$				'K'
$m[7] = 'C'$			'C'	

	0	2	7	9
$m[0] = 'A'$	'A'			
$m[2] = 'U'$		'U'		
$m[9] = 'M'$				'M'
$m[7] = 'R'$			'R'	
$m[0] = 'D'$	'D'			
$m[9] = 'Y'$				'Y'
$m[9] = 'K'$				'K'
$m[7] = 'C'$			'C'	

Execution trace: **indexes and values are private values**

# Batching Updates

$m[0] = 'A'$   
 $m[2] = 'U'$   
 $m[9] = 'M'$   
 $m[7] = 'R'$   
 $m[0] = 'D'$   
 $m[9] = 'Y'$   
 $m[9] = 'K'$   
 $m[7] = 'C'$

Sort by Key

stable sort!

$m[0] = 'A'$   
 $m[0] = 'D'$   
 $m[2] = 'U'$   
 $m[7] = 'R'$   
 $m[7] = 'C'$   
 $m[9] = 'M'$   
 $m[9] = 'Y'$   
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Sort by Key  
stable sort!

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 $m[9] = 'Y'$   
 $m[9] = 'K'$

# Batching Updates

'A'  
'U'  
'M'  
'R'  
'D'  
'Y'  
'K'  
'C'

Sort by Key

stable sort!

$m[0] = 'A'$   
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 $m[2] = 'U'$   
 $m[7] = 'R'$   
 $m[7] = 'C'$   
 $m[9] = 'M'$   
 $m[9] = 'Y'$   
 $m[9] = 'K'$

Compare  
Adjacent

$m[0] = 'A'$   
 $m[0] = 'D'$   
 $m[2] = 'U'$   
 $m[7] = 'R'$   
 $m[7] = 'C'$   
 $m[9] = 'M'$   
 $m[9] = 'Y'$   
 $m[9] = 'K'$

# Batching Updates

'A'  
'U'  
'M'  
'R'  
'D'  
'Y'  
'K'  
'C'

Sort by Key

stable sort!

$m[0] = 'A'$   
 $m[0] = 'D'$   
 $m[2] = 'U'$   
 $m[7] = 'R'$   
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 $m[9] = 'Y'$   
 $m[9] = 'K'$

# Batching Updates

'A'  
'U'  
'M'  
'R'  
'D'  
'Y'  
'K'  
'C'

Sort by Key

stable sort!

$m[0] = 'A'$   
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Adjacent

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Sort by Key

stable sort!

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Compare  
Adjacent

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 $m[0] = 'D'$   
 $m[2] = 'U'$   
 $m[7] = 'R'$   
 $m[7] = 'C'$   
 $m[9] = 'M'$   
 $m[9] = 'Y'$   
 $m[9] = 'K'$

output  
wires

Discarded

$m[0] = 'D'$   
 $m[2] = 'U'$   
 $m[7] = 'C'$   
 $m[9] = 'K'$   
 $m[0] = 'A'$   
 $m[7] = 'R'$   
 $m[9] = 'M'$   
 $m[9] = 'Y'$

Sort by  
Liveness

# Associative Map Cost

Oblivious Stable Sort



Comparisons and Liveness  
Marking



Oblivious Sort  
Liveness/Key

Circuit Size:

$\Theta(n \log n) \times$  comparison cost  
 $\Theta(n \log^2 n)$

# Example Application: DBScan

Martin Ester, Hans-Peter Kriegel,  
Jörg Sander, Xiaowei Xu. *KDD* 1996

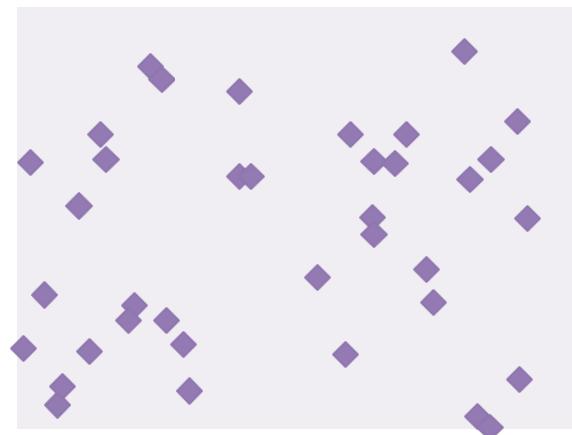
Density-based clustering:  
depth-first search to find dense clusters



Alice's Data



Bob's Data



Joint Clusters

```

 $n \leftarrow |P|$ 
 $c \leftarrow 0$ 
 $s \leftarrow \text{emptyStack}$ 
 $cluster \leftarrow [0, 0, \dots]$ 
for  $i \leftarrow [1, n]$  do
    if  $cluster[i] \neq 0$  then
        continue
     $V \leftarrow \text{getNeighbors}(i, P, minpts, radius)$ 
    if  $\text{count}(V) < minpts$  then
        continue
     $c \leftarrow c + 1$                                  $\triangleright$  Start a new cluster
    for  $j \leftarrow [1, n]$  do
        if  $V[j] = \text{true} \wedge cluster[j] \neq 0$  then
             $cluster[j] \leftarrow c$ 
             $s.\text{push}(j)$ 
while  $s \neq \emptyset$  do
     $k \leftarrow s.\text{pop}()$ 
     $V \leftarrow \text{getNeighbors}(k, P, minpts, radius)$ 
    if  $\text{count}(V) < minpts$  then
        continue
    for  $j \leftarrow [1, n]$  do
        if  $V[j] = \text{true} \wedge cluster[j] \neq 0$  then
             $cluster[j] \leftarrow c$ 
             $s.\text{push}(j)$ 

```

**Private Input:**  $P$  – array of points (combines private points from both parties)

**Public inputs:**  $minpts, radius$

**Output:** cluster number for each point

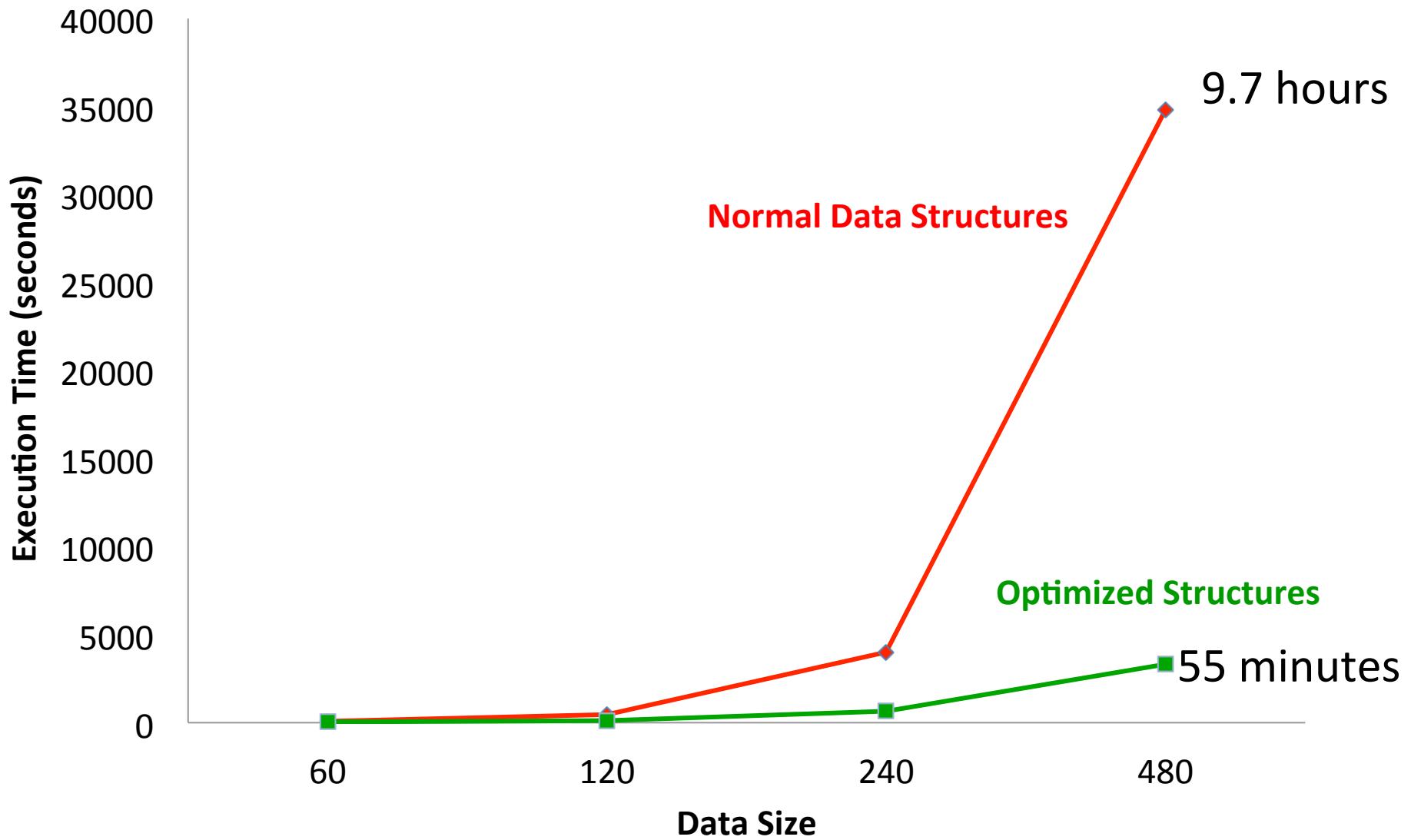
$\triangleright$  Start a new cluster

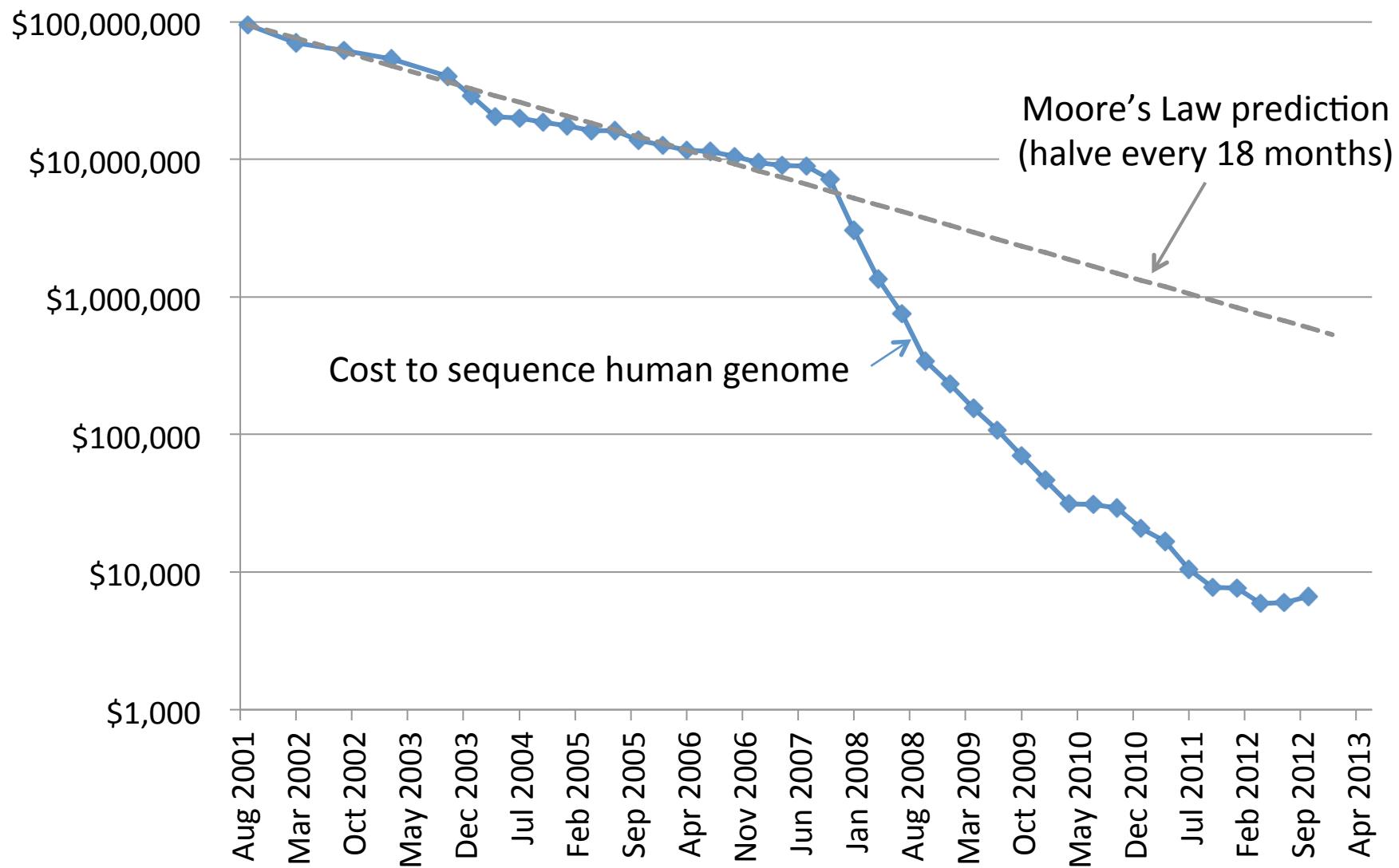
Conditional Push!

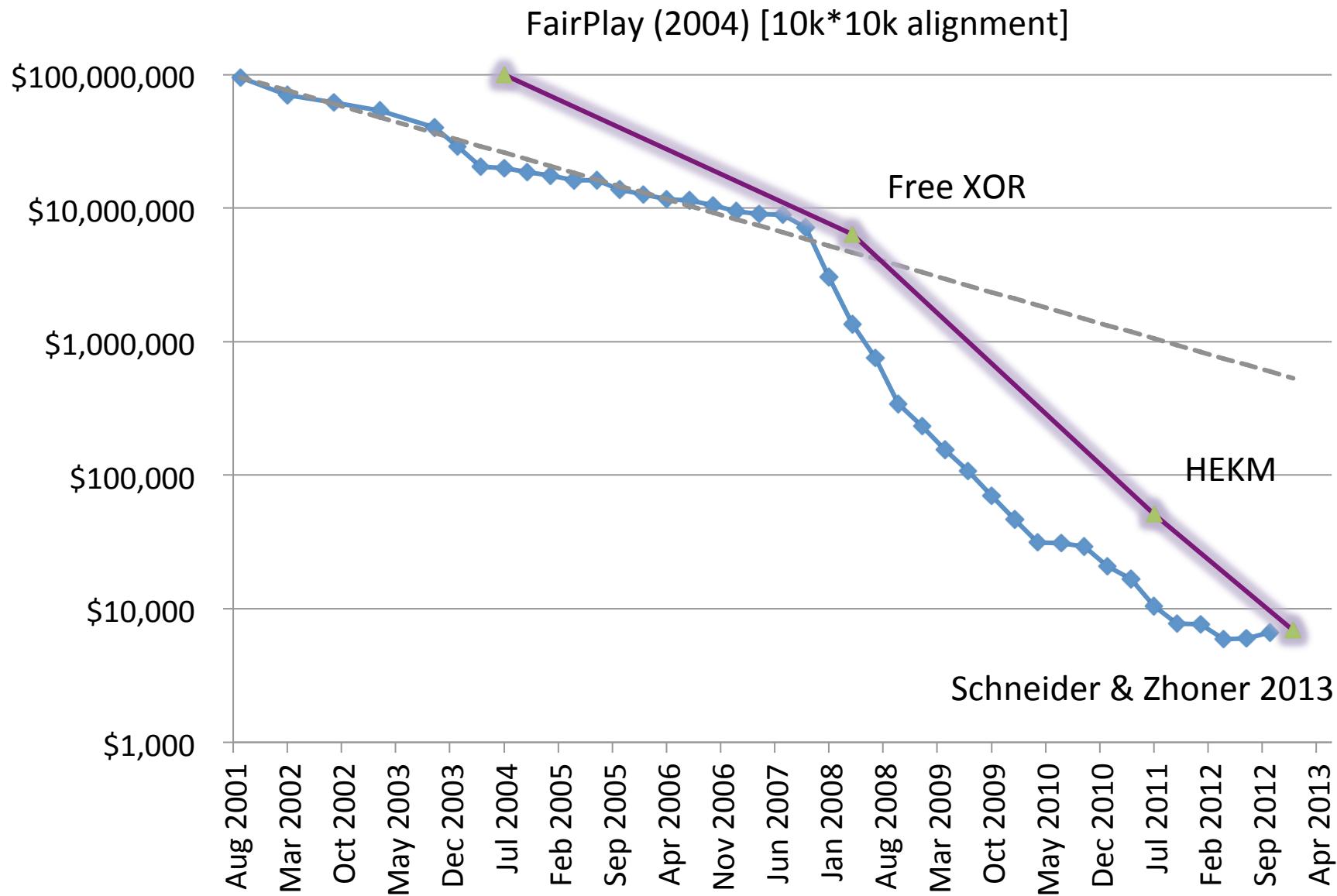


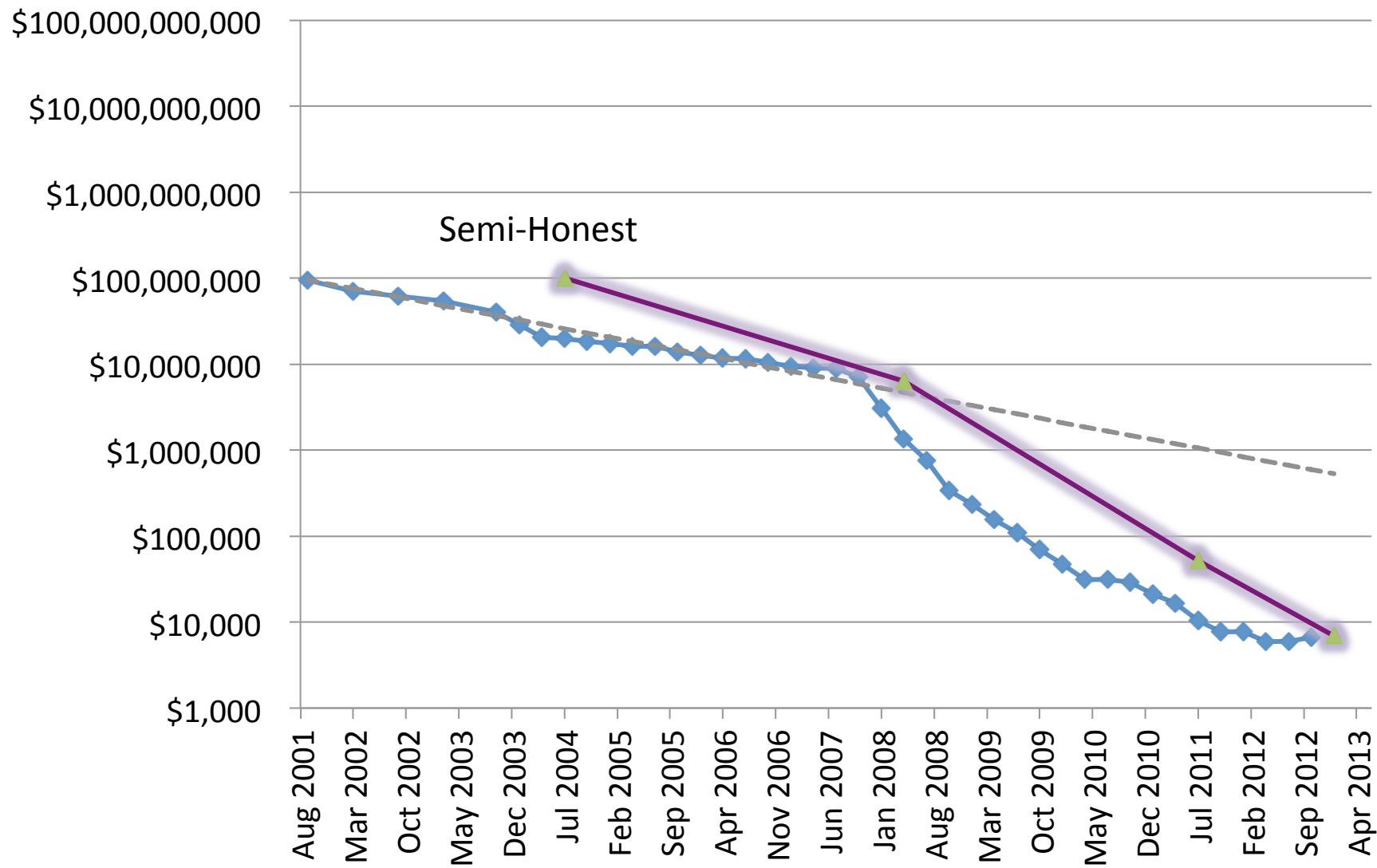
Array update!

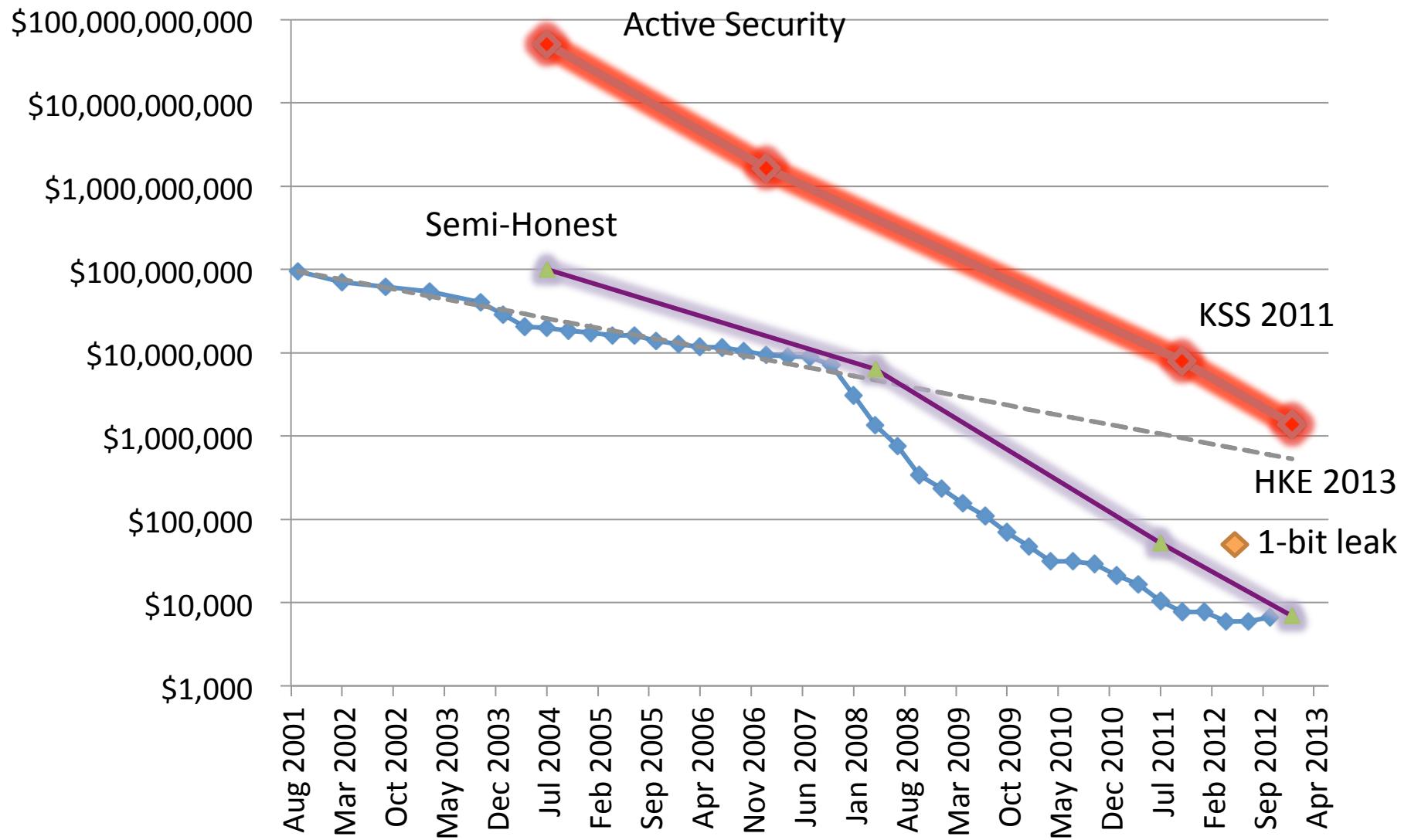












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Questions?