The Light Field I

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1 Radiometry and Photometry

Radiometry and Photometry are two related fields which study light. Radiometry is the general study of measuring light, wheras Photometry relates more to how light is seen and perceived by a human viewer.

There are several concepts important to the study of light and vision that are used throughout this lecture. For the most part, these can be described simply in terms of units. The first of these concepts is energy. Energy (with relation to the study of light) the ammount of light posessed by a certain ray or incident upon a certain point on a surface. It is measured in Joules in Radiometry, or Talbots in Photometry. The second concept, power, is measured in Joules per second (or Talbots per second in photometry), and can be described as rate energy is produced by a light source or radiative body. If we assume that the light in a given scene is at a sort of steady-state, where light at any given point is fairly constant over time, we can for the most part ignore energy and simply focus on how power is distributed from the light source(s) throughout the scene.

2 Luminous Efficiency

Light is essentially an electromagnetic wave which travels through space and which we choose to describe as quantized particles known as photons. However, only a small band of the spectrum of possible electromagnetic waves are actually visible and perceived by the eye as light. Furthermore, not all frequencies within that band of visible light are detected equally by the eye, as it is sensetive to some wavelengths significently more than others. In fact, the response is almost more of a bell curve, centered on about a 550nm wavelength. This curve of senestivity is known as the Visual Response curve, and is shown in Figure 1. What this curve says, roughly, is that given red (750nm), blue (450nm) and green (550nm) lasers of equal power, the eye would perceive the dot from the green one as being far brighter than either the red or blue.

Based on this curve, we can calculate the actual photometric brightness of a light sourced based on the following equation:

$$Y = \int_{380}^{780} \Phi(\lambda) L(\lambda)\, d\lambda$$

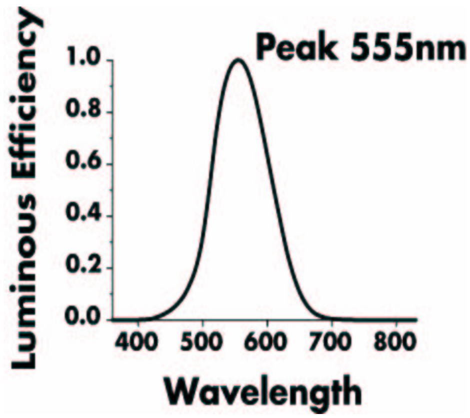

Figure 1: The Visual Response curve. Note that the eye is far more sensitive to green light (about 550nm) than it is to either red (750nm) or blue (450nm).

where $\Phi(\lambda)$ is the power from the light emitted at that wavelength, and $L(\lambda)$ is the visual response to that wavelength. Because of this, overall brightness of a light source does not necessarily scale with radiated power, but with the sensitivity to the wavelengths emitted as well. It should be noted, however, that this is not a perfectly linear mapping; human photon sensitivity begins to taper off at levels above a certain brightness. Thus, above a certain limit, the actual perceived brightness scales with respect to $Y^{1/3}$ and not directly with Y. This relationship corresponds to the Photometric measure of brightness, known as the brill.

2.1 Power Spectra

The relationship between the Power Spectrum (the spectrum of wavelengths emitted by a light source) and the perceived brightness of a light can be can be illustrated by comparing the power spectra of a tungsten lamp to that of a flourescent light (Figure 2).

3 Radiant Intensity

Radiant Intensity refers to the concept of power emitted from a point light over a three-dimensional arc (known as a “solid angle”). The units for radiant intensity are Power per Unit solid angle ($\frac{d\Phi}{d\omega}$). For Radiometry, these units are Watts per Stradians; for Photometry, they are Lumens per Stradians. These both correspond to the SI unit for light intensity, the Candella.

3.1 Solid Angle

The solid angle is an extrapolation of the one-dimensional angle that sweeps out an arc along the circumfrence of a sphere into three dimensions. A solid angle represents some projection

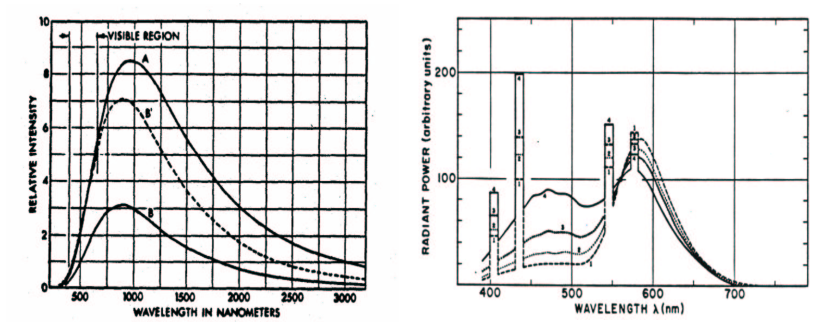

Figure 2: As can be seen from the images above, much of the energy in the spectrum of the tungsten lamp (left) lies outside of the visible spectrum and so does not contribute to the perceived brightness. A flourescent bulb (right) by contrast emits almost exclusively within the range of visible wavelengths, allowing it to be much more efficient.

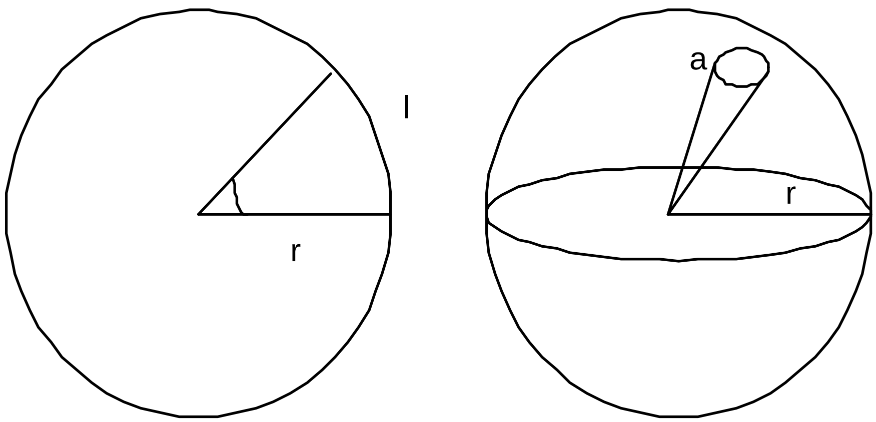

Figure 3: For an angle θ that sweeps out an arc l in a circle of radius r (left) can be derived using the equation $\theta = \frac{l}{r}$. In a similar fashion, the solid angle ω that sweep out a patch of area A on a sphere of radius r (right) can be found using the equation $\omega = \frac{A}{r^2}$.

that sweeps out a patch of area A on the surface of a unit sphere. This angle is usually denoted as ω. Its relationship with a one-dimensional angle is explained graphically in figure 3. Firthermore, if we want to express ω as a differential value $d\omega$ that is the function of a pair of spherical coordinates, we can derive it, as shown below:

$$A = (r \sin\theta \, d\phi)(r \, d\theta); \; \omega = \frac{A}{r^2}$$

Rearranging, we get

$$d\omega = \sin\theta \, d\theta d\phi$$

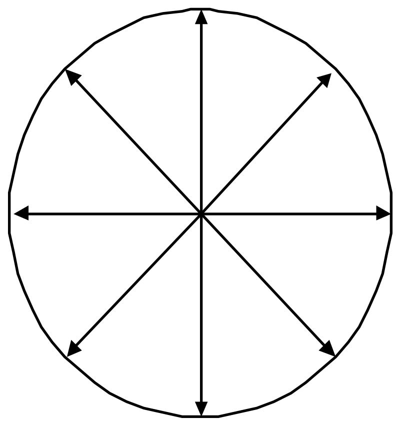

Figure 4: An isotropic point light with total power output Φ.

3.2 Example: Point Light

Let us assume our light source is a point light whose radiant intensity function is isotropic – that is, uniform in all directions (figure 4). If we let Φ be the total power output of the light, and I be the radiant intensity over a given solid angle $d\omega$, then we can derive the total power as a function of I as follows:

$$\Phi = \int_{s^2} I \, d\omega$$

Since the light is isotropic, I is constant in all directions. Therefore

$$\Phi = I \int_{s^2} d\omega = 4\pi I$$

3.3 Example: Warn's Spotlight

A spotlight(figure 5) is a slightly more complex lighting model, but its power output too can be calculated with a little work. First, we define I as a finction of the solid angle ω:

$$I(\omega) = (\cos\theta)^s$$

$$I(\omega) = (\vec{S} \cdot \vec{A})^s$$

Now we can compute the power output Φ:

$$\Phi = \int_0^{2\pi} \int_0^{\pi/2} I(\omega) \sin\theta \, d\theta d\phi$$

$$\Phi = \int_0^{2\pi} \int_0^{1} \cos^s\theta \, d(\cos\theta) d\phi$$

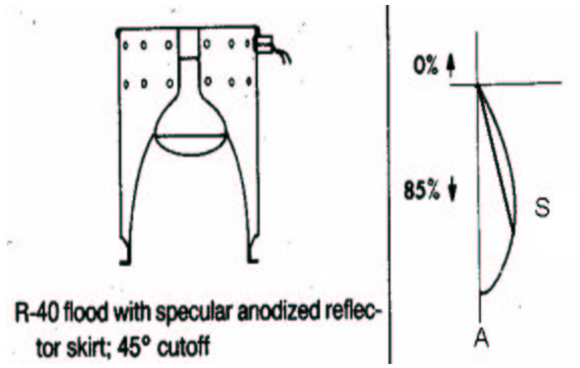

Figure 5: A spotlight aligned along axis $\vec{A}$.

$$\Phi = \frac{2\pi}{s+1}$$

This all yields an exact equation for the radiant intensity over a given solid angle ω:

$$I(\omega) = \Phi \cos^s \theta \frac{s+1}{2\pi}$$

3.4 Candlepower

Early photometric researchers reaized that, while the human eye is not very good for judging absolute apparent brightness of a light source, it is quite capable of comparing two sources relatively. Thus, experiments were conducted where the brightness of two light sources would be compared in a contraption where the user would move the lights towards and/or away from them until they appeared equally bright. After some experimentation, it became clear that the apparent brightness of a light source seems to scale proportionally to $\frac{1}{r^2}$, where r is the distance of the source to the eye. Initial experiments used what was known as a "standard candle" as their basis of comaring various light sources. Now, the unit has been standardized as $1candle = 550nm$ laser with $1/683\frac{W}{Sr}$.

4 Irradiance

Irradiance is the amount of power from a light source incident on a given surfacem in terms of power per unit area ($\frac{d\Phi}{dA}$). It is expressed in terms of Watts (Radiometry) or Lumens (Photometry) per meter squared, both of which correspond to a unit known as the lux.

The power falling on a surface can be calculated from the power output of the light source in that direction (Φ), the area of the surface (A), and the angle between the surface normal and the incident direction of the light (θ) (figure 6).

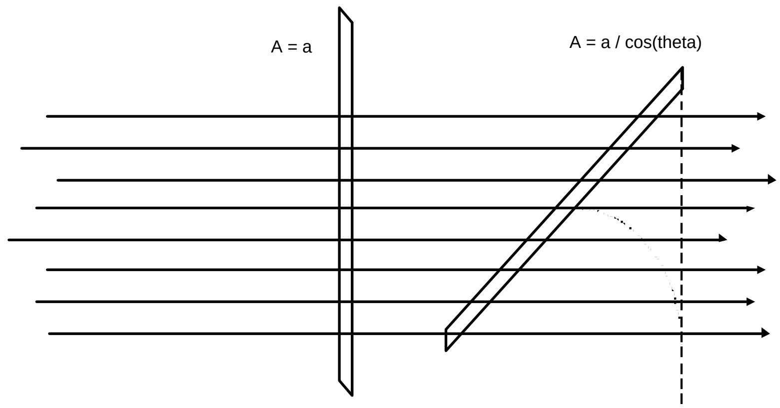

Figure 6: The power falling on a surface of area A depends on the angle of tilt θ relative to the light source.

$$E = \frac{\Phi}{A} = \frac{\Phi}{\frac{a}{\cos\theta}} = \frac{\Phi}{a}\cos\theta$$

It should be noted that this has nothing to do with the reflectivity of the surface material, but is instead a purely geometric result.

5 Radiance

Radiance is the intensity from a light source incident on a given surface, in terms of either intensity per unit area ($\frac{d\Phi}{dA}$) or power per unit area per unit solid angle ($\frac{d\Phi}{dAd\omega}$). This is often referred in Photometry as the "nit" unit. Radiance is of particular interest in ray-tracing because it is the the one measure conserved along the path of a ray, and since it is the property that relates directly to how the eye's sensors respond to the light reflected from a given surface. Radiance can also be referred to as an integral equation:

$$\frac{d\Phi}{dA} = \int_{\Omega} L(x,\omega)d\omega$$

And, in reality, this can be converted to yet another parametric equation (this one in 4 dimensions):

$$L(x,\omega) = L(u,v,\theta,\phi)$$

Also, the radiance for most surfaces is constant relative to distance. This does not, as one might think, violate the $\frac{1}{r^2}$ law stated earlier. As shown in figure 7, though the distance r from the wall increases, the amount of wall visible (and thus, the light reflected off the wall) increases with r^2, counteracting the $\frac{1}{r^2}$ effect. For example, let us consider the following situation:

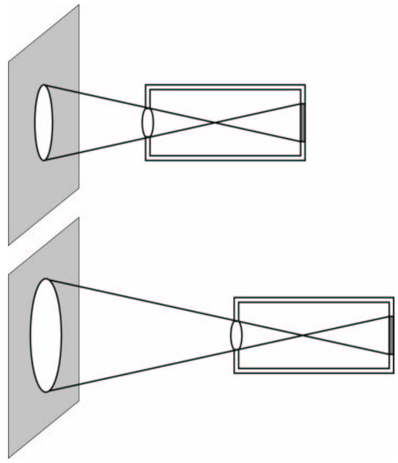

Figure 7: The radiance from a flat wall stays constant over increasing distance because the visible surface area is increasing as well.

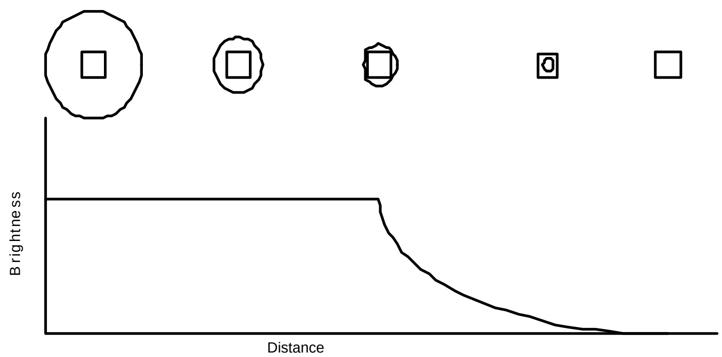

Figure 8: Even as the moon gets farther away, the brightness still remains constant; in only drops when the moon no longer fills the center pixel.

A camera is mounted on a moon lander, facing towards the moon. As the lander leaves the moon, the camera records pictures of the moon as they move apart over time. What would the brightiness of the center pixel look like, as a function of time?

As shown in figure 8, the brightness level stays constant for as long as the moon fills the center pixel. Once the moon gets smaller than the center pixel (assuming it reaches such a distance), the brightness begins to fall off proportional to $\frac{1}{r^2}$.