**Image Processing and Sampling**

Greg Humphreys  
University of Virginia  
CS445, Fall 2003

---

**Overview**

- Image representation
  - What is an image?
- Halftoning and dithering
  - Trade spatial resolution for intensity resolution
  - Reduce visual artifacts due to quantization
- Sampling and reconstruction
  - Key steps in image processing
  - Avoid visual artifacts due to aliasing

---

**What is an Image?**

- An image is a 2D rectilinear array of pixels

<table>
<thead>
<tr>
<th>Continuous image</th>
<th>Digital image</th>
</tr>
</thead>
</table>

**What is an Image?**

- An image is a 2D rectilinear array of pixels

<table>
<thead>
<tr>
<th>Continuous image</th>
<th>Digital image</th>
</tr>
</thead>
</table>

A pixel is a sample, not a little square!
What is an Image?

- An image is a 2D rectilinear array of pixels

A pixel is a sample, not a little square!

Image Acquisition

- Pixels are samples from continuous function
  - Photoreceptors in eye
  - CCD cells in digital camera
  - Rays in virtual camera

Image Display

- Re-create continuous function from samples
  - Example: cathode ray tube

Image Resolution

- Intensity resolution
  - Each pixel has only "Depth" bits for colors/intensities
- Spatial resolution
  - Image has only "Width" x "Height" pixels
- Temporal resolution
  - Monitor refreshes images at only "Rate" Hz

<table>
<thead>
<tr>
<th>Typical Resolutions</th>
<th>Width x Height</th>
<th>Depth</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>NTSC</td>
<td>640 x 480</td>
<td>8</td>
<td>30</td>
</tr>
<tr>
<td>Workstation</td>
<td>1280 x 1024</td>
<td>24</td>
<td>75</td>
</tr>
<tr>
<td>Film</td>
<td>3000 x 2000</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>Laser Printer</td>
<td>6600 x 5100</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>
## Sources of Error

- **Intensity quantization**
  - Not enough intensity resolution

- **Spatial aliasing**
  - Not enough spatial resolution

- **Temporal aliasing**
  - Not enough temporal resolution

\[
E^2 = \sum_{(x,y)} (I(x, y) - P(x, y))^2
\]

## Overview

- **Image representation**
  - What is an image?

- **Halftoning and dithering**
  - Reduce visual artifacts due to quantization

- **Sampling and reconstruction**
  - Reduce visual artifacts due to aliasing

## Quantization

- **Artifacts due to limited intensity resolution**
  - Frame buffers have limited number of bits per pixel
  - Physical devices have limited dynamic range

<table>
<thead>
<tr>
<th>Value</th>
<th>Value</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>255</td>
<td>150</td>
<td>75</td>
</tr>
<tr>
<td>255</td>
<td>150</td>
<td>75</td>
</tr>
<tr>
<td>255</td>
<td>150</td>
<td>75</td>
</tr>
</tbody>
</table>

- **Uniform Quantization**

\[
P(x, y) = \text{trunc}(I(x, y) + 0.5)
\]

- **Input**
  - \(I(x, y)\)
  - \(P(x, y)\) (2 bits per pixel)
### Uniform Quantization

- Images with decreasing bits per pixel:

  - 8 bits
  - 4 bits
  - 2 bits
  - 1 bit

### Reducing Effects of Quantization

- **Halftoning**
  - Classical halftoning

- **Dithering**
  - Random dither
  - Ordered dither
  - Error diffusion dither

### Classical Halftoning

- Use dots of varying size to represent intensities
  - Area of dots proportional to intensity in image

- Newspaper image from North American Bridge Championships Bulletin, Summer 2003
Halftone patterns

- Use cluster of pixels to represent intensity
  - Trade spatial resolution for intensity resolution

- Dithering
  - Distribute errors among pixels
  - Exploit spatial integration in our eye
  - Display greater range of perceptible intensities

**Random Dither**

- Randomize quantization errors
  - Errors appear as noise

\[ P(x, y) = \text{trunc}(I(x, y) + \text{noise}(x, y) + 0.5) \]
Ordered Dither

• Pseudo-random quantization errors
  - Matrix stores pattern of thresholds

\[ i = x \mod n \]
\[ j = y \mod n \]
\[ e = I(x,y) - \text{trunc}(I(x,y)) \]
if \( e > D(i,j) \)
\[ P(x,y) = \text{ceil}(I(x, y)) \]
else
\[ P(x,y) = \text{floor}(I(x,y)) \]

\[ D_2 = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \]

• Bayer’s ordered dither matrices

\[ D_o = \begin{bmatrix} 4D_{y2} + D_{1,1} & 4D_{y2} + D_{1,2} \\ 4D_{y2} + D_{2,1} & 4D_{y2} + D_{2,2} \end{bmatrix} \]

\[ D_2 = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \]

\[ D_2 = \begin{bmatrix} 15 & 13 & 5 \\ 3 & 11 & 9 \\ 12 & 14 & 6 \\ 0 & 8 & 10 \end{bmatrix} \]

Ordered Dither

Original (8 bits)

Uniform Quantization (1 bit)

4x4 Ordered Dither (1 bit)

Error Diffusion Dither

• Spread quantization error over neighbor pixels
  - Error dispersed to pixels right and below

\[ \alpha + \beta + \gamma + \delta = 1.0 \]

Figure 14.42 from H&B
Overview

- Image representation
  - What is an image?
- Halftoning and dithering
  - Reduce visual artifacts due to quantization
  - Sampling and reconstruction
  - Reduce visual artifacts due to aliasing
Aliasing

- In general:
  - Artifacts due to under-sampling or poor reconstruction
- Specifically, in graphics:
  - Spatial aliasing
  - Temporal aliasing

Spatial Aliasing

- Artifacts due to limited spatial resolution

Temporal Aliasing

- Artifacts due to limited temporal resolution
  - Strobing
  - Flickering
### Temporal Aliasing

- Artifacts due to limited temporal resolution
  - Strobing
  - Flickering

### Antialiasing

- Sample at higher rate
  - Not always possible
  - Doesn’t always solve problem
- Pre-filter to form bandlimited signal
  - Form bandlimited function (low-pass filter)
  - Trades aliasing for blurring

---

Must consider sampling theory!
Sampling Theory

- How many samples are required to represent a given signal without loss of information?
- What signals can be reconstructed without loss for a given sampling rate?

Spectral Analysis

- Spatial domain:
  - Function: \( f(x) \)
  - Filtering: convolution
- Frequency domain:
  - Function: \( F(u) \)
  - Filtering: multiplication

Any signal can be written as a sum of periodic functions.

Fourier Transform

- Fourier transform:
  \[
  F(u) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i u x} \, dx
  \]
- Inverse Fourier transform:
  \[
  f(x) = \int_{-\infty}^{\infty} F(u) e^{2\pi i u x} \, du
  \]
### Sampling Theorem
- A signal can be reconstructed from its samples if the original signal has no frequencies above 1/2 the sampling frequency.
- Nyquist rate (or Nyquist limit)

A signal is bandlimited if its highest frequency is bounded. The frequency is called the bandwidth.

### Convolution
- Convolution of two functions (= filtering):

\[ g(x) = f(x) \otimes h(x) = \int_{-\infty}^{\infty} f(\lambda) h(x - \lambda) d\lambda \]

- Convolution theorem
  - Convolution in frequency domain is same as multiplication in spatial domain, and vice-versa

### Image Processing
- **Quantization**
  - Uniform Quantization
  - Random dither
  - Ordered dither
  - Floyd-Steinberg dither
- **Pixel operations**
  - Add random noise
  - Add luminance
  - Add contrast
  - Add saturation
- **Filtering**
  - Blur
  - Detect edges
- **Warping**
  - Scale
  - Rotate
  - Warps
- **Combining**
  - Morphs
  - Composite

### Consider reducing the image resolution
- **Original image**
- **1/4 resolution**
Image Processing

• Image processing is a resampling problem

Antialiasing in Image Processing

• General Strategy
  - Pre-filter transformed image via convolution with low-pass filter to form bandlimited signal

• Rationale
  - Prefer blurring over aliasing

 thou shalt avoid aliasing!
Image Processing

Real world
Sample
Discrete samples (pixels)
Reconstruct
Reconstructed function
Transform
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display

Discrete Samples

Real world
Sample
Discrete samples (pixels)
Reconstruct
Reconstructed function
Transform
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display

Reconstructed Function

Real world
Sample
Discrete samples (pixels)
Reconstruct
Reconstructed function
Transform
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display

Transformed Function

Real world
Sample
Discrete samples (pixels)
Reconstruct
Reconstructed function
Transform
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display

Bandlimited Function
**Image Processing**

- Sample
- Discrete samples (pixels)
- Reconstruct
- Reconstructed function
- Transform
- Transformed function
- Filter
- Bandlimited function
- Sample
- Discrete samples (pixels)
- Reconstruct
- Display

**Ideal Low-Pass Filter**

- Frequency domain
- Spatial domain

\[ Sinc(x) = \frac{\sin \pi x}{\pi x} \]

*Figure 4.5 Wolberg*

**Practical Image Processing**

- Finite low-pass filters
  - Point sampling (bad)
  - Triangle filter
  - Gaussian filter

- Real world
- Sample
- Discrete samples (pixels)
- Reconstruct
- Reconstructed function
- Transform
- Transformed function
- Filter
- Bandlimited function
- Sample
- Discrete samples (pixels)
- Reconstruct
- Display
**Triangle Filter**
- Convolution with triangle filter

**Gaussian Filter**
- Convolution with Gaussian filter

**Image Processing**
- Quantization
  - Uniform Quantization
  - Random dither
  - Ordered dither
  - Floyd-Steinberg dither
- Pixel operations
  - Add random noise
  - Add luminance
  - Add contrast
  - Add saturation
- Filtering
  - Blur
  - Detect edges
- Warping
  - Scale
  - Rotate
  - Warps
- Combining
  - Morphs
  - Composite

**Brightness**
- Simply scale pixel components
  - Must clamp to range (e.g., 0 to 255)
- Trick: interpolate (extrapolate) from a black image
Contrast

- Compute mean luminance $\bar{L}$ for all pixels
  - $\text{luminance} = 0.30*r + 0.59*g + 0.11*b$
- Scale deviation from $\bar{L}$ for each pixel component
- Interpolate (extrapolate) from an average gray image

Image Processing

- Quantization
  - Uniform Quantization
  - Random dither
  - Ordered dither
  - Floyd-Steinberg dither
- Pixel operations
  - Add random noise
  - Add luminance
  - Add contrast
  - Add saturation
- Filtering
  - Blur
  - Detect edges
- Warping
  - Scale
  - Rotate
  - Warps
- Combining
  - Morphs
  - Composite

Blur and Sharpen

- Convolve with a filter whose entries sum to one
  - Each pixel becomes a weighted average of its neighbors
- Trick: extrapolate from blurry image = sharpen!
  - "Unsharp mask" in Photoshop

Saturation

- Interpolate (extrapolate) from grayscale version

Filter =
\[
\begin{bmatrix}
\frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\
\frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\
\frac{1}{6} & \frac{1}{6} & \frac{1}{6}
\end{bmatrix}
\]
Edge Detection
• Convolve with a filter that finds differences between neighbor pixels

Original Detect edges

Filter = [-1 -1 -1]
[-1 8 -1]
[-1 -1 -1]

Image Processing
• Quantization
  ◦ Uniform Quantization
  ◦ Random dither
  ◦ Ordered dither
  ◦ Floyd-Steinberg dither

• Pixel operations
  ◦ Add random noise
  ◦ Add luminance
  ◦ Add contrast
  ◦ Add saturation

• Filtering
  ◦ Blur
  ◦ Detect edges

• Warping
  ◦ Scale
  ◦ Rotate
  ◦ Warps

• Combining
  ◦ Morphs
  ◦ Composite

Scaling
• Resample with triangle or Gaussian filter

Original ½ resolution 2x resolution

More on this next lecture!

Image Processing
• Image processing is a resampling problem
  ◦ Avoid aliasing
  ◦ Use filtering
Summary

- Image representation
  - A pixel is a sample, not a little square
  - Images have limited resolution

- Halftoning and dithering
  - Reduce visual artifacts due to quantization
  - Distribute errors among pixels
    » Exploit spatial integration in our eye

- Sampling and reconstruction
  - Reduce visual artifacts due to aliasing
  - Filter to avoid undersampling
    » Blurring is better than aliasing