<table>
<thead>
<tr>
<th>Overview</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Image representation</td>
</tr>
<tr>
<td>• What is an image?</td>
</tr>
<tr>
<td>• Halftoning and dithering</td>
</tr>
<tr>
<td>• Trade spatial resolution for intensity resolution</td>
</tr>
<tr>
<td>• Reduce visual artifacts due to quantization</td>
</tr>
<tr>
<td>• Sampling and reconstruction</td>
</tr>
<tr>
<td>• Key steps in image processing</td>
</tr>
<tr>
<td>• Avoid visual artifacts due to aliasing</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Image Processing and Sampling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Greg Humphreys</td>
</tr>
<tr>
<td>CS445: Intro Graphics</td>
</tr>
<tr>
<td>University of Virginia, Fall 2004</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>What is an Image?</th>
</tr>
</thead>
<tbody>
<tr>
<td>• An image is a 2D rectilinear array of pixels</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Image Acquisition</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Pixels are samples from continuous function</td>
</tr>
<tr>
<td>• Photoreceptors in eye</td>
</tr>
<tr>
<td>• CCD cells in digital camera</td>
</tr>
<tr>
<td>• Rays in virtual camera</td>
</tr>
</tbody>
</table>

| A pixel is a sample, not a little square! |

<table>
<thead>
<tr>
<th>Image Acquisition Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Camera</td>
</tr>
<tr>
<td>View Plane</td>
</tr>
</tbody>
</table>
Image Display

- Re-create continuous function from samples
  - Example: cathode ray tube

  ![Image is reconstructed by displaying pixels with finite area (Gaussian)]

Image Resolution

- Intensity resolution
  - Each pixel has only “Depth” bits for colors/intensities
- Spatial resolution
  - Image has only “Width” x “Height” pixels
- Temporal resolution
  - Monitor refreshes images at only “Rate” Hz

<table>
<thead>
<tr>
<th>Typical Resolutions</th>
<th>Width x Height</th>
<th>Depth</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>NTSC</td>
<td>640 x 480</td>
<td>8</td>
<td>30</td>
</tr>
<tr>
<td>Workstation</td>
<td>1280 x 1024</td>
<td>24</td>
<td>75</td>
</tr>
<tr>
<td>Film</td>
<td>3000 x 2000</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>Laser Printer</td>
<td>6600 x 5100</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>

Sources of Error

- Intensity quantization
  - Not enough intensity resolution
- Spatial aliasing
  - Not enough spatial resolution
- Temporal aliasing
  - Not enough temporal resolution

\[ E^2 = \sum_{(x,y)} (I(x,y) - P(x,y))^2 \]

Overview

- Image representation
  - What is an image?
  - Halftoning and dithering
    - Reduce visual artifacts due to quantization
- Sampling and reconstruction
  - Reduce visual artifacts due to aliasing

Quantization

- Artifacts due to limited intensity resolution
  - Frame buffers have limited number of bits per pixel
  - Physical devices have limited dynamic range

Uniform Quantization

\[ P(x, y) = \text{trunc}(I(x, y) + 0.5) \]

P(x, y) = \text{trunc}(I(x, y) + 0.5)

I(x,y)

P(x,y)

(2 bits per pixel)
Uniform Quantization

- Images with decreasing bits per pixel:
  - 8 bits
  - 4 bits
  - 2 bits
  - 1 bit

Reducing Effects of Quantization

- Halftoning
  - Classical halftoning
- Dithering
  - Random dither
  - Ordered dither
  - Error diffusion dither

Classical Halftoning

- Use dots of varying size to represent intensities
  - Area of dots proportional to intensity in image

Classical Halftoning

Newspaper image from North American Bridge Championships Bulletin, Summer 2003

Halftone patterns

- Use cluster of pixels to represent intensity
  - Trade spatial resolution for intensity resolution

Dithering

- Distribute errors among pixels
  - Exploit spatial integration in our eye
  - Display greater range of perceptible intensities

Figure 14.37 from H&B
Random Dither

- Randomize quantization errors
  - Errors appear as noise

\[ P(x, y) = \text{trunc}(I(x, y) + \text{noise}(x, y) + 0.5) \]

Ordered Dither

- Pseudo-random quantization errors
  - Matrix stores pattern of thresholds

\[
\begin{align*}
i &= x \mod n \\
 j &= y \mod n \\
e &= I(x, y) - \text{trunc}(I(x, y)) \\
\text{if } (e > D(i, j)) \\
P(x, y) &= \text{ceil}(I(x, y)) \\
\text{else} \\
P(x, y) &= \text{floor}(I(x, y))
\end{align*}
\]

Error Diffusion Dither

- Spread quantization error over neighbor pixels
  - Error dispersed to pixels right and below

\[ \alpha + \beta + \gamma + \delta = 1.0 \]

Figure 14.42 from H&B
Dither Comparison

Original (8 bits)  |  Random Dither (1 bit)  |  Ordered Dither (1 bit)  |  Floyd-Steinberg Dither (1 bit)

Overview

• Image representation
  ▶ What is an image?
• Halftoning and dithering
  ▶ Reduce visual artifacts due to quantization
  ▶ Sampling and reconstruction
    ▶ Reduce visual artifacts due to aliasing

Sampling and Reconstruction

Sampling and Reconstruction

Sampling

Reconstruction

Figure 19.9 FvDFH

Aliasing

• In general:
  ▶ Artifacts due to under-sampling or poor reconstruction
• Specifically, in graphics:
  ▶ Spatial aliasing
  ▶ Temporal aliasing

Spatial Aliasing

• Artifacts due to limited spatial resolution

Figure 14.17 FvDFH

Figure 14.17 FvDFH

Under-sampling
### Spatial Aliasing
- Artifacts due to limited spatial resolution
  - *Jaggies*

### Temporal Aliasing
- Artifacts due to limited temporal resolution
  - Strobing
  - Flickering

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### Temporal Aliasing
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### Antialiasing
- Sample at higher rate
  - Not always possible
  - Doesn’t always solve problem
- Pre-filter to form bandlimited signal
  - Form bandlimited function (low-pass filter)
  - Trades aliasing for blurring

---

Must consider sampling theory!
### Sampling Theory
- How many samples are required to represent a given signal without loss of information?
- What signals can be reconstructed without loss for a given sampling rate?

### Spectral Analysis
- **Spatial domain:**
  - Function: \( f(x) \)
  - Filtering: convolution

- **Frequency domain:**
  - Function: \( F(u) \)
  - Filtering: multiplication

Any signal can be written as a sum of periodic functions.

### Fourier Transform
- **Fourier transform:**
  \[ F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi xu} \, dx \]

- **Inverse Fourier transform:**
  \[ f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi xu} \, du \]

### Convolution
- **Convolution of two functions (= filtering):**
  \[ g(x) = f(x) \ast h(x) = \int_{-\infty}^{\infty} f(\lambda) h(x-\lambda) \, d\lambda \]

- **Convolution theorem**
  - Convolution in frequency domain is same as multiplication in spatial domain, and vice-versa

A signal is bandlimited if its highest frequency is bounded. The frequency is called the bandwidth.
### Image Processing

<table>
<thead>
<tr>
<th>Quantization</th>
<th>Filtering</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform Quantization</td>
<td>Blur</td>
</tr>
<tr>
<td>Random dither</td>
<td>Detecedges</td>
</tr>
<tr>
<td>Ordered dither</td>
<td></td>
</tr>
<tr>
<td>Floyd-Steinberg dither</td>
<td></td>
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<thead>
<tr>
<th>Pixel operations</th>
<th>Warping</th>
</tr>
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<tr>
<td>Add random noise</td>
<td>Scale</td>
</tr>
<tr>
<td>Add luminance</td>
<td>Rotate</td>
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<tr>
<td>Add contrast</td>
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</tr>
<tr>
<td>Add saturation</td>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Morphs</td>
<td></td>
</tr>
<tr>
<td>Composite</td>
<td></td>
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</table>

### Image Processing

- Consider reducing the image resolution
- Image processing is a resampling problem

![Image Processing Diagram](image.png)

**Thou shalt avoid aliasing!**

### Antialiasing in Image Processing

- General Strategy
  - Pre-filter transformed image via convolution with low-pass filter to form bandlimited signal
- Rationale
  - Prefer blurring over aliasing

### Image Processing

- Real world
  - Sample
  - Discrete samples (pixels)
  - Reconstruct
  - Reconstructed function
  - Transform
  - Transformed function
  - Filter
  - Bandlimited function
  - Sample
  - Discrete samples (pixels)
  - Reconstruct
  - Display

- Continuous Function

![Image Processing Diagram](image.png)
Image Processing

Real world
Sample
Discrete samples (pixels)
Reconstruct
Transform
Reconstructed function
Transformed function
Filter
Bandlimited function
Sample
Discrete samples (pixels)
Reconstruct
Display

Image Processing

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**Ideal Low-Pass Filter**

- **Frequency domain**
  
- **Spatial domain**

Sinc(x) = \(\frac{\sin \pi x}{\pi x}\)

**Practical Image Processing**

- **Finite low-pass filters**
  - Point sampling (bad)
  - Triangle filter
  - Gaussian filter

**Triangle Filter**

- Convolution with triangle filter

**Gaussian Filter**

- Convolution with Gaussian filter

**Image Processing**

- **Quantization**
  - Uniform Quantization
  - Random dither
  - Ordered dither
  - Floyd-Steinberg dither

- **Pixel operations**
  - Add random noise
  - Add luminance
  - Add contrast
  - Add saturation

- **Filtering**
  - Blur
  - Detect edges

- **Warping**
  - Scale
  - Rotate
  - Warps

- **Combining**
  - Morphs
  - Composite

**Brightness**

- Simply scale pixel components
  - Must clamp to range (e.g., 0 to 255)
- Trick: interpolate (extrapolate) from a black image

**Figure 4.5 Wolberg**

**Figure 2.4 Wolberg**
### Contrast

- Compute mean luminance $L$ for all pixels
  - Luminance = $0.30*r + 0.59*g + 0.11*b$
- Scale deviation from $L$ for each pixel component
- Interpolate (extrapolate) from an average gray image

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  - Blur
  - Detect edges
- Warping
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  - Rotate
  - Warps
- Combining
  - Morphs
  - Composite

### Blur and Sharpen

- Convolve with a filter whose entries sum to one
  - Each pixel becomes a weighted average of its neighbors
- Trick: extrapolate from blurry image = sharpen!
  - "Unsharp mask" in Photoshop

### Saturation

- Interpolate (extrapolate) from grayscale version

### Edge Detection

- Convolve with a filter that finds differences between neighbor pixels

- Filter = 

\[
\begin{bmatrix}
-1 & -1 & -1 \\
-1 & 8 & -1 \\
-1 & -1 & -1 \\
\end{bmatrix}
\]
Scaling

- Resample with triangle or Gaussian filter

```
Original  0.5 resolution  2x resolution
```

More on this next lecture!

Image Processing

- Image processing is a resampling problem
  - Avoid aliasing
  - Use filtering

Summary

- Image representation
  - A pixel is a sample, not a little square
  - Images have limited resolution
- Halftoning and dithering
  - Reduce visual artifacts due to quantization
  - Distribute errors among pixels
    » Exploit spatial integration in our eye
- Sampling and reconstruction
  - Reduce visual artifacts due to aliasing
  - Filter to avoid undersampling
    » Blurring is better than aliasing