Midterm Exam
Introduction to Computer Graphics
CS 445/645
March 2, 2004
Professor Brogan

Time Limit: 1 hour and 15 minutes
Reference Materials: Closed book, closed notes, no calculator

Please write and sign the honor pledge:

Notational convention: bold lower case means vector and bold upper case means matrix. (so \( q \) is a scalar, \( \mathbf{q} \) is a vector, and \( \mathbf{Q} \) is a matrix)

All coordinate systems will be assumed right-handed.

Note that some questions take longer than others and some questions are worth more than others. Use your time wisely.

Because you're not allowed to use calculators, you should feel free to leave your answers in intermediate forms: unmultiplied, including trig functions, including square roots, cross products etc.

Display Technology (20 points)

1. (2 points) How does a cathode ray tube convert electricity into light?
   a) Light hits pixels
   b) Photons pass through a prism
   c) Pixels are turned on
   d) Electrons hit phosphors
   e) A filament glows
2. (2 points) Provide one explanation for why Macromedia Flash uses vector-based graphics?

Reduce transmission time
Simplicity of image scaling

3. (2 points) What happens if the shadow mask is not aligned properly?

Energy intended for one phosphor strikes another ... blurry, wrong colors

4. (3 points) How many times must the framebuffer be updated for each refresh of a non-interlaced display device in order to produce a proper interface between the computer and the monitor?

Once

What happens if the framebuffer update occurs at the proper frequency, but it is not synchronized with the display refresh?

Images in time

\[
\begin{array}{c|c}
0 & t_0 \\
--- & --- \\
t & 1 \\
\end{array}
\]

5. (5 points)

American television is shown in (approximately) 30 frames/second
There are 2 fields per frame in an interlaced monitor

American movies are display at 24 frames/second
There are 2 flickers per frame in American movies

Critical fusion frequency is (approximately) 60 frames/second

6. (2 points) True or false, a liquid crystal display emits polarized light?

7. (2 points) If my LCD has a passive matrix, can my computer provide constant illumination to a specific pixel, yes or no?

8. (2 points) If your display controller is using a color lookup table, what information is stored with each pixel in the framebuffer?

Index into a color lookup table where the RGB values of the color are stored.
Mathematical Foundations (16 points)

9. (5 points) Write the definition of $x'$ and $y'$ as functions of $x$, $y$, and theta (and trig functions).

$$x' = x \cdot \cos(\theta) - y \cdot \sin(\theta)$$

$$y' = x \cdot \sin(\theta) + y \cdot \cos(\theta)$$

10. (6 points) You are given the implicit equation of a plane, $Ax + By + Cz + D = 0$ (such that $\| (A, B, C) \| = 1$).
   
a. Compute a point on the plane.
      
      Examples: \((-\frac{D}{A}, 0, 0), (0, -\frac{D}{B}, 0), (0, 0, -\frac{D}{C})\),
      
      \((-\frac{D}{A}, -\frac{D}{B}, -\frac{D}{C})\)
   
   b. Compute a vector that is in the plane and parallel to the x/z plane (the ground).
      
      \((0, 1, 0) \times (A, B, C) \text{ unless } (A, B, C) \text{ is parallel to } (0, 1, 0)\)

11. (5 points) Let $\mathbf{v} = [3, 5, 2]$
   
a. What is the length of $\mathbf{v}$?
      
      $$\sqrt{3^2 + 5^2 + 2^2} = \sqrt{38}$$
   
   b. What angle does $\mathbf{v}$ make with the x axis $= [1, 0, 0]$?
      
      $$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \cdot \|\mathbf{v}\| \cos(\theta)$$
      
      $$\cos^{-1}(\theta) = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \cdot \|\mathbf{v}\|} = \frac{3}{\sqrt{38}}$$
Transformations (8 points)

12. (3 points) When we rotate \( q \) degrees about the x-axis, what direction do we turn?

- a) Counter clockwise looking along the negative x-axis
- b) Clockwise looking along the negative x-axis

13. (5 points) Use a homogeneous, two-dimensional coordinate space to combine transformations such that a house positioned at \((14, 4)\) is scaled to be half the original size, rotated 90 degrees clockwise, and moved to \((2, 10)\). Write your answer as a sequence of 3x3 matrix transformations followed by a multiplication by a 3x1 column vector corresponding to a vertex’s coordinates. You need not multiply them together.

\[
\begin{pmatrix}
1 & 0 & 2 \\
0 & 1 & 10 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\cos (-90) & -\sin (-90) & 0 \\
\sin (-90) & \cos (-90) & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\frac{1}{2} & 0 & 0 \\
0 & \frac{1}{2} & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & -14 \\
0 & 1 & -4 \\
0 & 0 & 1
\end{pmatrix}
\]
Rotation Representations (9 points)

14. (3 points) What is gimbal lock? Provide an example Euler Angle (and sequence of operations) that produces a gimbal lock.

Rotation about one axis aligns two others such that rotations about the aligned axes have the same effect. One degree of freedom is lost. If order is x, y, z,... any rotation like x, 90, z

15. (3 points) Why is it sensible to use the nine numbers of a 3x3 rotation matrix to represent the three terms corresponding to an object’s orientation in three-dimensional space? We would typically find it impossible (underconstrained) to derive the nine unknowns in the rotation matrix from the three orientation parameters alone.

Ambiguous Question. Not graded.

16. (3 points) Is every orientation uniquely defined by one Euler Angle, yes or no?

Rasterization and Clipping (17 points)

17. (3 points) Which clipping algorithm is best suited to rendering a scene composed of millions of triangles (say a CAD model of a battleship) and a camera that has a viewing frustum containing only the captain’s instrument cluster, Cohen-Sutherland or Cyrus-Beck?

18. (4 points) Describe in English when the clipping of a line segment is considered to be a trivial case?

When both points are inside the viewport (window) or both endpoints are on the outside of any one of the four lines defining the viewport.

Provide the Cohen-Sutherland out codes (region code) of two end points of such a segment.

Any such that outcode A & & & outcode B ≠ 0000

19. (3 points) In the clipping programming assignment, the original models were composed of triangles. Were the clipped polygons triangles, yes or no?
20. (3 points) When we studied line rasterization, we first considered an algorithm that used one segment of code to render all lines (no rotation of the line by 90 degrees). It did not render lines with slopes greater than one correctly. In the worst case, a line would be represented by how many pixels no matter how long it was?

21. (5 points) When building an efficient polygon rasterization algorithm like the active edge table:

a. We first compute the set of edges that have what relationship with the current scanline?

Intersection

b. We then order the edges in that set according to what property?

X-coordinate value of the intersection point

c. We then consider pixels on the scanline from left to right using the following operation to decide if we are inside or outside the polygon:

Parity count

OpenGL (18 points)

22. (3 points) Which best describes the OpenGL camera?
   a) Looks down the z-axis of a right-hand coordinate system
   b) Looks down the negative z-axis of a left-hand coordinate system
   c) Looks down the x-axis of a right-hand coordinate system
   d) Looks down the negative y-axis of a left-hand coordinate system
   e) Looks down the negative z-axis of a right-hand coordinate system

23. (3 points) What are the names of two stacks in the OpenGL environment?
24. (6 points) For assignment 1, you had to use transformation matrices to move a firetruck’s ladder. You are provided with a function called `drawLadder()` that places the ladder in the following position when called with a MODELVIEW matrix equal to identity.

\[ a = (-2, 0, 0) \quad b = (2, 0, 0) \]

You must write the sequence of OpenGL transformations that will:

1) Move endpoint \( a \) to the coordinate \((10, 10, 0)\)
2) Raise the ladder (so endpoint \( b \) is above \( a \)) by 30 degrees
3) Rotate the ladder about the y axis by 45 degrees so endpoint \( b \) comes out of the page towards the reader

```gl
glTranslatef(10, 10, 0)
glRotatef(30, 0, 0, 1)
glRotatef(-45, 0, 1, 0)
glTranslatef(2, 0, 0)
drawLadder()
```

25. (3 points) When calling `glPushMatrix()`
   a) the identity matrix is put on top of the stack
   b) the matrix passed as an argument is put on top of the stack
   c) the matrix on the top of the stack is replaced with the identity matrix
   d) the top matrix on the stack is duplicated and put on top of the stack

Circled: d)
26. (3 points) Which of the following sequences of matrix multiplies accurately depicts what happens as OpenGL parses the following code:

\[
\begin{align*}
glLoadIdentity(); \quad & \text{\textbackslash \textbar{} represented as } \mathbf{I} \\
glTranslatef (4, 3, -2); \quad & \text{\textbackslash \textbar{} represented at } \mathbf{T} \\
glRotatef (87, 0, 1, 0); \quad & \text{\textbackslash \textbar{} represented as } \mathbf{R} \\
glScalef (1.0, 0.4, 1.3); \quad & \text{\textbackslash \textbar{} represented as } \mathbf{S} \\
glBegin (POINTS); \\
glVertex3f (\mathbf{v}); \quad & \text{\textbackslash \textbar{} represented as } \mathbf{v} \\
glEnd ( ); \\
a) \mathbf{vSTRS} \\
b) \mathbf{vSRTI} \\
c) \mathbf{SRTIV} \\
d) \mathbf{vITRS}
\end{align*}
\]

**Viewing in 3-D (11 points)**

27. (5 points) What is the transformation matrix, \(\mathbf{V}\), that converts points from world coordinates to camera coordinates such that the camera’s y-axis (up vector) is the world’s y-axis, the camera looks down the world’s x-axis, and the camera has been translated to \((5, 0, 5)\). For example, the world coordinate \((5, 0, 5)\) should be transformed to \((0, 0, 0)\) according to: \(\mathbf{V}\)

\[
\begin{bmatrix}
5 \\
0 \\
5 \\
1
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 0 & 0 & -5 \\
0 & 1 & 0 & 0 \\
-1 & 0 & 0 & 5 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Method 1:

\[
\begin{bmatrix}
\cos (90) & \sin (90) & 0 \\
0 & 1 & 0 \\
-\sin (90) & \cos (90) & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & 0 & -5 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & -5 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Rotate \(-2 \Rightarrow x\)  
Move \((0, 0, 5)\) to origin

Method 2: \(\mathbf{up} = [0, 1, 0]\), \(\mathbf{gaze} = [1, 0, 0]\), \(\mathbf{eye} = [5, 0, 5]\)

\[
\begin{align*}
\mathbf{l} \cdot \frac{\mathbf{gaze}}{\|\mathbf{gaze}\|} &= [1, 0, 0] \\
\mathbf{r} &= \mathbf{l} \times \mathbf{up} = [1, 0, 0] \times [0, 1, 0] = 0, 0, 1 \\
\mathbf{u} &= \mathbf{r} \times \mathbf{l} = [0, 0, 1] \times [1, 0, 0] = [0, 1, 0]
\end{align*}
\]

\[
\mathbf{V} = \begin{bmatrix}
\mathbf{r} & -\mathbf{r} \cdot \mathbf{eye} \\
\mathbf{u} & -\mathbf{u} \cdot \mathbf{eye} \\
\mathbf{l} & \mathbf{e} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Page 8 of 9
28. (3 points) When did artists first demonstrate mastery of a perspective effect?
   a) 1000 – 1250
   b) 1250 – 1500  
   c) 1500 – 1750
   d) 1750 – 2000

29. (3 points) What is an example of a perspective effect?

   Converging lines
   Vanishing point

Extra Credit

Ex 1. (3 points) The Cavalier Projection is a parallel (orthographic) projection. The lengths of lines parallel to the view plane are preserved. But why does the Cavalier Projection cause the length of lines perpendicular to the view plane to be preserved as well? For example, this rendering of a unit cube was generated with a Cavalier Projection.

Ex 2. (3 points) Professor Brogan referred to quaternions as a four-dimensional hypersphere, which he then related to what kind of a ball?

Hairy