feature detection; hough transform
edges

\[ I(x, y) \]

\[ \left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right) \]
corners

\[ I(x, y) \]

\[ \left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right) \]
gradient covariance matrix

\[ C = \begin{pmatrix}
\sum I_x^2 & \sum I_x I_y \\
\sum I_x I_y & \sum I_y^2
\end{pmatrix} \]
case #1: uniform

\[
\left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right) = (?, ?)
\]
case #2: single edge

\[
\left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right) = (a, b)
\]
case #3: corner

\[ \left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right) = (a, b) \text{ and } (c, d) \]
Tomasi-Kanade corner detector

- C has one large eigenvalue $\Rightarrow$ edge
- C has two large eigenvalues $\Rightarrow$ corner
implementation

• compute image gradient

• for each M x M neighborhood, compute C

• if smaller eigenvalue is larger than threshold, record a corner (MATLAB: eig)

• nonmaximum suppression: keep strongest corner in each M x M window
corner detection

- Application: good features for tracking, image correspondence, etc.
- Why do corners make better features than edges?
- Other corner detectors
  - Curvature in edge detector output
  - Color segmentation in neighborhoods
  - Others...
“good” image features (small neighborhoods of pixels)

[Aliaga et al. 2003]
camera calibration
3d scene reconstruction from image sequences
Photo Tourism project

http://phototour.cs.washington.edu/
detecting lines
detecting lines
50% threshold

70% threshold

Image Credits: Bob Fisher
Least-squares minimization
total least squares
total least squares
outliers

- least squares assumes Gaussian errors
- **outliers**: points with extremely low probability of occurrence (according to Gaussian statistic)
  - Can result from data association errors
  - Strongly influence least squares
robust estimation

• Goal: develop parameter estimation methods insensitive to small numbers of large errors

• General approach: try to give large deviations less weight

• M-estimators: minimize some function other than $(y-f(x,a,b,...))^2$
least absolute value fitting

- Minimize $\sum |y_i - f(x_i, a, b, ...)|$

  instead of $\sum (y_i - f(x_i, a, b, ...))^2$

- Points far away from trend get comparatively less influence
example: constant

- For constant function $y = a$, minimizing squares gives $a = \text{mean}$
- Minimizing absolute value gives $a = \text{median}$
RANSAC

- **RAN**dom **SA**mple **C**onsensus: designed for bad data (in best case, up to 50% outliers)
- Take many random subsets of data
  - Compute least squares fit for each sample
  - See how many points agree: \((y_i - f(x_i))^2 < \text{threshold}\)
  - Threshold user-specified or estimated from more trials
  - At end, use fit that agreed with most points
  - Can perform one final LS with all inliers