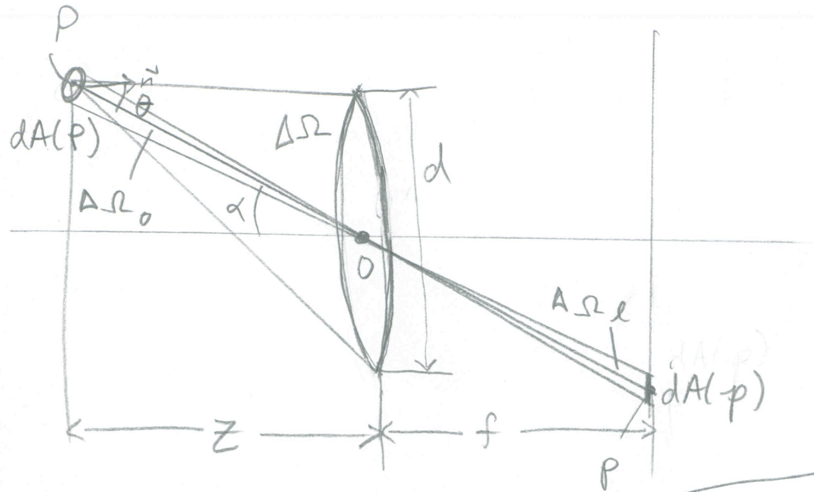

Wish to compute: irradiance at image sensor location p

$$E = \frac{d\Phi}{dA(p)}$$

← flux at p

↖ differential area at p (projection of diff area dA(P))

as function of:

f : focal length of lens

d : lens diameter

L(P) : radiance leaving P, traveling in direction toward lens (assume constant)

① compute flux at lens

$$d\Phi = dA(P)\, L(P)\, \Delta\Omega \cos\theta$$

- dA(P): area of "light source"
- L(P): radiance of source
- $\Delta\Omega$: solid angle of lens
- $\cos\theta$: foreshortening of dA(P) as seen from lens

② compute

$$\Delta\Omega = \frac{\pi}{4} d^2 \cos\alpha \left(\frac{\cos^2\alpha}{Z^2}\right)$$

- $\frac{\pi}{4} d^2$: area
- $\cos\alpha$: foreshortening term
- $\left(\frac{\cos^2\alpha}{Z^2}\right)$: $\frac{1}{r^2}$

③ compute $\Delta\Omega_\ell$ solid angle of dA(p) as seen from lens

$$\Delta\Omega_\ell = \frac{dA(p)\cos\alpha}{(f/\cos\alpha)^2}$$

④ compute $\Delta\Omega_0$ solid angle of dA(P) as seen from lens.

$$\Delta\Omega_0 = \frac{dA(P)\cos\theta}{(Z/\cos\alpha)^2}$$

⑤ note $\Delta\Omega_0 = \Delta\Omega_\ell$ so

$$\frac{dA(P)}{dA(p)} = \frac{\cos\alpha}{\cos\theta}\left(\frac{Z}{f}\right)^2$$

$$\therefore E = L(P)\,\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4\alpha$$

value at sensor is function of L(P) and depends on $\left(\frac{d}{f}\right)^2$ (f-number) and $\cos^4\alpha$ falloff