Progressive Meshes

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Garland & Heckbert 97

- Surface simplification using quadric error metrics
- Presented at SIGGRAPH in 1997
- Greedy decimation algorithm
- Pair collapse (allow edge + non-edge collapses)

Quadric error metrics:
- Evaluate potential collapses
- Determine optimal new vertex locations
General Strategy

- Contract valid pair of vertices w/ lowest error
- Based on point-to-plane distance
Recall distance from point to plane

\[ p = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \]

\[ v = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \]

\[ E_p(v) = D_p^2(v) \]

\[ E_p(v) = (ax + by + cz + d)^2 \]

\[ E_p(v) = p^T v \]
Preliminaries

Associate to each vertex a set of planes

\[ P_i = \{A, B\} \]
\[ P_j = \{B, C\} \]
Quadric Error Metric

Define error at each vertex as sum of point to plane distances:

\[ E_{P_i}(v) = \sum_{p \in P_i} (p^T v)^2 \]

\[ E_{P_i}(v) = \sum_{p \in P_i} v^T p p^T v \]

\[ E_{P_i}(v) = v^T \left( \sum_{p \in P_i} p p^T \right) v \]

\[ E_{P_i}(v) = v^T Q_i v \]
Error of Pair Collapse

\[
E \{(v_i, v_j) \rightarrow \bar{v}\} = \bar{v}^T Q_i \bar{v} + \bar{v}^T Q_j \bar{v}
\]

\[
E \{(v_i, v_j) \rightarrow \bar{v}\} = \bar{v}^T (Q_i + Q_j) \bar{v}
\]

\[
E \{(v_i, v_j) \rightarrow \bar{v}\} = \bar{v}^T (\bar{Q}) \bar{v}
\]
Using Quadrics

How do we compute this?

$$\min_{\vec{v}} \vec{v}^T \bar{Q} \vec{v}$$

$$Q = \begin{bmatrix}
q_{11} & q_{12} & q_{13} & q_{14} \\
q_{21} & q_{22} & q_{23} & q_{24} \\
q_{31} & q_{32} & q_{33} & q_{34} \\
q_{41} & q_{42} & q_{43} & q_{44}
\end{bmatrix}$$

$$\bar{v} = \begin{bmatrix}
q_{11} & q_{12} & q_{13} & q_{14} \\
q_{21} & q_{22} & q_{23} & q_{24} \\
q_{31} & q_{32} & q_{33} & q_{34} \\
0 & 0 & 0 & 1
\end{bmatrix}^{-1} \begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}$$
Update Quadric

- Remember: every vertex needs a quadric!
- Quadric for new vertex is simply the sum of the quadrics for the two vertices it replaces
- Effectively captures the “history” of the vertices removed previously

\[ \bar{Q} = Q_i + Q_j \]
Why “Quadric”? 

- Quadric Surface = locus of zeros of a quadratic polynomial
- The isocontours of the quadric error metric trace out ellipsoids (Exercise 1)

\[ \bar{v}^T \bar{Q} \bar{v} = k \]
Summary

- Input: triangle mesh
- Compute the quadric of each vertex
- Determine valid pairs
- Compute optimal contraction target and associated quadric error for each pair
- Place pairs on a heap, ordered by smallest error

Repeat:
- Remove pair \((v_i,v_j)\) with least error from heap
- Contract pair and remove degenerate planes
- Update cost for all pairs involving \(v_i\) and \(v_j\)

Until done.
View-Dependent Extension

- Simplify dynamically according to viewpoint

Hoppe
View-Dependent Extension

```
1m 0 [GL sme ap ]
n faces=213 pixel_tol=0.29
```
Preserving Appearance

7,809 tris

3,905 tris

1,951 tris

488 tris

975 tris

Caltech & Stanford Graphics Labs and Jonathan Cohen