Shading and Visibility

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CS 4810: Graphics

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3D Rendering Pipeline (for direct illumination)

- 3D Primitives
  - 3D Modeling Coordinates
    - Modeling Transformation
    - 3D World Coordinates
      - Camera Transformation
      - 3D Camera Coordinates
        - Lighting
        - 3D Camera Coordinates
          - Projection Transformation
          - 2D Screen Coordinates
            - Clipping
            - 2D Screen Coordinates
              - Viewport Transformation
              - 2D Image Coordinates
                - Scan Conversion
                - 2D Image Coordinates
                  - Image

3D Model

2D Window

2D Screen
3D Rendering Pipeline (for direct illumination)

3D Primitives
  → Modeling Transformation
    → Camera Transformation
      → Lighting
        → Projection Transformation
          → Clipping
            → Viewport Transformation
              → Scan Conversion
                → Image

3D Modeling Coordinates
3D World Coordinates
3D Camera Coordinates
3D Camera Coordinates
2D Screen Coordinates
2D Screen Coordinates
2D Image Coordinates
2D Image Coordinates

3D Model
2D Window
2D Screen
3D Rendering Pipeline (for direct illumination)

3D Primitives

- 3D Modeling Coordinates
  - Modeling Transformation
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  - 3D Camera Coordinates
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  - 2D Screen Coordinates
  - Viewport Transformation
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  - Scan Conversion
  - 2D Image Coordinates
  - Image

3D Model

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3D Rendering Pipeline (for direct illumination)

3D Primitives

Modeling Transformation

3D Modeling Coordinates

Camera Transformation

3D World Coordinates

Lighting

3D Camera Coordinates

Projection Transformation

3D Camera Coordinates

Clipping

2D Screen Coordinates

Viewport Transformation

2D Screen Coordinates

Scan Conversion

2D Image Coordinates

Image

3D Model

2D Window

2D Screen
Overview

- **Scan conversion**
  - Figure out which pixels to fill

- **Shading**
  - Determine a color for each filled pixel

- **Depth test**
  - Determine when the color of a pixel comes from the front-most primitive
Polygon Shading

- Simplest shading approach is to perform independent lighting calculation for every pixel.
  - When is this unnecessary?

\[ I = I_E + K_A I_{AL} + \sum_i \left( K_D (N \cdot L_i) I_i + K_S (V \cdot R_i)^n I_i \right) \]
Polygon Shading

- Can take advantage of spatial coherence
  - Illumination calculations for pixels covered by same primitive are related to each other

\[ I = I_E + K_A I_{AL} + \sum_i (K_D (N \cdot L_i) I_i + K_S (V \cdot R_i)^n I_i) \]
Polygon Shading Algorithms

- Flat Shading
- Gouraud Shading
- Phong Shading
Flat Shading

- Can take advantage of spatial coherence
  - Make the lighting equation constant over the surface of each primitive

\[ I = I_E + K_A I_{AL} + \sum_i (K_D (N \cdot L_i) I_i + K_S (V \cdot R_i)^n I_i) \]
Flat Shading

• Can take advantage of spatial coherence
  o Make the lighting equation constant over the surface of each primitive
    • If the normal is constant over the primitive, and
    • if the light is directional, the diffuse component is the same for all points on the primitive

\[ I = I_E + K_A I_{AL} + \sum_i (K_D (N \cdot L_i) I_i + K_S (V \cdot R_i)^n I_i) \]
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\[
I = I_E + K_A I_{AL} + \sum_i (K_D (N \cdot L_i) I_i + K_S (V \cdot R_i)^n I_i)
\]
Flat Shading

- Can take advantage of spatial coherence
  - Make the lighting equation constant over the surface of each primitive
    - If the normal is constant over the primitive, and
    - if the light is directional, the diffuse component is the same for all points on the primitive
    - If the normal is constant over the primitive, and
    - if the light is directional, and
    - if the direction to the viewer is constant over the primitive the specular component is the same for all points on the primitive

\[
I = I_E + K_A I_{AL} + \sum_i (K_D (N \cdot L_i) I_i + K_S (V \cdot R_i)^n I_i)
\]
Flat Shading

• Can take advantage of spatial coherence
  o Make the lighting equation constant over the surface of each primitive

• If the normal is constant over the primitive, and
• if the light is directional, the diffuse component is the same for all points on the primitive

• If the normal is constant over the primitive, and
• if the light is directional, and
• if the direction to the viewer is constant over the primitive the specular component is the same for all points on the primitive

\[ I = I_E + K_A I_{AL} + \sum_i (K_D (N \cdot L_i) I_i + K_S (V \cdot R_i)^n I_i) \]
Flat Shading

- Illuminate as though all light sources are directional, the polygon is flat, and is viewed from infinitely far away
  - $N \cdot L_i$ constant over polygon
  - Attenuation function constant over polygon
  - $V \cdot R$ constant over surface

$$I = I_E + K_A I_{AL} + \sum_i (K_D (N \cdot L_i) I_i + K_S (V \cdot R_i)^n I_i)$$
Flat Shading

- One lighting calculation per polygon
  - Assign all pixels inside each polygon the same color
Flat Shading

- Objects look like they are composed of polygons
  - OK for polyhedral objects
  - Not so good for smooth surfaces
Polygon Shading Algorithms

- Flat Shading
- **Gouraud Shading**
- Phong Shading
Gouraud Shading

- What if smooth surface is represented by polygonal mesh with a normal at each vertex?

\[ I = I_E + K_A I_{AL} + \sum_{i} (K_D (N \cdot L_i) I_i + K_S (V \cdot R_i)^n I_i) \]
Gouraud Shading

• One lighting calculation per vertex
  o Assign pixel colors inside polygon by interpolating colors computed at vertices
Gouraud Shading

- Bilinearly interpolate colors at vertices down and across scan lines.

\[ A = \alpha l_1 + (1-\alpha)l_3 \]
\[ B = \beta l_2 + (1-\beta)l_3 \]
\[ I = \varphi A + (1-\varphi)B \]
Gouraud Shading

• Bilinearly interpolate colors at vertices down and across scan lines

\[ A = \alpha l_1 + (1-\alpha) l_3 \]

Note: The values of \( \alpha \) and \( \beta \) only need to be updated as we move to the next scan-line. The value of \( \varphi \) needs to be updated as we advance along the scan-line.
Gouraud Shading

- Produces smoothly shaded polygonal mesh
  - Smooth shading over adjacent polygons
  - Need fine mesh to capture subtle lighting effects
Gouraud Shading

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  - Smooth shading over adjacent polygons
  - Need fine mesh to capture subtle lighting effects

What happens with large polygon & spotlight?
Gouraud Shading

- Produces smoothly shaded polygonal mesh
  - Smooth shading over adjacent polygons
  - Need fine mesh to capture subtle lighting effects

What happens with large polygon & spotlight?
Polygon Shading Algorithms

- Flat Shading
- Gouraud Shading
- Phong Shading
Phong Shading

- One lighting calculation per pixel
  - Approximate surface normals for points inside polygons by bilinear interpolation of normals from vertices

\[
I = I_E + K_A I_{AL} + \sum_i (K_D (N \cdot L_i) I_i + K_S (V \cdot R_i)^n I_i)
\]
Phong Shading

- Bilinearly interpolate surface normals at vertices down and across scan lines

\[ A = \alpha N_1 + (1-\alpha)N_3 \]
\[ B = \beta N_2 + (1-\beta)N_3 \]
\[ I = \phi A + (1-\phi)B \]
Polygon Shading Algorithms

Wireframe

Flat

Gouraud

Phong
3D Rendering Pipeline (for direct illumination)

3D Primitives → 3D Modeling Coordinates
   Modeling Transformation
   3D World Coordinates
   Camera Transformation
   3D Camera Coordinates
   Lighting
   3D Camera Coordinates
   Projection Transformation
   3D Camera Coordinates
   Clipping
   2D Screen Coordinates
   Viewport Transformation
   2D Screen Coordinates
   Scan Conversion
   2D Image Coordinates
   Image

3D Model
2D Window
2D Screen
Overview

• Scan conversion
  o Figure out which pixels to fill

• Shading
  o Determine a color for each filled pixel

• Depth test
  o Determine when the color of a pixel comes from the front-most primitive
Hidden Surface Removal

• Motivation

• Algorithms for HSR
  • Back-face detection
  • Depth sort
  • Ray casting
  • Z-buffer
Motivation

In general, we don’t want to draw surfaces that are not visible to the viewer:

• Surfaces may be back-facing.
• Surfaces may intersect in 3D.
• Surfaces may intersect in the image plane.
Somewhere in here we have to decide which objects are visible, and which objects are hidden.
Overview

• Motivation

• Algorithms for HSR
  o Back-face detection
  o BSP-Trees
  o Ray casting
  o Z-buffer
Visibility algorithms

Figure 29. Characterization of ten opaque-object algorithms & Comparison of the algorithms.
Back-face detection

Q: How do we test for back-facing polygons?
Back-face detection

Q: How do we test for back-facing polygons?
A: Dot product of the normal and view directions.

If $V \cdot N > 0$, then polygon is back-facing
Back-face detection

This method breaks down for:
- Overlapping primitives
- Non-solid models and/or models without a well defined orientation.

In general, back-face removal expected to remove $\approx$ half of polygon surfaces from further visibility tests.
3D Rendering Pipeline

3D Primitives

- Modeling Transformation
  - Trivial Reject
  - 3D World Coordinates
  - 3D Camera Coordinates

- Camera Transformation
  - 3D Camera Coordinates

- Lighting
  - 3D Camera Coordinates

- Projection Transformation
  - 2D Screen Coordinates

- Clipping
  - 2D Screen Coordinates

- Viewport Transformation
  - 2D Image Coordinates

- Scan Conversion
  - 2D Image Coordinates

Image

A polygon is backfacing if $V \cdot N > 0$
3D Rendering Pipeline

3D Primitives → 3D Model Coordinates ➔ Modeling Transformation ➔ 3D World Coordinates ➔ Camera Transformation ➔ 3D World Coordinates ➔ Lighting ➔ 3D Camera Coordinates ➔ Projection Transformation ➔ 2D Screen Coordinates ➔ Clipping ➔ 2D Screen Coordinates

A polygon is back-facing if \( \mathbf{V} \cdot \mathbf{N} > 0 \)

Note: When your graphics card does this, it does not use the normals you provide at the vertices. Instead it uses the cross-product of the triangle vertices, so make sure that the ordering of the vertices is consistent (e.g. CCW).
Ideal Solution

Painter’s Algorithm:

• Sort primitives front to back and draw the back ones first, over-writing pixel values with information from the front primitives as they are processed.
Ideal Solution

Painter’s Algorithm:

• Sort primitives front to back and draw the back ones first, over-writing pixel values with information from the front primitives as they are processed.

Problem:

• You can’t always sort the primitives.
Ideal Solution

Painter’s Algorithm:

• Sort primitives front to back and draw the back ones first, over-writing pixel values with information from the front primitives as they are processed.

Problem:

• You can’t always

However, in some cases you can sort the primitives – e.g. if all the vertices of one primitive are in front of all the vertices of the second.
BSP-Tree Rendering (Object Precision)

- BSP-Trees recursively partition space by planes
  - Given two primitives on either side of a plane, the one on the opposite side from the camera will always be further away.
  - Draw the further side first, and then draw the closer one
BSP-Tree Rendering (Object Precision)

- Draw further half first, then the closer one.
  - Draw right side of 1
  - Draw left side of 1
BSP-Tree Rendering (Object Precision)

• Draw further half first, then the closer one.
  • Draw right side of 1
  • Draw left side of 3
  • Draw right side of 3
  • Draw left side of 1
BSP-Tree Rendering (Object Precision)

- Draw further half first, then the closer one.
  - Draw right side of 1
    - Draw left side of 3
      - Draw D
    - Draw right side of 3
  - Draw left side of 1
BSP-Tree Rendering (Object Precision)

- Draw further half first, then the closer one.
  - Draw right side of 1
    - Draw left side of 3
      - Draw D
    - Draw right side of 3
      - Draw left side of 5
      - Draw right side of 5
  - Draw left side of 1
BSP-Tree Rendering (Object Precision)

- Draw further half first, then the closer one.
  - Draw right side of 1
    - Draw left side of 3
      - Draw D
    - Draw right side of 3
      - Draw left side of 5
        - Draw E
        - Draw right side of 5
  - Draw left side of 1
BSP-Tree Rendering (Object Precision)

• Draw further half first, then the closer one.
  • Draw right side of 1
    • Draw left side of 3
      • Draw D
    • Draw right side of 3
      • Draw left side of 5
        • Draw E
    • Draw right side of 5
      • Draw F
  • Draw left side of 1
BSP-Tree Rendering (Object Precision)

- Draw further half first, then the closer one.
  - Draw right side of 1
    - Draw left side of 3
    - Draw D
  - Draw right side of 3
    - Draw left side of 5
    - Draw E
    - Draw right side of 5
    - Draw F
  - Draw left side of 1
    - Draw left side of 2
    - Draw right side of 2
BSP-Tree Rendering (Object Precision)

• Draw further half first, then the closer one.
  • Draw right side of 1
    • Draw left side of 3
      • Draw D
    • Draw right side of 3
      • Draw left side of 5
        • Draw E
      • Draw right side of 5
        • Draw F
  • Draw left side of 1
    • Draw left side of 2
      • Draw left side of 4
      • Draw right side of 4
    • Draw right side of 2
BSP-Tree Rendering (Object Precision)

- Draw further half first, then the closer one.
  - Draw right side of 1
    - Draw left side of 3
      - Draw D
  - Draw right side of 3
    - Draw left side of 5
      - Draw E
    - Draw right side of 5
      - Draw F
  - Draw left side of 1
    - Draw left side of 2
      - Draw left side of 4
        - Draw A
      - Draw right side of 4
    - Draw right side of 2
BSP-Tree Rendering (Object Precision)

• Draw further half first, then the closer one.
  • Draw right side of 1
    • Draw left side of 3
      • Draw D
    • Draw right side of 3
      • Draw left side of 5
        • Draw E
      • Draw right side of 5
        • Draw F
  • Draw left side of 1
    • Draw left side of 2
      • Draw left side of 4
        • Draw A
      • Draw right side of 4
        • Draw B
    • Draw right side of 2
BSP-Tree Rendering (Object Precision)

- Draw further half first, then the closer one.
  - Draw right side of 1
    - Draw left side of 3
      - Draw D
  - Draw right side of 3
    - Draw left side of 5
      - Draw E
    - Draw right side of 5
      - Draw F
  - Draw left side of 1
    - Draw left side of 2
      - Draw left side of 4
        - Draw A
      - Draw right side of 4
        - Draw B
    - Draw right side of 2
      - Draw C
Building BSP-Trees

- Choose polygon (arbitrary)
- Split its cell using plane on which polygon lies
  - May have to chop polygons in two (Clipping!)
- Continue until each cell contains only one polygon fragment
- Splitting planes could be chosen in other ways, but there is no efficient optimal algorithm for building BSP trees
  - Optimal means minimum number of polygon fragments in a balanced tree

Slide courtesy UWisconsin CS559
Ray Casting

- Fire a ray for every pixel
  - If ray intersects multiple objects, take the closest
Ray Casting Pipeline

Ray casting comments
- $O(p \log n)$ for $p$ pixels
- May (or not) use pixel coherence
- Simple, but generally not used
Z-Buffer

- Store color & depth of closest object at each pixel
  - Initialize depth of each pixel to $\infty$
  - Update only pixels whose depth is closer than in buffer
Z-Buffer

- Store color & depth of closest object at each pixel
  - Initialize depth of each pixel to $\infty$
  - Update only pixels whose depth is closer than in buffer

---

**Case 1:**
- **Blue $\rightarrow$** $(d=1)<(d=\infty)$:
  - Set to $(0, 0, 1), d=1$
- **Red $\rightarrow$** $(d=2)>(d=1)$:
  - Don’t change pixel

---

**Case 2:**
- **Red $\rightarrow$** $(d=2)<(d=\infty)$:
  - Set to $(1, 0, 0), d=2$
- **Blue $\rightarrow$** $(d=1)<(d=2)$:
  - Set to $(0, 0, 1), d=1$
Z-Buffer

- Store color & depth of closest object at each pixel
  - Initialize depth of each pixel to $\infty$
  - Update only pixels whose depth is closer than in buffer
  - Depths are interpolated from vertices, just like colors

$$A = \alpha l_1 + (1-\alpha)l_3$$
$$B = \beta l_2 + (1-\beta)l_3$$
$$I = \varphi A + (1-\varphi)B$$
A-Buffer

- Alpha values can cause problems:
  - Z-buffer can only find one visible surface at each pixel
  - A-buffer supports linked list of surfaces at each pixel for better transparency support
  - A-buffer also helps with anti-aliasing
3D Rendering Pipeline

Z-buffer comments
- Polygons rasterized in any order
- Requires additional memory
  - Z-buffer size \(\approx\) frame buffer
  - Was expensive, cheap now
- Has problems with Alpha (A-buffer)
- Very common in hardware
3D Rendering Pipeline (for direct illumination)

3D Primitives

Modeling Transformation

Camera Transformation

Lighting

Projection Transformation

Clipping

Viewport Transformation

Scan Conversion

Image

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3D Primitives
3D Modeling Coordinates

3D World Coordinates

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2D Screen Coordinates

2D Screen Coordinates

2D Image Coordinates

2D Image Coordinates

2D Image Coordinates
Scan Conversion

How do we average information from the three vertices of a triangle?

- Interpolate using weights determined by the screen space projection?
- Interpolate using weights determined by the 3D locations?

It’s easier to do the interpolation in 2D.

Is there a difference?
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

How should we interpolate the information from vertices $p_1$ and $p_2$ at the pixel corresponding to ray $R$?
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

- $R$ intersects the projected line segment in the middle:
  - We should use equal contributions from $p_1$ and $p_2$. 

\[ z = 1 \]
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

• $R$ intersects the projected line segment in the middle:
  - We should use equal contributions from $p_1$ and $p_2$.

• $R$ intersects the 2D line segment closer to $p_1$:
  - We should use more information from $p_1$ than from $p_2$. 
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

- How do we interpolate correctly?
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

- How do we interpolate correctly?

Recall: The 2D point \((x, z)\) maps to the point \((x/z)\) in 1D.
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

- How do we interpolate correctly?

**Recall:** The 2D point \((x, z)\) maps to the point \((x/z)\) in 1D.

If \(p_1 = (x_1, z_1)\) and \(p_2 = (x_2, z_2)\), to find the blending value for a pixel at position \(x\) in the screen we need to solve for \(\alpha\) s. t.:

\[
(1 - \alpha)(x_i, z_i) + \alpha(x_2, z_2) \rightarrow (x, 1)
\]
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

- How do we interpolate correctly?

Recall: The 2D point \((x, z)\) maps to the point \((x/z)\) in 1D.

If \(p_1=(x_1, z_1)\) and \(p_2=(x_2, z_2)\), to find the blending value for a pixel at position \(x\) in the screen we need to solve for \(\alpha\) s. t.:

\[
(1 - \alpha)(x_i, z_i) + \alpha(x_2, z_2) \rightarrow (x, 1)\\
((1 - \alpha)x_i + \alpha x_2, (1 - \alpha)z_i + \alpha z_2) \rightarrow (x, 1)
\]
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

- How do we interpolate correctly?

Recall: The 2D point \((x, z)\) maps to the point \((x/z)\) in 1D.

If \(p_1=(x_1, z_1)\) and \(p_2=(x_2, z_2)\), to find the blending value for a pixel at position \(x\) in the screen we need to solve for \(\alpha\) s. t.:

\[
(1 - \alpha)(x_i, z_i) + \alpha(x_2, z_2) \rightarrow (x, 1)
\]

\[
((1 - \alpha)x_i + \alpha x_2, (1 - \alpha)z_i + \alpha z_2) \rightarrow (x, 1)
\]

\[
\frac{(1-\alpha)x_1 + \alpha x_2}{(1-\alpha)z_1 + \alpha z_2} = x
\]
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

- How do we interpolate correctly?

Recall: The 2D point \((x, z)\) maps to the point \((x/z)\) in 1D.

To compute the interpolation weights correctly, we need to perform a perspective divide!

If \(p_1 = (x_1, z_1)\) and \(p_2 = (x_2, z_2)\), to find the blending value for a pixel at position \(x\) in the screen we need to solve for \(\alpha\) s. t.:

\[
(1 - \alpha)(x_i, z_i) + \alpha(x_2, z_2) \rightarrow (x, 1)
\]

\[
((1 - \alpha)x_i + \alpha x_2, (1 - \alpha)z_i + \alpha z_2) \rightarrow (x, 1)
\]

\[
\frac{(1 - \alpha)x_1 + \alpha x_2}{(1 - \alpha)z_1 + \alpha z_2} = x
\]
A line segment in 2D projected onto a 1D screen.

- How do we interpolate correctly?

Recall: The 2D point $(x, z)$ maps to the point $(x/z)$ in 1D.

If $p_1 = (x_1, z_1)$ and $p_2 = (x_2, z_2)$, to find the blending value for a pixel at position $x$ in the screen we need to solve for $\alpha$ s. t.:

$$p_2 - p_1 = (x/z_2 - x/z_1) = 0$$

$$z_1 = 1$$

To compute the interpolation weights correctly, we need to perform a perspective divide!

Note that this is not the same as solving for the blending value in the image plane:

$$\frac{(1-\alpha)x_1 + \alpha x_2}{(1-\alpha)z_1 + \alpha z_2} = x$$

$$\frac{x_1}{z_1} + \alpha \frac{x_2}{z_2} = x$$