shape from shading; photometric stereo

Most slides courtesy Szymon Rusinkiewicz
shape from shading

* pioneered in 1970s by Berthold K.P. Horn

\[ \vec{n}(x, y) \]

\[ h(x, y) \]

[Forsth and Ponce]
shading ambiguity
Lambertian reflectance model

- diffuse surfaces appear equally bright from all directions

- for point illumination, brightness proportional to $\cos(\theta)$
Lambertian reflectance model

nothing is a perfect Lambertian reflector, but some...
non-Lambertian surfaces

nothing is a perfect Lambertian reflector, but some...better than others
Lambertian reflectance model

* diffuse surfaces appear equally bright from all directions

* for point illumination, brightness proportional to $\cos(\theta)$
therefore, for a constant-colored object with distant illumination, we can write

\[ E = L \rho (\mathbf{l} \cdot \mathbf{n}) \]

\( E \) = observed brightness
\( L \) = brightness of light source
\( \rho \) = reflectance (albedo) of surface
\( \mathbf{l} \) = direction to light source
\( \mathbf{n} \) = surface normal
shape from shading

- the above equation contains some information about shape (surface orientation), and in some cases is enough to recover shape completely (in theory) if $L$, $\rho$, and $l$ are known
- similar to integration (surface normal is like a derivative), but only know a part of derivative
- have to assume surface continuity
shape from shading

• assume surface is given by \( Z(x,y) \)
• let
  \[
p = \frac{\partial Z}{\partial x} \quad q = \frac{\partial Z}{\partial y}
\]
• in this case, surface normal is
  \[
n = \frac{q}{\sqrt{1 + p^2 + q^2}} \begin{pmatrix} -p \\ -q \\ 1 \end{pmatrix}
\]
shape from shading

- so, write

\[
E = L\rho (\vec{l} \cdot \vec{n})
\]

\[
E = \frac{L\rho}{\sqrt{1 + p^2 + q^2}} \begin{pmatrix} l_x & l_y & l_z \end{pmatrix} \begin{pmatrix} -p \\ -q \\ 1 \end{pmatrix}
\]

- discretize: end up with one equation per pixel
- but this is p equations in 2p unknowns...
shape from shading

- integrability constraint:

\[
\frac{\partial^2 Z}{\partial x \partial y} = \frac{\partial^2 Z}{\partial y \partial x} \rightarrow \frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}
\]

- wind up with system of 2p (nonlinear) differential equations

- no solution in presence of noise or depth discontinuities
estimating illumination and albedo

- need to know surface reflectance and illumination brightness and direction
- in general, can’t compute from single image
- certain assumptions permit estimating these:
  - assume uniform distribution of normals, look at distribution of intensities in image
  - insert known reference object into image
  - slightly specular object: estimate lighting from specular highlights, then discard pixels in highlights
variational shape from shading

• approach: energy minimization

• given observed $E(x,y)$, find shape $Z(x,y)$ that minimizes energy:

$$\varepsilon = \int \left( (E(x, y) - L\rho (l \cdot n(x, y)))^2 + \lambda (p_x^2 + p_y^2 + q_x^2 + q_y^2) \right) dx \, dy$$

• regularization: minimize combination of difference with data, surface curvature
difficulties with shape from shading

- robust estimation of $L$, $\rho$, $I$?
- shadows
- non-Lambertian surfaces
- more than one light, or area light sources
- interreflections
shape from shading results

Figure 9.2 Two images of the same Lambertian surface seen from above but illuminated from different directions and 3-D rendering of the surface. Practically all the points in the top left image receive direct illumination ($i = [0.20, 0, 0.98]^T$); some regions of the top right image are in the dark due to self-shadowing effects ($i = [0.94, 0.31, 0.16]^T$).
active shape from shading

- idea: several (user-controlled) light sources

- more data:
  - allows determining surface normal directly
  - allows spatially-varying reflectance
  - redundant measurements: discard shadows and specular highlights

- often called photometric stereo
shape from shading results

Figure 9.4  Reconstructions of the surface in Figure 9.2 after 100 (a), 1000 (b) and 2000 (c) iterations. The initial surface was a plane of constant height. The asymmetry of the first two reconstruction is due to the illuminant direction.
photometric stereo setup
photometric stereo setup

[Rushmeier et al. 1997]
for each point $p$, can write

$$\rho_p \begin{bmatrix} l_{1,x} & l_{1,y} & l_{1,z} \\ l_{2,x} & l_{2,y} & l_{2,z} \\ l_{3,x} & l_{3,y} & l_{3,z} \end{bmatrix} \begin{bmatrix} n_{p,x} \\ n_{p,y} \\ n_{p,z} \end{bmatrix} = \alpha \begin{bmatrix} E_{p,1} \\ E_{p,2} \\ E_{p,3} \end{bmatrix}$$

constant $\alpha$ incorporates light source brightness, camera sensitivity, etc.
photometric stereo math

- solving above equation gives \((\rho / \alpha)n\)
- \(n\) must be unit-length => uniquely determined
- determine \(\rho\) up to global constant
- with more than 3 light sources:
  - discard highest and lowest measurements
  - if still more, solve by least squares
photometric stereo results

input images

recovered normals (re-lit)

recovered color

[Rushmeier et al. 1997]
photometric stereo results

- input data:
case study: surface enhancement

Malzbender et al., “Surface Enhancement using Real-Time Photometric Stereo and Reflectance Transformation”
case study: surface enhancement

input images

stylized rendering of normal field
case study: surface enhancement

various reflectance transformations