3D Polygon Rendering Pipeline

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CS 4810: Graphics

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Road Map for Next Lectures

• Leaving ray-tracing

• Moving on to polygon-based rendering
  o Rendering pipeline (today)
  o Clipping
  o Scan conversion & shading
  o Texture-mapping
  o Hidden-surface removal

• Polygon-based rendering is what happens on your PC (think NVIDIA, etc.)
3D Polygon Rendering

- Many applications use rendering of 3D polygons with direct illumination
3D Polygon Rendering

• Many applications use rendering of 3D polygons with direct illumination

Half-Life 2
(Valve Software)
3D Polygon Rendering

- Many applications use rendering of 3D polygons with direct illumination

Star Wars: Knights of the Old Republic

(BioWare)
Ray Casting Revisited

• For each sample …
  - Construct ray from eye position through view plane
  - Find first surface intersected by ray through pixel
  - Compute color of sample based on surface radiance

More efficient algorithms utilize spatial coherence!
3D Polygon Rendering

• Logical inverse of ray casting

• Idea: Instead of sending rays from the camera into the scene, send rays from the scene into the camera.
3D Polygon Rendering

- Ray casting: pick pixel and figure out what color it should be based on what object its ray hits
- Polygon rendering: pick polygon and figure out what pixels it should affect
3D Rendering Pipeline (direct illumination)

This is a pipelined sequence of operations to draw a 3D primitive into a 2D image.
3D Rendering Pipeline (direct illumination)

3D Geometric Primitives

- Modeling Transformation
- Camera Transformation
- Lighting
- Projection Transformation
- Clipping
- Scan Conversion

Image

Transform from current (local) coordinate system into 3D world coordinate system.
3D Rendering Pipeline  (for direct illumination)

- 3D Geometric Primitives
  - Modeling Transformation
    - Transform into 3D world coordinate system
  - Camera Transformation
    - Transform into 3D camera coordinate system
  - Lighting
  - Projection Transformation
  - Clipping
  - Scan Conversion
  - Image

Monday, October 17, 11
**3D Rendering Pipeline** (for direct illumination)

1. **3D Geometric Primitives**
2. **Modeling Transformation**
3. **Camera Transformation**
4. **Lighting**
5. **Projection Transformation**
6. **Clipping**
7. **Scan Conversion**
8. **Image**

Transform into 3D world coordinate system

Transform into 3D camera coordinate system

Illuminate according to lighting and reflectance
3D Rendering Pipeline (for direct illumination)

3D Geometric Primitives

Modeling Transformation

Transform into 3D world coordinate system

Camera Transformation

Transform into 3D camera coordinate system

Lighting

Illuminate according to lighting and reflectance

Projection Transformation

Transform into 2D camera coordinate system

Clipping

Scan Conversion

Image
3D Rendering Pipeline (for direct illumination)

3D Geometric Primitives

Modeling Transformation
- Transform into 3D world coordinate system

Camera Transformation
- Transform into 3D camera coordinate system

Lighting
- Illuminate according to lighting and reflectance

Projection Transformation
- Transform into 2D camera coordinate system

Clipping
- Clip (parts of) primitives outside camera’s view

Scan Conversion

Image
3D Rendering Pipeline (for direct illumination)

3D Geometric Primitives

- Modeling Transformation
  - Transform into 3D world coordinate system

- Camera Transformation
  - Transform into 3D camera coordinate system

- Lighting
  - Illuminate according to lighting and reflectance

- Projection Transformation
  - Transform into 2D camera coordinate system

- Clipping
  - Clip (parts of) primitives outside camera’s view

- Scan Conversion
  - Draw pixels (includes texturing, hidden surface, ...)

Image
Transformations

3D Geometric Primitives

- **Modeling Transformation**
  - Transform into 3D world coordinate system

- **Camera Transformation**
  - Transform into 3D camera coordinate system
  - Illuminate according to lighting and reflectance

- **Lighting**

- **Projection Transformation**
  - Transform into 2D camera coordinate system
  - Clip primitives outside camera’s view

- **Clipping**

- **Scan Conversion**
  - Draw pixels (includes texturing, hidden surface, etc.)

Image
Transformations

\[ p(x,y,z) \]

- **Modeling Transformation**
  - 3D Object Coordinates

- **Camera Transformation**
  - 3D World Coordinates
  
  - 3D Camera Coordinates

- **Projection Transformation**
  - 2D Screen Coordinates

- **Window-to-Viewport Transformation**
  - 2D Image Coordinates

\[ p'(x',y') \]

Transformations map points from one coordinate system to another.

- 3D World Coordinates
  
  - 3D Camera Coordinates
  
  - 3D Object Coordinates

- 2D Screen Coordinates

- 2D Image Coordinates

- 3D World Coordinates
Viewing Transformations

\[ p(x,y,z) \]

- **Modeling Transformation**
  - 3D Object Coordinates
  - 3D World Coordinates

- **Camera Transformation**
  - 3D Camera Coordinates

- **Projection Transformation**
  - 2D Screen Coordinates

- **Window-to-Viewport Transformation**
  - 2D Image Coordinates

\[ p'(x',y') \]
Viewing Transformation

- Mapping from world to camera coordinates
  - Eye position maps to origin
  - Right vector maps to X axis
  - Up vector maps to Y axis
  - Back vector maps to Z axis
Camera Coordinates

- Canonical coordinate system
  - Convention is right-handed (looking down -z axis)
  - Convenient for projection, clipping, etc.

- Camera right vector maps to X axis
- Camera back vector maps to Z axis (pointing out of page)
- Camera up vector maps to Y axis
Finding the Viewing Transformation

• We have the camera (in world coordinates)

• We want $T$ taking objects from world to camera

\[ p^c = T \cdot p^w \]

• Trick: find $T^{-1}$ taking objects in camera to world

\[ p^w = T^{-1} p^c \]

\[
\begin{bmatrix}
  x' \\
  y' \\
  z' \\
  w'
\end{bmatrix}
= 
\begin{bmatrix}
  a & b & c & d \\
  e & f & g & h \\
  i & j & k & l \\
  m & n & o & p
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z \\
  w
\end{bmatrix}
\]
Finding the Viewing Transformation

• Trick: map from camera coordinates to world
  - Origin maps to eye position
  - Z axis maps to Back vector
  - Y axis maps to Up vector
  - X axis maps to Right vector

\[
\begin{bmatrix}
  x' \\
y' \\
z' \\
w'
\end{bmatrix} =
\begin{bmatrix}
  R_x & U_x & B_x & E_x \\
  R_y & U_y & B_y & E_y \\
  R_z & U_z & B_z & E_z \\
  0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
w
\end{bmatrix}
\]

\[p^w = T^{-1} p^c\]

• This matrix is \(T^{-1}\) so we invert it to get \(T\) … easy!
Finding the Viewing Transformation

• Trick: map from camera coordinates to world
  - Remember, with homogeneous coordinates, we divide through by \( w \) values…
  - So if we know actual point in 3D, \( w = 1 \)
  - Easy to find code to invert a matrix

\[
p^w = T^{-1} p^c
\]

\[
\begin{bmatrix}
x' \\
y' \\
z' \\
1
\end{bmatrix} =
\begin{bmatrix}
R_x & U_x & B_x & E_x \\
R_y & U_y & B_y & E_y \\
R_z & U_z & B_z & E_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
\]

• This matrix is \( T^{-1} \) so we invert it to get \( T \) … easy!
Viewing Transformations

\[ p(x, y, z) \]

\[ \text{Modeling Transformation} \rightarrow 3D \text{ Object Coordinates} \]

\[ \text{Camera Transformation} \rightarrow 3D \text{ World Coordinates} \]

\[ \text{Projection Transformation} \rightarrow 3D \text{ Camera Coordinates} \]

\[ \text{Window-to-Viewport Transformation} \rightarrow 2D \text{ Screen Coordinates} \]

\[ p'(x', y') \]

\[ \text{2D Image Coordinates} \]
Projection

• General definition:
  - A linear transformation of points in $n$-space to $m$-space ($m<n$)

• In computer graphics:
  - Map 3D camera coordinates to 2D screen coordinates
Taxonomy of Projections

Planar geometric projections

Parallel

Orthographic

- Top (plan)
- Front elevation
- Axonometric
  - Side elevation
  - Isometric

Oblique

Cabinet

One-point

- Two-point
- Three-point

Perspective

- Other
- Other

FvDFH Figure 6.13
Projection

• Two general classes of projections, both of which shoot rays from the scene, through the view plane:
  o Parallel Projection:
    » Rays converge at a point at infinity and are parallel
  o Perspective “Projection”:
    » Rays converge at a finite point, giving rise to perspective distortion
Taxonomy of Projections

Planar geometric projections

Parallel

Orthographic

Top (plan)

Front elevation

Axonometric

Side elevation

Isometric

Oblique

Cabinet

Cavalier

One-point

Two-point

Three-point

Perspective

Other

Other

FvDFH Figure 6.13
Parallel Projection

- Center of projection is at infinity
  - Direction of projection (DOP) same for all points

Angel Figure 5.4
Parallel Projection

- Parallel lines remain parallel
- Relative proportions of objects preserved
- Angles are not preserved
- Less realistic looking
  - Far away objects don’t get smaller
Taxonomy of Projections

Planar geometric projections

Parallel

Orthographic

Top (plan)  Front elevation  Isometric

Axonometric

Side elevation  Other

Oblique

Cabinet  Cavalier  Other

One-point

Two-point  Three-point

Perspective

FvDFH Figure 6.13
Orthographic Projections

- DOP perpendicular to view plane

Angel Figure 5.5
Orthographic Projections

• DOP perpendicular to view plane

• Lines perpendicular to the view plane vanish
• Faces parallel to the view plane are un-distorted.
Taxonomy of Projections

Planar geometric projections

Parallel

Orthographic

Oblique

Perspective

One-point

Two-point

Three-point

Front elevation

Axonometric

Cabinet

Cavalier

Top (plan)

Side elevation

Isometric

Other

Other

FvDFH Figure 6.13
Oblique Projections

- DOP not perpendicular to view plane

Cavalier
(DOP $\alpha = 45^\circ$)

Cabinet
(DOP $\alpha = 63.4^\circ$)

- $\phi$ describes the angle of the projection of the view plane’s normal
- $L$ represents the scale factor applied to the view plane’s normal

H&B Figure 12.21
Parallel Projection Matrix

- General parallel projection transformation:

\[
\begin{bmatrix}
    x_p \\
    y_p
\end{bmatrix}
= \begin{bmatrix}
    1 & 0 & L \cos \phi \\
    0 & 1 & L \sin \phi
\end{bmatrix}
\begin{bmatrix}
    x \\
    y \\
    z
\end{bmatrix}
\]

Cavalier
(DOP $\alpha = 45^\circ$)

Cabinet
(DOP $\alpha = 63.4^\circ$)

H&B Figure 12.21
Parallel Projection View Volume
Taxonomy of Projections

Planar geometric projections

Parallel

Orthographic

Top (plan)
Front elevation
Side elevation

Axonometric

Isometric

Oblique

Cabinet

Cavalier

One-point

Two-point

Three-point

Perspective

Other

Other
Perspective “Projection”

- Map points onto “view plane” along “projectors” emanating from “center of projection” (COP)

Angel Figure 5.9
**Perspective Projection**

- How many vanishing points?

Number of vanishing points determined by number of axes parallel to the view plane

Angel Figure 5.10
Perspective Projection

• Perspective “projection” is not really a projection because it is not a linear map from 3D to 2D.
  • Parallel lines do not remain parallel!
Perspective Projection View Volume

Frustum View Volume

View Plane

Back Plane

Front Plane

Window

Projection Reference Point

H&B Figure 12.30
Perspective Projection

- What are the coordinates of the point resulting from projection of \((x_0, y_0, z_0)\) onto the view plane at a distance of \(D\) along the z-axis?
Perspective Projection

- Use the fact that for any point \((x_0, y_0, z_0)\) and any scalar \(\alpha\), the points \((x_0, y_0, z_0)\) and \((\alpha x_0, \alpha y_0, \alpha z_0)\) map to the same location:

\[
(2x_0, 2y_0, 2z_0)
\]
Perspective Projection

- Use the fact that for any point \((x_0, y_0, z_0)\) and any scalar \(\alpha\), the points \((x_0, y_0, z_0)\) and \((\alpha x_0, \alpha y_0, \alpha z_0)\) map to the same location.

- Since we want the position of the point on the line that intersect the image plane at a distance of \(D\) along the z-axis:

\[
(x_0, y_0, z_0) \rightarrow \left( \frac{D}{z_0} x_0, \frac{D}{z_0} y_0, D \right)
\]
Perspective Projection Matrix

- 4x4 matrix representation?

\[
\begin{align*}
    x_s &= x_c \frac{D}{z_c} \\
    y_s &= y_c \frac{D}{z_c} \\
    z_s &= D \\
    w_s &= 1
\end{align*}
\]

\[
\begin{bmatrix}
    x_s \\
    y_s \\
    z_s \\
    w_s
\end{bmatrix} = \begin{bmatrix}
    ? & ? & ? & ?
\end{bmatrix} \begin{bmatrix}
    x_c \\
    y_c \\
    z_c \\
    1
\end{bmatrix}
\]
Perspective Projection Matrix

- 4x4 matrix representation?

\[
\begin{align*}
    x_s &= x_c \frac{D}{z_c} \\
    y_s &= y_c \frac{D}{z_c} \\
    z_s &= D \\
    w_s &= 1
\end{align*}
\]

We want to divide by the \( z \) coordinate. How do we do that with a 4x4 matrix?
Perspective Projection Matrix

- 4x4 matrix representation?

\[
\begin{align*}
    x_s &= x_c \frac{D}{z_c} \\
    y_s &= y_c \frac{D}{z_c} \\
    z_s &= D \\
    w_s &= 1
\end{align*}
\]

We want to divide by the \( z \) coordinate. How do we do that with a 4x4 matrix?

Recall that in homogenous coordinates:
\[(x, y, z, w) = (x/w, y/w, z/w, 1)\]
Perspective Projection Matrix

- 4x4 matrix representation?

\[
x_s = x_c \frac{D}{z_c} \\
y_s = y_c \frac{D}{z_c} \\
z_s = D \\
w_s = 1
\]

We want to divide by the \( z \) coordinate. How do we do that with a 4x4 matrix?

Recall that in homogenous coordinates:

\((x, y, z, w) = (x/w, y/w, z/w, 1)\)

\[
\begin{pmatrix}
x_c D \\ y_c D \\ z_c \\ D 
\end{pmatrix} = \begin{pmatrix}
\end{pmatrix} \begin{pmatrix}
x_c \\ y_c \\ z_c \\ 1
\end{pmatrix}
\]
Perspective Projection Matrix

• 4x4 matrix representation?

\[ \begin{align*}
  x_s &= x_c \frac{D}{z_c} \\
  y_s &= y_c \frac{D}{z_c} \\
  z_s &= D \\
  w_s &= 1
\end{align*} \]

We want to divide by the z coordinate. How do we do that with a 4x4 matrix?

Recall that in homogenous coordinates:
\((x, y, z, w) = (x/w, y/w, z/w, 1)\)

\[
\begin{pmatrix}
  \frac{x_c D}{z_c}, & \frac{y_c D}{z_c}, & D, & 1
\end{pmatrix}
\]

\[
\begin{pmatrix}
  x \\ y \\ z \\ w
\end{pmatrix}
\]

\[
\begin{pmatrix}
  x_s \\
  y_s \\
  z_s \\
  w_s
\end{pmatrix}
\]

\[
\begin{bmatrix}
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 1/D & 0
\end{bmatrix}
\begin{pmatrix}
  x_c \\
  y_c \\
  z_c \\
  D
\end{pmatrix}
\]
Perspective vs. Parallel

• Perspective projection
  + Size varies inversely with distance - looks realistic
  – Distance and angles are not preserved
  – Only parallel lines that are parallel to the view plane remain parallel

• Parallel projection
  + Good for exact measurements
  + Parallel lines remain parallel
  + Angles are preserved on faces parallel to the view plane
  – Less realistic looking