Low-Latency Multi-flow Broadcasts in Fading Wireless Networks

Chenxi Qiu, Lei Yu and Haiying Shen
Department of Electrical and Computer Engineering
Clemson University, Clemson, USA
{chenxiq, leiy, shenh}@clemson.edu

Sohraab Soltani
Intelligent Automation, Inc.
Rockville, USA
ssoltani@i-a-i.com

***Abstract*—Cooperative broadcast, in which a packet receiver cooperatively combines received weak signal power from different senders to decode the original packet, has gained increasing attention. However, existing approaches are developed based on the assumption that there is a single flow in the network; thus, they are not suitable for multi-flow broadcasting in which broadcasts are initiated by different nodes and consist of more than one packet at any point in time. In this paper, we aim to achieve low-latency multi-flow broadcast in wireless multi-hop networks with fading channels. We formulate this problem as a Minimum Slotted Delay Cooperative Broadcast (MSDCB) problem, and prove that it is NP-complete and $o(\log N)$ inapproximable. We then propose two heuristic algorithms named PCBH-S and PCBH-M to solve MSDCB. Our experimental results show that our algorithms outperform previous methods.**

I. Introduction

In wireless networks, broadcasting is a particularly important mechanism for disseminating a message from one source to all other nodes. Blind flooding, in which each node forwards the broadcast packet exactly once has been the simplest way to implement broadcasting. The major drawback of blind flooding is its high cost and excessive redundant transmissions that lead to increased contention and collisions. Thus, many approaches have been proposed for efficient broadcasting, based on broadcast tree [1], [2], Minimum Connected Dominated Set (MCDS) [3], [4] and cooperative communication [5]–[10].

Broadcast tree and MCDS based approaches [2]–[4] improve the broadcast efficiency by determining a small subset of connected nodes as forwarders from which all other nodes in the network can be reached. However, most of these works use simple deterministic topology or link metrics to determine the relay nodes. They suffer a low delivery rate in fading environments [2]–[4] where the transmissions between relay nodes are vulnerable to failure. To mitigate the detrimental effects of fading, cooperative diversity [5]–[10] has been exploited. Due to the broadcast nature of the wireless channel, packets transmitted between senders and receivers can be overheard by some other nodes. If these nodes are utilized for *cooperative relaying*, i.e., relaying the overheard packets to their intended recipients, error rates can be significantly reduced. Cooperative relaying enables broadcasting without the usual multiplexing loss because of intrinsic retransmissions in broadcasting. Thus, it is particularly promising to exploit cooperative diversity for improving broadcast efficiency. However, none of these works can guarantee successful communication with high probability in fading environment. Recently, a cooperation diversity approach [1] has been proposed to address broadcasting in fading environments, which firstly uses the probability of successful communication as a metric for constructing broadcast backbone. It aims to reduce the broadcast delay, defined as the number of slots required for one packet to be distributed throughout the entire network. It incorporates the Rayleigh fading model [1] into tree construction and keeps the size of the tree low in order to minimize the broadcast delay. However, [1] assumes that only a single broadcast by a single node is in progress at any point in time. Hence, it has limited practical use. In fact, broadcast communication is an essential operation in many distributed network applications, and broadcasts can be initiated by different nodes and consist of more than one packet at any point in time. That is, multiple packet flows are broadcast from different sources simultaneously. This is termed *multi-flow broadcast*. In such cases, the allocations of cooperative relay nodes for different data flow broadcasts have to be considered. Previous works did not consider a method for allocating relay nodes in multi-flow cases. Determining relay nodes with their "single-flow" methods can lead to degraded performance of multi-flow broadcasting.

In this paper, we propose a cooperative diversity scheme for multi-flow broadcast in fading wireless networks to minimize broadcast delay. In our scheme a group of nodes, termed a cooperative relaying set, is chosen and acts together during the same time slot to forward a packet; this contrasts with previous work [1] in which only a single node is chosen. Thus, packets are forwarded from relay-set to relay-set, instead of from node to node. A challenging problem for our scheme is determining the allocations of relay nodes to different flows while minimizing the broadcast delay, as arbitrarily allocating some relay nodes to one flow may cause unbalanced high latency for other flows in the multi-flow case. As opposed to previous works, we need to determine a *cooperative relay set* rather than a single relay node in each slot. To address this challenge, we introduce a probabilistic mechanism and formulate the allocation problem as the *Minimum Slotted Delay Cooperative Broadcast (MSDCB) problem*. We prove that MSDCB is NP-complete and $o(\log N)$ inapproximable under some restrictions. Finally, we develop two heuristic algorithms for this problem and implement simulations to examine the performance of the algorithms. The experimental results demonstrate that the heuristic algorithm we propose performs better than some typical schemes (e.g., PCDB [1]).

The remainder of this paper is organized as follows. Section II builds the mathematical model. Section III identifies a problem called MSDCB and derived numerous properties of

MSDCB. In Section IV, we propose two heuristic algorithms (PCBH-S and PCBH-M) for MSDCB. Section V presents the simulation results for PCBH-S and PCBH-M. Section VI concludes this paper.

II. System Model

Network model: We consider a wireless network consisting of a set of nodes $\mathcal{V} = \{v_1, v_2, ..., v_N\}$, and a set of packet flows $\mathcal{F} = \{f_1,\ f_2,\ ...,\ f_M\}$, where each flow, say f_j, is broadcast from one source node s_j to all other nodes denoted by set $\mathcal{D}_j$, (i.e., $D_j = \mathcal{V}/s_j$ $(j = 1, 2, ..., M)$). We consider a time-slotted system in which the nodes that have received and decoded the packet are allowed to transmit it in future slots. All nodes are assumed to operate in full-duplex mode, i.e. they can transmit and receive simultaneously.

Channel model: We assume a frequency-flat time-varying wireless channel [1]. For the transmitted signal from sender v_i at receiver v_j, the channel effect can be modeled by a single, complex, random channel coefficient $h_{i,j}$. We consider a Rayleigh fading channel in which all $|h_{i,j}|^2$ are independent and exponentially distributed with a mean value $\sigma^2_{i,j} = P_i d^{-\alpha}_{i,j}$ (with transmitter power P_i and distance $d_{i,j}$ between v_i and v_j). The instantaneous signal power $S_{i,j}$ received at v_j from v_i is a random variable with Cumulative Distribution Function (CDF) $F_{S_{i,j}} = 1 - e^{-x/\sigma^2_{i,j}}$. For a single transmission, whether or not a packet is successfully received depends on the instantaneous SNR at the receiver, which is given by $\frac{S_{i,j}}{N_0}$ (N_0 is the noise power density). We use a non-negative random variable $X_{i,j}$ to represent the SNR received at v_j from v_i ($X_{i,j} \sim X_{j,i}$), and v_j can successfully receive a packet iff $\sum_{v_i \in \mathcal{R}} X_{i,j} \geq \gamma_{\mathrm{th}}$, where $\mathcal{R}$ is the set of nodes sending the packet to v_j and γ_{th} is the fixed decoding threshold. $X_{i,j}$ has CDF $F_{X_{i,j}} = 1 - e^{-N_0 x/\sigma^2_{i,j}}$, and the probability of the packet being correctly received by v_j from v_i is $e^{-N_0\gamma_{\mathrm{th}}/\sigma^2_{i,j}}$.

Diversity and combining: The reliability of a packet signal can be improved by diversity schemes which use two or more stochastically independent communication channels to transmit copies of a packet to one receiver [11]. Diversity schemes can exploit independent channels in time, frequency and space to obtain a decrease in error probability, which is called a *diversity gain*. Here we assume the Maximal Ratio Combining (MRC) filter commonly used in diversity receivers [12]. If the sum of all the received instantaneous SNRs is above the decoding threshold γ_{th}, the original packet can be successfully decoded from the packet copies. Assuming that $X_{i,1}$, $X_{i,2}$, ..., X_{i,L_i} are independent, the sum SNR that v_j receives follows a distribution [13] that has Probability density function (PDF):

$$f_{\sum_{v_i \in \mathcal{R}} X_{i,j}} = \sum_{v_i \in \mathcal{R}} \beta_{i,j} e^{-\beta_{i,j} x} \prod_{v_k \in \mathcal{R}, k \neq i} \frac{\beta_{k,j}}{\beta_{k,j} - \beta_{i,j}} \qquad (1)$$

where $\beta_{i,j} = N_0/\sigma^2_{i,j}$. The probability that v_j cannot correctly receive the packet can be calculated as:

$$\Pr\left[\sum_{v_i \in \mathcal{R}} X_{i,j} < \gamma_{\mathrm{th}}\right] = \int_0^{\gamma_{\mathrm{th}}} f_{\sum_{v_i \in \mathcal{R}} X_{i,j}} \mathrm{d}x \qquad (2)$$

III. Problem formulation

A. Serial Transmission of Cooperative Relay Set

For a packet which is generated at slot t_0 and has not reached its destination, we define the packet's *age* at a specified time slot t is $t - t_0$, i.e., the number of time slots the packet has been transferred in the network since t_0. We define *packet delay* of flow f_j at node v_i as the minimum number of slots the packets of flow f_j need to be transmitted from source s_j to v_i, and define *Broadcast delay* of f_j, denoted as L_j, as the maximum packet delay of f_j among all the nodes in $\mathcal{D}_j$. In addition, the k^{th} cooperative relay set of a flow f_j is defined as the set of nodes which are responsible for forwarding the packets of f_j at age k $(1 \leq k \leq L_i - 1)$. We use $\mathcal{R}_{j,k}$ to denote the k^{th} cooperative relay set of flow f_j and use K to denote the size constraint of all the cooperative relay sets: $|\mathcal{R}_{j,k}| \leq K$. For f_j, we say $\mathcal{R}_{j,l}$ is $\mathcal{R}_{j,k}$'s *previous set* if $l < k$. We use $\mathcal{B}_{j,k}$ to represent $\cup_{l=1}^{k} \mathcal{R}_{j,l}$.

Property 3.1: Let $\mathcal{S}_{j,k}$ be the set of nodes at which the packet delay of f_j equals k and let $\mathcal{A}_{j,k} = \cup_{l=1}^{k} \mathcal{S}_{j,l}$. Then $\mathcal{R}_{j,k} \subset \mathcal{B}_{j,k} \subseteq \mathcal{A}_{j,k}$.

Definition 3.1: (*Serial transmission*): for any packet flow, say f_j, its cooperative relay sets form a sequence $\mathcal{R}_{j,1}$, $\mathcal{R}_{j,2}$, ..., $\mathcal{R}_{j,L_j}$ such that the nodes in $\mathcal{R}_{j,l}$ $(1 \leq l \leq L_i - 1)$ finish transmitting a packet before the nodes in $\mathcal{R}_{j,l+1}$ start transmitting the same packet. To guarantee serial transmission, the cooperative relay sets must satisfy the following two properties:

Property 3.2: $\mathcal{R}_{j,k}$ cannot be empty $(1 \leq k \leq L_j)$, and also its size cannot exceed K: $0 < |\mathcal{R}_{j,k}| \leq K$. We say a cooperative relay set $R_{i,j}$ is *saturated* if $|R_{i,j}| = K$.

Property 3.3: For any two cooperative relay sets in the same flow, say $\mathcal{R}_{j,k}$ and $\mathcal{R}_{j,l}$, the intersection of two cooperative relay sets is empty: $\mathcal{R}_{j,k} \cap \mathcal{R}_{j,l} = \phi$.

B. Probabilistic Allocation of Relay Nodes

When the packets of multi-flow are transmitted simultaneously, competition might happen among the cooperative relay sets of different flows. As a solution, we propose a *probabilistic mechanism* for allocating cooperative relay nodes, in which each relay node probabilistically forwards the packets in the flow. We use random variable $Y_{i,j,k}$ ($Y_{i,j,k}$ follows Bernoulli distribution and $\mathrm{E}\left(Y_{i,j,k}\right) = p_{i,j,k}$) to represent whether v_i needs to forward the packet for $\mathcal{R}_{j,k}$. In this case we would like $Y_{i,j,k}$ to be 1 if v_i is the cooperative relay node for $\mathcal{R}_{j,k}$, and 0 otherwise.

Definition 3.2: (*Cooperative relay set schedule*): The cooperative relay set schedule is defined as the schedule that determines the probability of each node serving a cooperative relay set. We use $\mathcal{P}$ to represent a cooperative relay set schedule, which can be also represented as a matrix: $\mathbf{P} = \{p_{i,j,k}\}_{N \times M \times L}$, where $p_{i,j,k}$ denotes the probability that v_i serves for $\mathcal{R}_{j,k}$ and $L \geq \max\{L_1, L_2, L_3, ..., L_M\}$. Let $p_{i,j,k} = 0$ when v_i is not in $\mathcal{R}_{j,k}$ or $k > L_j$. For simplicity, in the following we use *schedule* instead of *cooperative relay set schedule*, and a *schedule* is *optimal* if $\max\{L_1,$ $L_2, L_3, ..., L_M\}$ is minimized.

Suppose v_i is a relay node in $\mathcal{R}_{j,k}$. When previous $k-1$ cooperative relay sets complete the transmission of a packet in f_j, the sum SNR that v_i receives can be represented by $Z_{i,j,k} = \sum_{l=1}^{k-1} \sum_{v_r \in \mathcal{R}_{j,l}} X_{r,i} Y_{r,j,l}$. Assuming $X_{r,i}$ and $Y_{r,j,l}$ are independent, we can get the closed form of $Z_{i,j,k}$'s PDF using a Fourier transform (details are introduced in our technical report [14]):

$$f_{Z_{i,j,k}} = \sum_{l<k, v_r \in \mathcal{R}_{j,l}} E_{r,i,j} e^{-\beta_{r,i} x} u(x) + E_{i,j,k} \delta(x) \quad (3)$$

where $E_{i,j,k} = \prod_{l<k, v_r \in \mathcal{R}_{j,l}} (1 - p_{r,j,l})$ and $E_{r,i,j}$ is given by Equ. (13) in [14]. The probability that node v_i cannot correctly receive the packet before forwarding it in f_j is given by

$$\Pr[Z_{i,j,k} < \gamma_{\mathrm{th}}] = \int_0^{\gamma_{\mathrm{th}}} f_{Z_{i,j,k}} \mathrm{d}x \quad (4)$$

$$= \sum_{l<k, v_r \in \mathcal{R}_{j,l}} \frac{E_{r,i,j}\left(1 - e^{-\beta_{r,i}\gamma_{\mathrm{th}}}\right)}{\beta_{r,i}} + E_{i,j,k} \quad (5)$$

We use the constant ε to represent the acceptable error probability for the network. We say a node can *successfully receive* a packet if the sum SNR it has received, denoted as Z, satisfies $Z > \gamma_{\mathrm{th}}$. For any flow f_j ($j = 1, 2, 3, ..., M$), we say a node is *informed* in f_j if the probability it cannot successfully receive the packet in f_j is smaller than ε, i.e., $\Pr[Z < \gamma_{\mathrm{th}}] < \varepsilon$; otherwise we say the node is *uninformed* in f_j. The candidate of f_j is defined as the node that has been informed by f_j but hasn't been selected as relay node for f_j.

C. Problem Statement

For a schedule that can successfully broadcast packets in each flow, the following three conditions must be satisfied.

Condition 1: for any node $v_i \in \mathcal{R}_{j,k}$, the probability that v_i cannot correctly receive the packet in f_j before forwarding the packet should be smaller than ε: $\Pr[Z_{i,j,k} < \gamma_{\mathrm{th}}] < \varepsilon$.

Condition 2: For each flow f_j, after all the nodes in $\mathcal{B}_{j,L_j}$ forward the packet in f_j, for any node $v_i \in \mathcal{D}_j$, the probability that v_i cannot correctly receive the packet should be smaller than ε: $\Pr\left[Z_{i,L_j+1,k} < \gamma_{\mathrm{th}}\right] < \varepsilon$.

Condition 3: Any $p_{i,j,k}$ in $\mathbf{P}$ has the following restrictions:

- $\sum_{j=1}^{M} \sum_{k=1}^{L} p_{i,j,k} \leq 1$ ($1 \leq i \leq N$), i.e., the sum of the probabilities that v_i allocates in all cooperative relay sets cannot exceed 1 (we say a node v_i is *fully used* if $\sum_{j=1}^{M} \sum_{k=1}^{L} p_{i,j,k} = 1$);
- $\sum_{i=1}^{N} p_{i,j,k} > 0$ ($1 \leq j \leq M$ and $1 \leq k \leq L_j$), i.e., any cooperative relay set cannot be empty (according to *Property 3.2*);
- $\sum_{k=1}^{L} \lceil p_{i,j,k} \rceil \leq 1$ ($1 \leq i \leq N$ and $1 \leq k \leq L_j$), i.e., the cooperative relay sets for the same flow do not intersect with each other (according to *Property 3.3*);
- $\sum_{i=1}^{N} \lceil p_{i,j,k} \rceil \leq K$ ($1 \leq j \leq M$ and $1 \leq k \leq L_j$), i.e., the size of the cooperative relay cannot exceed K.

Our objective is to find the minimized broadcast delay for each flow. The problem can be formulated as follows:

Definition 3.3: (*Minimum Slotted Delay Cooperative Broadcast problem (MSDCB)*): For instance $I(\varepsilon, \mathcal{V}, \mathcal{F}, L, K)$, existence of a *cooperative relay set schedule* $\mathbf{P}$ satisfying *Condition 1*, *Condition 2* and *Condition 3*, where ε is the acceptable error probability for the network, $\mathcal{V}$ denotes the set of nodes, $\mathcal{F}$ denotes the set of flows, L is the max broadcast delay among $\{L_1, L_2, ..., L_M\}$ and K denotes the cooperative relay set constraint (or *relay set constraint* for short).

D. Problem analysis

In the following, we derived some theorems, corollaries and properties for MSDCB. Due to space constraints, detailed proofs of these results are omitted here and can be found in our technical report [14].

Theorem 3.1: For any instance $I(\varepsilon, \mathcal{V}, \mathcal{F}, L, K)$, *MSDCB* is in NP if the magnitude of any signal fades according to a Rayleigh distribution and is independent to others.

Property 3.4: None of the following three operations decreases the value of $\Pr[Z_{i,j,k} < \gamma_{\mathrm{th}}]$ for any $Z_{i,j,k}$ given that restrictions that the operation neither violates *Condition 3* nor decreases the value of any element in $\mathbf{P}$: 1) increasing an element's value in $\mathbf{P}$; 2) moving a node from any cooperative relay set, say $\mathcal{R}_{j,k}$, to $\mathcal{R}_{j,k}$'s previous set with the probability unchanged 3) adding a candidate to any cooperative relay set.

Corollary 3.1: If MSDCB has a solution, there always exists an optimal schedule $\mathbf{P}$ for which $\forall v_i \in \mathcal{V}$ satisfies either $\sum_{j=1}^{M} \sum_{k=1}^{L} p_{i,j,k} = 1$ or $\sum_{j=1}^{M} \sum_{k=1}^{L} p_{i,j,k} = 0$.

Corollary 3.2: If $\mathcal{F} = \{f_1\}$ (single-flow case) and MSDCB has a schedule, there exists an optimal schedule $\mathbf{P}$ where $p_{i,1,k}$ must be either 1 or 0.

Corollary 3.3: If MSDCB has a schedule, there exists an optimal schedule such that any cooperative relay set, say $\mathcal{R}_{j,k}$, satisfies at least one of the following three conditions: 1) the cooperative relay set is saturated: $|\mathcal{R}_{j,k}| = K$; 2) all the nodes informed are in $\mathcal{B}_{j,k} = \cup_{l=1}^{k} \mathcal{R}_{j,l}$; and 3) all the nodes fully used are in $\mathcal{R}_{j,k}$.

Theorem 3.2: MSDCB is NP-complete given the following restrictions: 1) the magnitude of any signal fades according to a Rayleigh distribution and is independent to others; 2) the relay set size constraint is K (constant value); and 3) $F = \{f_1\}$ (single-flow case).

Theorem 3.3: MSDCB remains NP-complete for the multi-flow case given the following restrictions: 1) the magnitude of any signal fades according to an independently Rayleigh distribution; 2) the relay set size constraint is K (constant value); and 3) the number of flows M has constant complexity.

Corollary 3.4: MSDCB is $o(\log N)$ inapproximable given the restrictions of *Theorem 3.2* or *Theorem 3.3*:

IV. Algorithm Design and Analysis

In Section III, we present the *Minimized Slotted Delay Cooperative Broadcast problem (MSDCB)*. Solving this problem leads to minimizing the broadcast delay of the network. However, given some restrictions, the problem has been proved NP-complete and $o(\log N)$ inapproximable in both the single-flow case and the multi-flow case. Thus, we need to design a scalable heuristic algorithm rather than finding an optimal solution. Fortunately, though the corollaries and properties proved in Section III-D mainly discuss the properties of an *optimal schedule*, they also provide some foundations for the design of a heuristic algorithm. In this section we develop two time efficient heuristic algorithms, named *Probabilistic Cooperative Broadcast Heuristic algorithm (PCBH)* for the single-flow case and multi-flow case.

A. PCBH-S

In the case of single-flow ($\mathcal{F} = \{f_1\}$), we use $\mathcal{S}_{1,k}$ to represent the informed set in the k^{th} iteration (or time slot) and use

$\mathcal{B}_{1,k}$ to represent the set of relay nodes that have been selected ($k = 1, 2, 3, ..., L$). Thus, $\mathcal{B}_{1,k} = \cup_{l=1}^{k} \mathcal{R}_{1,l}$. In addition, let $\mathcal{S}_{1,0} = \mathcal{B}_{1,0} = \{s_1\}$, and let $\mathbf{G} = \{\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3, ..., \mathcal{G}_C\}$ denote the set of $\mathcal{V}$'s subsets with cardinality no larger than K, where $C = \sum_{l=1}^{K} \frac{N!}{l!(N-l)!}$.

Algorithm 1 shows the pseudocode of PCBH-S. Here function Info($\mathcal{B}$) returns the set of nodes informed if the nodes in $\mathcal{B}$ are selected as relay nodes. In each iteration, PCBH-S always selects maximal number of nodes from the candidates (indicated by *Corollary 3.3*) and each relay node forward the packet with probability 1 (indicated by *Corollary 3.2*). If the number of candidates is larger than K, PCBH-S selects the K candidates that make the most uninformed nodes informed; otherwise, all the candidates are selected as relay nodes.

Algorithm 1: PCBH-S.

```
begin
    B_{1,0} ← {s_1}; S_{1,0} ← Info(B_{1,0}); P ← 0;
    k ← 0; flag ← 0;
    while S_{1,k} ⊊ V do
        for i ← 1 to C do
            if G_i ⊆ S_{1,k}/B_{1,k} and |Info(G_i ∪ B_{1,k})
            /Info(B_{1,k})| > flag then
                flag ← |Info(G_i ∪ B_{1,k})/Info(B_{1,k})|;
                R_{1,k+1} ← G_i;
        k ← k + 1;
        for i ← 1 to N do
            if v_i ∈ R_{1,k} then
                p_{i,1,k} ← 1;
        B_{1,k} ← B_{1,k} ∪ R_{1,k};
        S_{1,k} ← F_S(B_{1,k});
```

Property 4.1: Algorithm 1 can always find a solution for MSDCB if $F = \{f_1\}$ and MSDCB has a solution.

Proof: Detailed proof can be found in [14]. ■

B. PCBH-M

In the multi-flow case, we use $\mathcal{S}_{j,k}$ to represent the informed set for flow f_j in the k^{th} iteration and use $\mathcal{B}_{j,k}$ to represent the relay nodes that have been selected. In the k^{th} iteration, there are two cases when selecting new relay nodes for flow f_j: (1) $|\mathcal{S}_{j,k}/\mathcal{B}_{j,k}| \leq K$, i.e., the number of candidates in f_j is no larger than K; (2) $|\mathcal{S}_{j,k}/\mathcal{B}_{j,k}| > K$, i.e., the number of candidates in f_j is larger than K. For case (1), all the candidates are selected as relay nodes (indicated by *Property 3.1*); for case (2) at least K nodes should be selected as relay nodes from the candidates (indicated by *Corollary 3.3*).

In the k^{th} iteration, we say any K nodes in the candidates of f_j compose a *choice* for f_j (denoted as $\mathcal{C}_{l,j,k}$), then there are totally $A_{j,k}$ different choices $\mathbf{C}_{j,k} = \{\mathcal{C}_{1,j,k}, \mathcal{C}_{2,j,k}, ..., \mathcal{C}_{A_{j,k},j,k}\}$ for f_j, where $A_{j,k} = \frac{|\mathcal{S}_{j,k}/\mathcal{B}_{j,k}|!}{(|\mathcal{S}_{j,k}/\mathcal{B}_{j,k}|-K)! \times K!}$. Note that a choice equals the candidates when the number of candidates is smaller than K. A series of choices (one choice for one flow) compose a *choices collection*, denoted as $\mathbf{C}'_{u,k} = \{\mathcal{C}_{u_1,1,k}, \mathcal{C}_{u_2,2,k}, ..., \mathcal{C}_{u_M,M,k}\}$ ($\mathcal{C}_{u_j,j,k}$ is selected from $\mathbf{C}_{j,k}$). Then there are $D_k = \prod_{j=1}^{M} A_{j,k}$ different choice collections in the k^{th} iteration. Given a choices collection $\mathbf{C}'_{u,k}$, we need to determine the matrix $\mathbf{P}(k) = \{p_{i,j,k}\}_{N \times M}$ to make the total probability of failure reception of all the uninformed nodes as small as possible, which can be formulated as the following *non-linear programming* problem:

$$\min \sum_{j=1}^{M} \sum_{v_i \notin \mathcal{N}_j^{(k)}} \Pr\left[Z_{i,j,k} < \gamma_{\text{th}}\right] \tag{6}$$

$$\text{s.t.} \sum_{j=1}^{M} p_{i,j,k} \leq 1 - \sum_{j=1}^{M} \sum_{l=1}^{k-1} p_{i,j,l} \tag{7}$$

$$\sum_{i=1}^{N} p_{i,j,k} > 0 \tag{8}$$

$$p_{i,j,k} = 0 \text{ if } v_i \text{ is not in } \mathbf{C}'_{u,k} \tag{9}$$

where $p_{i,j,l}$ ($l = 1, 2, 3, ..., k-1$) is pre-calculated in the l^{th} iteration in Equ. (7) and $\forall j$ satisfies $\mathcal{S}_{j,l} \subsetneq \mathcal{V}$. We use the notation $[\mathbf{P}(k)\ Result] = \text{NLP}(\mathbf{P}, \mathbf{C}'_{u,k}, \gamma_{\text{th}})$ to refer to the solution of the above non-linear programming, where $Result$ represents the minimum value of the objective function. The Probabilistic Cooperative Broadcast Heuristic algorithm for multi-flow (PCBH-M) is introduced in *Algorithm 2*, where InfoMatrix($\mathbf{P}$) returns the set of nodes informed if $\mathbf{P}$ is the schedule.

Algorithm 2: PCBH-M.

```
begin
    B_{j,0} ← {s_j}; S_{j,0} ← Info(B_{j,0});
    P ← 0, k ← 0;
    while ∃S_{j,k} ⊊ V (j = 1, 2, 3, ..., M) do
        flag ← ∞;
        for j ← 1 to M do
            for u ← 1 to D_k do
                [P'(k) Result] ← NLP(P, C'_{u,k}, γ_th);
                if Result < flag then
                    P(k) ← P'(k), C ← C'_{u,k};
            for j ← 1 to M do
                B_{j,k} ← C(j) ∪ B_{j,k-1};
            S_{j,k} ← InfoMatrix(P);
        k ← k + 1;
```

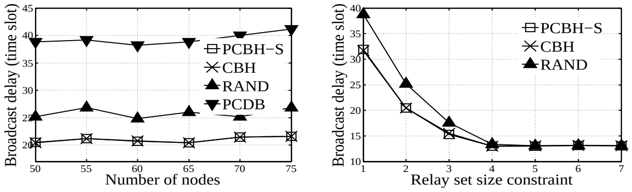
Property 4.2: Algorithm 2 can always find a solution for MSDCB with $F = \{f_1, f_2, f_3, ..., f_M\}$ within N iterations if in each iteration $\text{NLP}(\mathbf{P}, \mathbf{C}'_{u,k}, \gamma_{\text{th}})$ has a solution.

Proof: Detailed proof can be found in [14]. ■

V. Performance Evaluation

To test the performance of our schemes, in this section we implement a simulation using MATLAB. We compare our schemes with a cooperative broadcast heuristic algorithm (CBH), which is the same as PCBH except that each relay node is deterministically allocated to one flow, RAND, which randomly selects relay nodes for each flow, and PCDB [1], a typical cooperative broadcast scheme which serially selects one relay node in each iteration. We use discrete-event simulation where each node can finish sending and receiving one packet in one slot. We generate a random placement of nodes in the region with $1000\text{m} \times 1000\text{m}$ for the single-flow case and $700\text{m} \times 700\text{m}$ for the multi-flow case, and randomly choose a source node for each flow; $\alpha = 4$ (path loss exponent), $P_i = 20\text{dBm}$ (transmission power), $\gamma_{\text{th}} = 25.8\text{dB}$ (decoding threshold), $N_0 = 4.32 \times 10^{-18}\text{W/Hz}$ (Noise power density) and $R = 1\text{Mbit/s}$ (data rate). The metric we mainly evaluate is *broadcast delay*, which is defined as the number of slots required to inform all nodes in the network.

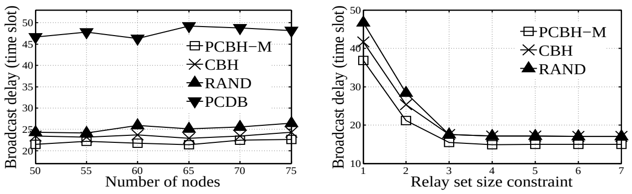
Fig. 1 (a) and Fig. 1 (b) compare the broadcast delay of PCBH-M, CBH, RAND and PCDB in single-flow case with



(a) Broadcast delay vs. number of nodes, $K = 2$

(b) Broadcast latency vs. relay set size constraint, $N = 60$

Fig. 1. Single-flow case



(a) Broadcast delay v.s. number of nodes, $K = 2$

(b) Broadcast delay v.s. relay set size constraint, $N = 60$

Fig. 2. Multi-flow case

different number of nodes (from 50 to 75) and different K (from 1 to 7) respectively. We do not display the results of PCDB in Fig. 1 (b) because PCDB only selects one relay node in each iteration, which means in PCDB K is always 1. From Fig. 1 (a) we find that the average broadcast delay follows PCDB $>$ RAND $>$ CBH $\approx$ PCBH-S. PCDB has the highest broadcast delay because it only selects one relay node in each time slot, which cannot fully utilize relay node resource to decrease delay. The broadcast delay of RAND is higher than that of CBH and PCBH-S because both PCBH-S and CBH try to inform maximized number of nodes in each time slot, while RAND just randomly selects relay nodes. The performances of PCBH-S and CBH are almost the same because in the single-flow case the schedules calculated by PCBH-S and CBH are the same (*Corollary 3.2*). From Fig. 1 (b) we can find that for broadcast delay RAND $>$ CBH $\approx$ PCBH, which is consistent with the result in Fig. 1 (a). Also, we find that for each algorithm, as the value of K increases, the broadcast delay decreases when $K \leq 4$, and remains nearly constant when $K > 4$. It is because that when K is small ($K \leq 4$), in each time slot the number of relay nodes is more likely bounded by K, and more relay nodes can be selected in each time slot if K is larger. While when K is large ($K > 4$), in each time slot the number of relay nodes is more likely bounded by the number of candidates, so K does not significantly affect the broadcast delay in this range.

Fig. 2 (a) and Fig. 2 (b) compare the broadcast delay of PCBH-M, CBH, RAND and PCDB in multi-flow case (M = 2) with different number of nodes (from 50 to 75) and different K (from 1 to 7) respectively. From Fig. 2 (a), we find that the broadcast delay follows PCDB $>$ RAND $>$ CBH $>$ PCBH-M, and from Fig. 2 (b), we find that as K increases, the broadcast delay decreases when $K \leq 3$, and remains the same level when $K > 3$, which are similar with the results in Fig. 1 (a) and Fig. 1 (b). The difference between Fig. 1 (single-flow) and Fig. 2 (multi-flow) is that in Fig. 1 CBH and PCBH-S has the same performance while in Fig. 2, PCBH-M outperforms CBH. This is because in the single-flow case, no competition exists among different flows, thus the results for selecting cooperative relays are the same between PCBH-S and CBH (indicated by *Corollary 3.2*). However, in multi-flow case, a node might be the key relay node for multiple flows, thus arbitrarily allocating the node to any flow would lead to the broadcast delay of other flows much higher.

VI. Conclusion

In this paper, we proposed a cooperative diversity scheme for low-latency multi-flow broadcast in fading wireless networks. We built a mathematical model considering probabilistic relay node allocation, multi-flow and relay-set based forwarding, and based on this model we identified a problem named the Minimum Slotted Delay Cooperative Broadcast (MSDCB) problem. We proved that MSDCB is NP-complete and $o(\log N)$ inapproximable given some restrictions and derived numerous properties of the problem. Guided by these properties we developed two heuristic algorithms named PCBH-S (single-flow) and PCBH-M (multi-flow). The experimental results demonstrate that our schemes outperform a typical previous approach. In our future work, we aim to develop a continuous time model instead of discrete time model and implement the algorithm in real-world test-bed.

Acknowledgements

This research was supported in part by U.S. NSF grants OCI-1064230, CNS-1249603, CNS-1049947, CNS-1156875, CNS-0917056 and CNS-1057530, CNS-1025652, CNS-0938189, CSR-2008826, CSR-2008827, Microsoft Research Faculty Fellowship 8300751, and U.S. Department of Energy's Oak Ridge National Laboratory including the Extreme Scale Systems Center located at ORNL and DoD 4000111689.

References

[1] H. S. Lichte, H. Frey, and H. Karl, "Fading-resistant low-latency broadcasts in wireless multihop networks: The probabilistic cooperation diversity approach," in *Proc. of MobiHoc*, 2010.

[2] P. Sinha, R. Sivakumar, and V. Bharghavan, "Enhancing ad hoc routing with dynamic virtual infrastructures," in *IEEE Conf. Comput. Commun*, 2001.

[3] J. Wu, M. Cardei, F. Dai, and S. Yang, "Extended dominating set and its applications in ad hoc networks using cooperative communication," *IEEE Trans. Parallel Distrib. Syst.*, 2006.

[4] S. Guha and S. Khuller, "Approximation algorithms for connected dominating sets," *Algorithmica*, 1998.

[5] B. S. Mergen, A. Scaglione, and G. Mergen, "Asymptotic analysis of multi-stage cooperative broadcast in wireless networks," *Joint special issue of the IEEE Transactions on Information Theory and IEEE/ACM Trans. On Networking*, 2006.

[6] M. Baghaie and B. Krishnamachari, "Delay constrained minimum energy broadcast in cooperative wireless networks," in *Proc. of Infocom*, 2011.

[7] H.-C. Lu and W. Liao, "On cooperative strategies in wireless relay networks," in *Proc. of Infocom*, 2011.

[8] S. Sharma, Y. Shi, Y. T. Hou, H. D. Sherali, and S. Kompella, "Cooperative communications in multi-hop wireless networks: joint flow routing and relay node assignment," in *Proc. of Infocom*, 2010.

[9] I. Maric and R. D. Yates, "Cooperative multicast for maximum network lifetime," *IEEE J. Sel. Areas Commun*, 2005.

[10] Y.-W. Hong and A. Scaglione., "Energy-efficient broadcasting with cooperative transmissions in wireless sensor networks.," *IEEE Trans. Wireless Communications*, 2006.

[11] J. N. Laneman, D. N. C. Tse, and G.W.Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior.," *IEEE Trans. Inf. Theory*, 2004.

[12] J. D. Proakis, *Digital Communications*. McGraw-Hill Science, 2000.

[13] S. M. Ross, *Introduction to Probability Models, 8th Edition*. Amsterdam: Academic Press, 2003.

[14] C. Qiu, L. Yu, and H. Shen, "Low-latency multi-flow broadcasts in fading wireless networks," tech. rep., 2012. [online] Available: http://people.clemson.edu/ shenh/.