Energy-efficient Cooperative Broadcast in Fading Wireless Networks

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***Abstract*—Cooperative broadcast, in which receivers are allowed to combine received packet from different senders to combat transmission errors, has gained increasing attention. Previous studies showed that broadcast optimization solutions are sufficient in non-fading environments but may suffer a low delivery ratio under wireless channel fading. Though previous work analyzed the tradeoff between energy and delay in cooperative broadcast, no works investigated the tradeoff in a fading environment. Thus, in this paper, we study this tradeoff with the consideration of fading. We formulate this problem as a Fading-resistant Delay-constrained Minimum Energy Cooperative Broadcast (FDMECB) problem, and prove that it is NP-complete. We then propose an approximation algorithm for theoretical interests. We further propose a heuristic algorithm that makes approximately optimal local decision to achieve global optimization. Our experimental results show that our algorithms outperform a previous non-fading resistant algorithm.**

I. Introduction

Various approaches have been proposed for efficient broadcasting in wireless multi-hop networks [1]–[10], in which a source node transmits a packet to all other nodes in the network. Particularly, cooperative broadcast has gained increasing attention, in which a packet receiver cooperatively combines received weak signal power from different senders to recover the original packet in broadcasting. Due to the broadcast nature of the wireless channel, a packet transmitted by a sender can be heard by all of its neighbor nodes. Thus, a node can receive multiple copies of a specific packet from multiple senders in broadcasting and cooperatively combine the signal power in an additive fashion using a cooperative diversity technique (e.g., maximal ratio combining (MRC)) [6] to recover this packet. The efficiency of broadcasting is improved by combining weak signals rather than discarding them.

In cooperative broadcast, the broadcast delay/energy cost is measured by the total time/energy needed for all nodes in the network to receive a broadcasted packet. Many works have been devoted to reducing the energy cost or delay [3], [5], [6], [9], [10], but not both. There exists a tradeoff between the energy consumption and the delay. Reducing the transmit power of senders saves energy but results in fewer nodes capable of decoding the signal, thus generating a longer broadcast delay. On the other hand, sending packets at the highest possible power maximizes the number of receivers capable of decoding the packet at each step, thus minimizing the broadcast delay.

To solve the tradeoff problem, Baghaie *et al.* [6] attempted to minimize the energy cost while meeting a desired delay constraint. They assumed a time-slotted and memoryless system, in which the MRC at the receiver is restricted to source transmissions from the present time slot. With this assumption, they formulated the delay constrained minimum energy broadcast (DMECB) problem to find which nodes broadcast together and what power level they use at each time slot. They proved that this problem is NP-hard and proposed an efficient polynomial time algorithm.

However, their problem formulation did not take into account the time-varying fading environments, where the transmissions between relay nodes are susceptible to random fluctuations in signal strength due to mobility in a multi-path propagation environment. Therefore, the optimal solution of DMECB lacks robustness and may not guarantee high delivery ratio (i.e., percent of nodes successfully receiving the packet) in fading environments. Indeed, previous studies showed that the optimization solutions are sufficient in non-fading environments but may suffer a low delivery ratio under wireless channel fading [1].

To overcome this limitation, in this paper, we study the tradeoff between energy cost and delay in cooperative broadcast with the consideration of fading. We aim to minimize the energy cost under delay constraint, while guaranteeing high delivery ratio in fading environment (fading-resistant). Accordingly, we formulate the Fading-resistant Delay-constrained Minimum Energy Cooperative Broadcast (FDMECB) problem. FDMECB takes into account the Rayleigh fading channel model [1] and ensures that each node's probability of successful packet decoding reaches a threshold. As DMECB, FDMECB assumes a deterministic successful decoding condition; that is, a receiver can successfully decode a packet if the sum of all of its received *expected* SNRs or SINRs [1] is above a decoding threshold. FDMECB also aims to find which nodes broadcast together and what power level they use in every time slot in a time-slotted and memory-less system.

We have proved that FDMECB is NP-complete and $o(\log(N))$ inapproximable. For theoretical interests, we also identify an approximation algorithm of the Diameter-bounded Directed Steiner Tree (DDST) problem [11] for FDMECB that can provide $O(n^{\epsilon})$ approximation. We then propose a time-

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efficient heuristic algorithm named Fading-Resistant Energy-Efficient Broadcast (FREEB) that makes approximately optimal local decision to achieve global optimization. The extensive experimental results demonstrate that FREEB outperforms the non-fading resistant algorithm and it nearly achieves the optimal performance compared to the optimal solution.

II. Related Work

Cooperative communication has attracted great attention from both research community and industry [1], [3], [5], [6], [9], [10], [12]–[14]. For example, Dejun Yang *et al.* [12] designed an auction scheme for the cooperative communications, where wireless nodes can trade relay services to be selected as relay nodes, which increases nodes' incentives for the participating wireless nodes to serve as relay nodes. Also, since some selfish nodes may cheat in cooperative relay to benefit themselves, Haifan Yao *et al.* [13] first proposed a cheat-proof scheme based on strategic game model for cooperative relay in cognitive radio networks in order to suppresses cheating behaviors. In cognitive radio technologies, licensed users have higher priority to access channel than unlicensed users, and motivated by the idea of cooperative communication, Jin Zhang *et al.* [14] proposed a cooperative framework where primary users may select some of unlicensed users to be the cooperative relays, and lease portion of the channel access time to unlicensed users for their own data transmission. Both analytical result and numerical result show that such cooperation can benefit both licensed users and unlicensed users for the utilization of channel. In [15], Liaoruo Wang *et al.* considered how cooperative communication improves the connectivity in wireless ad hoc networks and they demonstrated that even relatively simple physical layer cooperation in the form of noncoherent power summing can substantially improve the connectivity of large ad hoc networks.

Many cooperative broadcast methods [3], [5], [6], [9], [10] also have been proposed to reduce energy consumption or delay of broadcast. Maric *et al.* [9] considered cooperative data multicast with the objective to maximize the network lifetime. They propose an optimal algorithm to enable nodes to reliably receive a message by collecting energy during each retransmission when the message is forwarded through the network. Hong *et al.* [10] analyzed the energy savings provided by cooperative broadcast, and derived the optimal energy allocation policy that minimizes the total energy cost. They also proved that the optimum energy assignment for cooperative networks is an NP-complete and introduced several sub-optimal energy-saving solutions. Lichte *et al.* [1] considered the problem of minimizing broadcast delay while ensuring that a packet is successfully delivered to all nodes in the network (complete distribution) with high probability in fading environments. Mergen *et al.* [5] theoretically studied the effect of the source/relay transmission powers and the decoding threshold on the number of informed nodes. Wu *et al.* [3] proposed an Extended Minimum CDS (E-MCDS) approach in order to cover larger groups of uninformed nodes.

Most of the above methods try to reduce either energy consumption or broadcast delay, but not both. Finding an optimal tradeoff between energy cost and delay is essential for the applications that concern both. Baghaie *et al.* [6] formulated this tradeoff problem to the DMECB problem and found its solution. However, their formulation of successful packet decoding deterministically depends on the expected received signal strength at the receiver without considering the fading. Therefore, their derived solution may have a low delivery ratio in fading environments. Unlike this work, we study the DMECB problem with the consideration of fading by explicitly considering the distribution function of the received signal strength in fading environments.

III. System model

We consider a wireless network consisting of a set of nodes $\mathcal{V} = \{v_1, v_2, v_3, ..., v_N\}$. A packet is broadcasted from a source node, denoted by $v_{\rm s}$, to all other nodes ($\mathcal{V} - v_{\rm s}$). Time is assumed to be discretized into fixed duration time slots. The nodes that have received and decoded a packet are allowed to transmit it in the future time slots. The *packet delay* of node v_i is the minimum number of time slots for a packet to be broadcasted from source $v_{\rm s}$ to v_i. Then, the packet delay of $v_{\rm s}$ is 0. *Broadcast delay*, denoted by T, is the maximum packet delay among all the nodes in $\mathcal{V}$. Table I lists major notations used in this paper.

TABLE I
NOTATIONS

Notation	Description
$\mathcal{V}$	The set of sensor nodes
v_i	Node i
$v_{\rm s}$	Source node
$\mathcal{W}$	The set of power levels
w_k	Power level k
$h_{i,j,t}$	Channel coefficient from v_i to v_j at time slot t
$d_{i,j}$	Distance between v_i and v_j
$z_{i,k,t}$	Indicator variable indicating if v_i uses power w_k at time slot t
$\mathbf{Z}$	Schedule matrix
$X_{i,j,t}$	SNR transmitted from v_i to v_j at time slot t
N_0	Noise power density
N	The number of nodes in the network
K	The number of power levels
W	The total energy consumption of a broadcast be selected at each time slot
T	The constrained delay
$\gamma_{\rm th}$	Decoding threshold
ε	Acceptable error probability
α	Path loss exponent

We assume that the transmit power at each node is adjustable finitely, and there are K adjustable power levels denoted by $\mathcal{W} = \{w_1,\ w_2,\ w_3,\ ...,\ w_K\}$ where $w_1 = 0$ and $w_k \leq w_{k'}$ if $k \leq k'$. We consider time-varying and frequency-flat fading wireless channels. Channel effects from v_i to v_j (with distance $d_{i,j}$) at time slot t can be modeled by a single, complex, random channel coefficient $h_{i,j,k}$. We consider the Rayleigh fading channel model [1], in which all $|h_{i,j,k}|^2$ are independent and exponentially distributed with a mean value

$$\sigma^2_{i,j,k} = w_k d_{i,j}^{-\alpha} \tag{1}$$

where α is path loss exponent. We use $S_{i,j,k}$ to denote the instantaneous signal power received by v_j from v_i using power level w_k. $S_{i,j,k}$ is a random variable with Cumulative Distribution Function (CDF) of

$$F_{S_{i,j,k}} = \Pr\{S_{i,j,k} \le x\} = 1 - e^{-x/\sigma^2_{i,j,k}}. \tag{2}$$

In a single transmission, whether a packet can be successfully received by a receiver depends on the instantaneous SNR=$\frac{S_{i,j,k}}{N_0}$ at the receiver, where N_0 is the noise power density. We use a non-negative random variable $X_{i,j,k}$ to represent the SNR at v_j for the signal transmitted from v_i using power level w_k. Node v_j can successfully receive the packet from v_i iff $X_{i,j,k} \ge \gamma_{\text{th}}$ [1], where γ_{th} is a fixed decoding threshold. Based on Formula (2), suppose $X_{i,j,k}$ follows exponential distribution, we derive that the probability that v_j can successfully receive a packet from v_i with transmit power w_k equals $e^{-\gamma_{\text{th}} N_0/\sigma^2_{i,j,k}}$.

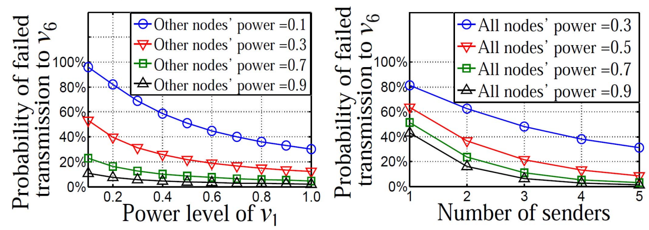
We then consider cooperative broadcast. We use MRC [6], a commonly used cooperative diversity technique, to combine the received powers at a receiver. We consider a memory-less system where nodes do not accumulate energy from transmissions occurred in previous time slots. Thus, the condition for successful decoding at receiver node v_j is $\sum_{v_i \in \mathcal{R}} X_{i,j,k_i} \ge \gamma_{\text{th}}$, where each node v_i uses power level w_{k_i} ($w_{k_i} \in \mathcal{W}$) and $\mathcal{R}$ is the set of relays transmitting packets to v_j in time slot t. Assume that X_{1,j,k_1}, X_{2,j,k_2}, ..., X_{N,j,k_N} are independent and let $\beta_{i,j,k} = N_0/\sigma^2_{i,j,k}$, the sum of SNRs that node v_j receives follows a hypoexponential distribution with the probability density function (PDF) [16]:

$$f_{\sum_{v_i \in \mathcal{R}} X_{i,j,k_i}} = \sum_{v_i \in \mathcal{R}} \beta_{i,j,k_i} e^{-\beta_{i,j,k_i} x} \prod_{v_l \in \mathcal{V}, l \neq i} \frac{\beta_{l,j,k_l}}{\beta_{l,j,k_l} - \beta_{i,j,k_i}}$$

The probability that v_j cannot successfully receive the packet in time slot t can be calculated by:

$$\Pr\left[\sum_{v_i \in \mathcal{R}} X_{i,j,k_i} < \gamma_{\text{th}}\right] = \int_0^{\gamma_{\text{th}}} f_{\sum_{v_i \in \mathcal{R}} X_{i,j,k_i}} \mathrm{d}x$$
$$= \sum_{v_i \in \mathcal{R}} (1 - e^{-\beta_{i,j,k_i}\gamma_{\text{th}}}) \prod_{v_l \in \mathcal{V}, l \neq i} \frac{\beta_{l,j,k_i}}{\beta_{l,j,k_l} - \beta_{i,j,k_i}} \tag{3}$$

We then show the relationship between Pr, the number of relays, and their used power levels in Formula (3). We consider an example with 5 relays $\{v_1, ..., v_5\}$ and 1 receiver v_6, and $\gamma_{\text{th}} = 1$ and $\alpha = 4$. Figure 1(a) shows the probability of failed transmission to v_6 (Pr) when v_1's power level is varied from 0.1 to 1 and all others' power is set to a fixed level. The result shows that Pr decreases with the increase of v_1's power level and also with the increase of all other relays' power level. This implies that higher power level of relays helps increase successful transmission to a receiver. In Figure 1(b), we vary the number of relays from 1 to 5. The figure shows that Pr decreases as the number of relays increases and also as the power level of all relays increases. The result implies that more relays help increase successful transmission to a receiver.



(a) (b)

Fig. 1. Curves of Formula (3)

IV. Problem Formulation and Analysis

A. Problem Formulation

Definition 4.1: (**broadcast relay schedule**) A broadcast relay schedule specifies which nodes in which time slot using which transmit power level to relay the broadcasted packet from a source.

A broadcast relay schedule can be represented by a three dimension matrix (named *schedule matrix*) $\mathbf{Z} = \{z_{i,k,t}\}_{N \times K \times T}$, where $z_{i,k,t}$ is an indicator variable indicating if node v_i is selected as relay in time slot t using power w_k. Then, the SNR of the signal received by v_j from v_i in time slot t is represented by $\sum_{k=1}^{K} X_{i,j,k} z_{i,k,t}$, and the sum of SNRs received at v_j is represented by $\sum_{i=1}^{N} \sum_{k=1}^{K} X_{i,j,k} z_{i,k,t}$. Formula (3) calculates the probability that a node cannot successfully decode the packet in time slot t. We assume an acceptable error probability ε. Our problem is to find an optimal broadcast relay schedule to minimize the total energy cost, while ensuring a high probability $(1-\varepsilon)$ of complete distribution with delay constraint of T. Then, the optimal relay schedule should ensure that for each receiver, say v_j, the probability that it cannot successfully decode the packet in time slot t ($\exists t \le T$) is smaller than ε, i.e., $\Pr[\sum_{i=1}^{N} \sum_{k=1}^{K} X_{i,j,k} z_{i,k,t} < \gamma_{\text{th}}] < \varepsilon$. We say v_j is *informed* in time slot t if $\Pr[\sum_{i=1}^{N} \sum_{k=1}^{K} X_{i,j,k} z_{i,k,t} < \gamma_{\text{th}}] < \varepsilon$; otherwise, we say v_j is *uninformed*. As a result, our problem can be formulated as a decision problem:

Definition 4.2: Fading-resistant Delay-constrained Minimum Energy Cooperative Broadcast problem (FDMECB): ***Instance***: *A finite set of nodes $\mathcal{V}$, a source node $v_s \in \mathcal{V}$, a set of power levels $\mathcal{W}$, non-negative random variables $X_{i,j,t}$ denoting the SNR of v_j's received signal from v_i using transmit power w_k, constants ε, T, and W (denoted by $I(\mathcal{V}, \mathcal{W}, T, v_s, \varepsilon, W)$).*

Question: *Existence of a broadcast relay schedule (or schedule matrix) such that:*

Condition1: *In each time slot t, any node v_j can be selected as relay only if v_j has been informed: $\exists \tau < t$, Pr [$\sum_{i=1}^{N} \sum_{k=1}^{K} X_{i,j,k}\ z_{i,k,\tau} < \gamma_{\text{th}}$] $< \varepsilon$.*

Condition2: *In each time slot t, any node v_i can only use one power level for transmission: $\sum_{k=1}^{K} z_{i,k,t} = 1$.*

Condition3: *By the end of the time slot T, all the nodes in $\mathcal{V}/v_s$ have been informed: $\exists \tau \le T$, $\forall v_i \in \mathcal{V}/v_s$, $\Pr[\sum_{i=1}^{N} \sum_{k=1}^{K} X_{i,j,k} z_{i,k,\tau} < \gamma_{\text{th}}] < \varepsilon$.*

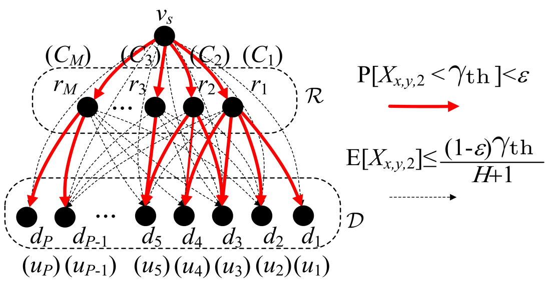


Fig. 2. Proof of Theorem 4.1

Condition4: *By the end of the time slot T, the sum of the energy consumption of all the nodes in $\mathcal{V}$ is no larger than W: $\sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{k=1}^{K} w_k z_{i,k,t} \le W$.*

We say a broadcast relay schedule is a *feasible schedule* for a problem instance $I(\mathcal{V}, \mathcal{W}, T, v_{\mathrm{s}}, \varepsilon, W)$ if the schedule can satisfy the above five conditions.

B. Hardness of FDMECB

In this section, we prove that finding an optimal solution for the FDMECB problem is NP-hard. Furthermore, we prove that finding any polynomial time algorithm that approximates the optimal solution within a factor of $o(log(N))$ is also NP-hard by using an approximation-preserving reduction from the Set Cover decision problem [17] (Set Cover in short) to the FDMECB problem.

Lemma 4.1: FDMECB is in NP.

Proof: Let n be the total number of symbols needed to represent an instance $I(\mathcal{V}, \mathcal{W}, T, v_{\mathrm{s}}, \varepsilon, W)$. To prove that FDMECB for the instance is in NP, we must prove that for a given broadcast relay schedule $\mathbf{Z}$, we can verify if it satisfies Conditions 1-4 in $O(q(n))$ and $q(\cdot)$ is a polynomial. The time complexities of additions/subtractions and multiplications /divisions of two n-symbol values are $O(n)$ and $O(n^2)$, respectively [18]. Obviously, it requires $O(NKT)$ additions, and $O(NKT)$ additions and multiplications to verify Conditions 2 and 4 respectively, which generate $O(NKTn)$ and $O(NKTn^2)$ time complexity, respectively, all of which are polynomial. To verify Conditions 1 and 3, we need to repeat the calculation of Equ. (3) which takes $O(N^2K^2n^2)$ time complexity [19] for $O(NT)$ and $O(N)$ times, respectively. Thus, the time complexity for verifying Conditions 1 and 3 are $O(N^3K^2Tn^2)$ and $O(N^3K^2n^2)$, respectively. Therefore, we can verify Conditions 1-4 in $O(N^3K^3Tn^2)$, which has a polynomial time complexity. ∎

Theorem 4.1: FDMECB is NP-complete.

Proof: We prove this theorem by constructing a polynomial time reduction from the NP-complete Set Cover [20] to FDMECB. Given a finite set of nodes $\mathcal{U} = \{u_1, u_2, ..., u_P\}$, a set of $\mathcal{U}$'s subsets $\mathcal{C} = \{C_1, ..., C_M\}$, and a positive integer H, Set Cover is: if there exists a subset $\mathcal{C}' \subseteq \mathcal{C}$ with $|\mathcal{C}'| \le H$ such that every element of $\mathcal{U}$ belongs to at least one member of $\mathcal{C}'$.

We construct the following FDMECB instance that maps to Set Cover (see Figure 2): $\mathcal{W} = \{w_1, w_2\}$, delay constrained $T = 2$, total energy consumption $W = Hw_2$, and $\mathcal{V} = \{v_s, \mathcal{R}, \mathcal{D}\}$, where relay set $\mathcal{R} = \{r_1, r_2, r_3, ..., r_M\}$ (r_i corresponds to $C_i \in \mathcal{C}$) and destination node set $\mathcal{D} = \{d_1, d_2, d_3, ..., d_P\}$ (d_i corresponds to $u_i \in \mathcal{U}$). For each pair of distinct nodes $x, y \in \mathcal{V}$, we select independent random variables $X_{x,y,k}$ such that the following conditions are satisfied: if $\{x, y\} = \{v_s, r_i\}$ or $\{x, y\} = \{r_i, d_j\}$ and $u_j \in C_i$, then $\Pr[X_{x,y,2} > \gamma_{\mathrm{th}}] < \varepsilon$ and $E[X_{x,y,1}] = 0$; otherwise, $E[X_{x,y,2}] \le \frac{(1-\varepsilon)\gamma_{\mathrm{th}}}{H+1}$ and $E[X_{x,y,1}] = 0$. The problem is, given v_{s}, $W = Hw_2$ and $T = 2$, whether a schedule $\mathbf{Z}$ exists such that Conditions 1-4 are met. This reduction process from Set Cover to FDMECB is performed in polynomial time. We then show the correctness of this reduction, i.e., a solution exists for Set Cover iff there exists a feasible schedule for the FDMECB instance.

$\Rightarrow$: Assume there exists a solution $\mathcal{C}' = \{C_{i_1}, C_{i_2}, ..., C_{i_m}\}$ ($m \le H$) for Set Cover. Then, we can construct a feasible schedule for the FDMECB instance that satisfies Conditions1-4: in the 1st time slot, v_s broadcasts the packet with transmit power w_2, and in the 2nd time slot $\mathcal{R}' = \{r_{i_1}, r_{i_2}, ..., r_{i_m}\}$ are selected as relays with transmit power w_2. Obviously, Condition1 and Condition2 are satisfied. Condition3 is satisfied because for each $r_i \in \mathcal{R}$

$$\Pr[X_{v_s,r_i,2} > \gamma_{\mathrm{th}}] < \varepsilon. \tag{4}$$

Because of the existence of a solution for Set Cover, $\forall u_j \in \mathcal{U}$, $\exists C_{i_l} \in \mathcal{C}'$ such that $u_j \in C_{i_l}$. Then, $\forall d_j \in \mathcal{D}$, $\exists r_{i_l} \in \mathcal{R}'$ ($1 \le l \le m$), such that

$$\Pr[\sum_{r=1}^{m} X_{r_{i_l},d_j,2} > \gamma_{\mathrm{th}}] < \Pr[X_{r_{i_l},d_j,2} > \gamma_{\mathrm{th}}] < \varepsilon. \tag{5}$$

Thus, every node in $\mathcal{V}$ is informed. Finally, Condition 4 is satisfied because the schedule only uses m relays and each node uses power w_2, then the total energy consumption $mw_2 \le Hw_2$.

$\Leftarrow$: Assume there exists a feasible schedule for FDMECB: in the 1st time slot, v_s broadcasts the packet with power w_2 and in the 2nd time slot, all the nodes in $\mathcal{R}' = \{r_{i_1}, r_{i_2}, ..., r_{i_m}\}$ forward the packet with power w_2. Consequently, all the nodes in $\mathcal{D}$ are informed. Then, we need to prove that we can also find a solution for Set Cover, which is $\mathcal{C}' = \{C_{i_1}, C_{i_2}, ..., C_{i_m}\}$. We first assume that this solution cannot solve Set Cover; that is, there exists $u_j \notin C_{i_l}$ for all $C_{i_l} \in \mathcal{C}'$. Hence, $\forall r_{i_l} \in \mathcal{R}'$, $E\left[X_{r_{i_l},d_j,2}\right] \le \frac{(1-\varepsilon)\gamma_{\mathrm{th}}}{H+1}$. Then, by MarKov's inequality, we can derive that

$$\Pr[\sum_{r_{i_l} \in \mathcal{R}'} X_{r_{i_l},d_j,2} > \gamma_{\mathrm{th}}] \le \frac{E(\sum_{r_{i_l} \in \mathcal{R}'} X_{r_{i_l},d_j,2})}{\gamma_{\mathrm{th}}}$$

$$\le \frac{E(\sum_{r_{i_l} \in \mathcal{R}'} X_{r_{i_l},d_j,2})}{\gamma_{\mathrm{th}}} = \frac{\sum_{r_{i_l} \in \mathcal{R}'} E(X_{r_{i_l},d_j,2})}{\gamma_{\mathrm{th}}} < 1 - \varepsilon$$

which implies that $\Pr\left[\sum_{r_{i_l} \in \mathcal{R}'} X_{r_{i_l},d_j,2} < \gamma_{\mathrm{th}}\right] > \varepsilon$. Thus, d_j is not informed after the 2nd time slot, which contradicts with the FDMECB schedule that informs all the nodes in $\mathcal{D}$.

Therefore, $\mathcal{C}' = \{C_{i_1}, C_{i_2}, ..., C_{i_m}\}$ is a solution for the Set Cover problem. ■

Corollary 4.1: FDMECB is $o(\log(N))$ inapproximable.

Proof: The reduction used in the construction of the instance in *Theorem 4.1* preserves the approximation factor. That is, if one can find an θ-approximation for FDMECB, by extension there must exist an θ-approximation for Set Cover. It was proved that Set Cover is $o(\log(N))$ inapproximable [17], thus FDMECB must be $o(\log(N))$ inapproximable. ■

V. Approximation Algorithm

In Section IV-B, we proved that FDMECB is NP-complete and $o(\log(N))$ inapproximable, therefore it is *hard* to approximate FDMECB to a factor no worse than $o(\log(N))$. It is of theoretical interest to know how close a polynomial time algorithm for FDMECB can approach the optimal solution. In this section, we consider a special case of FDMECB, in which the white noise has an exponential distribution, and we show that the existing approximation algorithm [6] for the DDST problem [11] can be used to provide $O(N^{\epsilon})$ approximation for FDMECB in this case.

Lemma 5.1: For each node v_j, if the white noise follows exponential distribution with mean value μ_0, then the probability that v_j can be informed in time slot t iff

$$\sum_{v_i \in \mathcal{R}} \delta_{i,j,k} \geq \ln(1/\varepsilon). \tag{6}$$

where $\delta_{i,j,k} = \ln\left(1 + \frac{w_k d_{i,j}^{-\alpha}}{\gamma_{\text{th}}\mu_0}\right)$ is called *relative SNR* of v_j received by v_i with power level w_k.

Proof: Define a random variable $B_j = \frac{N_0}{\sum_{v_i \in \mathcal{R}} S_{i,j,k_i}}$. Then v_j can be informed iff $B_j \leq 1/\gamma_{\text{th}}$, and the CDF of B_j is given by

$$\begin{aligned} F_{B_j}(x) &= Pr\left(B_j \leq x\right) \\ &= P(N_0 \leq x \sum_{v_i \in \mathcal{R}} S_{i,j,k_i}) \\ &= \int_0^{\infty} \int_0^{xz} f_{N_0}(y)\mathrm{d}y \cdot f_{\sum_{v_i \in \mathcal{R}} S_{i,j,k_i}}(z)\mathrm{d}z. \end{aligned} \tag{7}$$

By differentiating, we can obtain

$$\begin{aligned} f_{B_j}(x) &= \frac{\mathrm{d}}{\mathrm{d}x} F_{B_j}(x) \\ &= \int_0^{\infty} z f_{N_0}(xz) f_{\sum_{v_i \in \mathcal{R}} S_{i,j,k_i}}(z)\mathrm{d}z \\ &= \int_0^{\infty} \frac{z}{\mu_0} e^{-\frac{xz}{\mu_0}} f_{\sum_{v_i \in \mathcal{R}} S_{i,j,k_i}}(z)\mathrm{d}z. \end{aligned} \tag{8}$$

Then, the probability that v_j can be informed equals

$$\begin{aligned} &\Pr(B_j \geq 1/\gamma_{\text{th}}) \\ =& \int_0^{\infty} \int_{1/\gamma_{\text{th}}}^{\infty} \frac{z}{\mu_0} e^{-\frac{xz}{\mu_0}} f_{\sum_{v_i \in \mathcal{R}} S_{i,j,k_i}}(z)\mathrm{d}x\mathrm{d}z \\ =& \int_0^{\infty} e^{-\frac{z}{\gamma_{\text{th}}\mu_0}} f_{\sum_{v_i \in \mathcal{R}} S_{i,j,k_i}}(z)\mathrm{d}z \\ =& \mathcal{L}_{\sum_{v_i \in \mathcal{R}} S_{i,j,k_i}}\left(\frac{1}{\gamma_{\text{th}}\mu_0}\right) \end{aligned} \tag{9}$$

where $\mathcal{L}_{\sum_{v_i \in \mathcal{R}} S_{i,j,k_i}}(s)$ represents the Laplace transform of $f_{\sum_{v_i \in \mathcal{R}} S_{i,j,k_i}}(x)$. Because the Laplace transform of the exponential distribution with mean $1/\mu$ equals $\mu/(\mu + s)$, $\mathcal{L}_{\sum_{v_i \in \mathcal{R}} S_{i,j,k_i}}(s) = \prod_{v_i \in \mathcal{R}} \frac{1}{1 + w_k d_{i,j}^{-\alpha} s}$. So we can derive that,

$$\begin{aligned} \Pr\left[\sum_{v_i \in \mathcal{R}} X_{i,j,k_i} \geq \gamma_{\text{th}}\right] &= 1 - \Pr(B_j \geq 1/\gamma_{\text{th}}) \\ &= 1 - \prod_{v_i \in \mathcal{R}} \frac{1}{1 + \frac{w_k d_{i,j}^{-\alpha}}{\gamma_{\text{th}}\mu_0}}. \end{aligned} \tag{10}$$

Hence, v_j can be informed iff

$$\prod_{v_i \in \mathcal{R}} \frac{1}{1 + \frac{w_k d_{i,j}^{-\alpha}}{\gamma_{\text{th}}\mu_0}} \leq \varepsilon. \tag{11}$$

Take the logarithm on both sides of Equ. (12), we get that

$$\sum_{v_i \in \mathcal{R}} \ln\left(1 + \frac{w_k d_{i,j}^{-\alpha}}{\gamma_{\text{th}}\mu_0}\right) \geq \ln(1/\varepsilon). \tag{12}$$

■

Lemma 5.1 shows that to check whether v_j can be informed is actually to check whether the sum of relative SNR from all the senders is high than $\ln(1/\varepsilon)$. We now consider a restricted version of FDMECB, named integral version of FDMECB (FDMECB-int), which does not allow signals to be combined at receivers. Based on the conclusion in Lemma 5.1, FDMECB-int is similar to the integral version of DMECB introduced [6] that loses a factor of $\log(N)$ compared to the optimal DMECB. It is straightforward to derive that the integrality gap of FDMECB is also $\log(N)$. Now we turn our attention to how to build a polynomial time reduction from FDMECB-int to DDST. First, we introduce some concepts and notations in DDST.

In a graph, the *diameter* is the longest distance between any pair of nodes in the graph and the cost is the sum of the weights of the edges in the graph. Definition 5.1 describes the decision version of DDST:

Definition 5.1: Given a directed graph $G(\mathcal{A}, \mathcal{E})$, a set of terminals $\mathcal{A}_1 \subseteq \mathcal{A}$, a root $a_{\text{s}} \in \mathcal{A}_1$, and constants T' and W', the decision version of DDST, denoted by $\text{DDST}(G, \mathcal{A}_1, a_{\text{s}}, T', W')$, is to construct a Steiner tree rooted at a_{s}, spanning all the terminals in $\mathcal{A}_1$, with diameter no larger than T' and cost no larger than W'.

To make a polynomial time reduction from FDMECB to DDST, we construct an instance of DDST from FDMECB-int. Then, the approximation algorithm of DDST can be used to solve FDMECB. The approximation algorithm provides $O(|\mathcal{A}_1|^{\epsilon})$ approximation with time complexity $O(|\mathcal{A}_1||\mathcal{A}|^{\frac{1}{\epsilon}})$ [11].

First, we construct an *auxiliary weighted directed graph* $G(\mathcal{A}, \mathcal{E})$ for the FDMECB instance $I(\mathcal{V}, \mathcal{W}, T, v_{\text{s}}, \varepsilon, W)$. We define a graph vertex a_i for each node $v_i \in \mathcal{V}$ (named *node vertex*), and define a graph vertex $a_{i,k}$ for each power level w_k of node v_i (named *power vertex*). Then, $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2\}$, where $\mathcal{A}_1$ and $\mathcal{A}_2$ represent the sets of node vertices and power vertices, respectively. Suppose $\mathcal{V} = \{v_1, v_2, v_3, ..., v_7\}$ and $\mathcal{W} = \{w_1, w_2, w_3\}$. v_1 can inform $\{v_2, v_3, v_4\}$ using power w_1, can inform $\{v_4, v_5\}$ using power w_2, and can inform $\{v_6, v_7\}$ using power w_3. Then, the auxiliary graph formed by node vertices and power vertices is created (Figure 3). The number of vertices in $\mathcal{A}_1$ and $\mathcal{A}_2$ are $O(N)$ and $O(NK)$, respectively. We then build an edge from $a_{j,l}$ to a_i with $e(a_{j,l}, a_i) = 0$ if v_i can be informed by v_j using power level w_l, and build an edge from a_i to $a_{i,l}$ with $e(a_i, a_{i,l}) = w_l$. The set of edges in the former case (black arrows) are directed from power vertices to node vertices (denoted by $\mathcal{E}_1$), and those in the latter case (red arrows) are directed from node vertices to power vertices (denoted by $\mathcal{E}_2$). Then, $\mathcal{E} = \{\mathcal{E}_1, \mathcal{E}_2\}$. We define the tree-depth of a vertex as the distance between this vertex and the root.

Lemma 5.2: For any tree $G(\mathcal{A}, \mathcal{E})$ rooted at a node vertex, all the node vertices have odd tree-depth, and all the power vertices have even tree-depth.

Proposition 5.1: Let $G(\mathcal{A}, \mathcal{E})$ be FDMECB-int's auxiliary graph for instance $I(\mathcal{V}, \mathcal{W}, T, v_{\rm s}, \varepsilon, W)$, then FDMECB-int has a feasible schedule iff $DDST(G, \mathcal{A}_1, a_{\rm s}, 2T, W)$ has a solution.

Proof: $\Rightarrow$: For $I(\mathcal{V}, \mathcal{W}, T, v_{\rm s}, \varepsilon, W)$, suppose FDMECB-int has a feasible schedule $\mathbf{Z} = [\mathbf{Z}_1, \mathbf{Z}_2, \mathbf{Z}_3, ..., \mathbf{Z}_T]$ with $\sum_{t=1}^{T} |\mathbf{Z}_t| \leq W$. Then, we can construct a solution for $DDST(G, \mathcal{A}_1, a_{\rm s}, 2T, W)$ by serially adding vertices and edges to build a Steiner tree.

We first add $a_{\rm s}$ as the root of the tree. We then add the power vertex $a_{{\rm s},k}$ with $w_k = |\mathbf{Z}_1|$ to the tree and build edge $e(a_{\rm s}, a_{{\rm s},k})$. For each $v_i \in R_1$, we connect $a_{{\rm s},k}$ to a_i with $e(a_{{\rm s},k}, a_i) = 0$. Then, we build edge from a_i to $a_{i,l}$ if v_i uses power level w_l in $\mathbf{Z}$. After adding all the node vertices and power vertices corresponding to R_1, we add the vertices corresponding R_2 in the same matter. This process is repeated until R_T is reached. Each iteration increases the diameter of the tree by at most 2. The sum weights of all the edges equals $\sum_{t=1}^{T} |\mathbf{Z}_t| \leq W$ and the diameter is no larger than $2T$. We then add the node vertices corresponding to non-relays, which must be directed by power vertices with weight equals 0. Weight 0 does not increase the cost and connecting vertices to power vertices does not increase diameter. As a result, we find a Steiner tree spanning all the node vertices with diameter $\leq 2T$ and cost $\leq W$, a solution for $DDST(G, \mathcal{A}_1, a_{\rm s}, 2T, W)$.

$\Leftarrow$: Suppose there exists a Steiner tree rooted at $a_{\rm s}$, spanning all the terminals in the auxiliary graph, with diameter $\leq 2T$ and cost $\leq W$. Then, we prove there exists a feasible schedule for FDMECB-int.

According to Lemma 5.2, we can divide the vertices in the Steiner tree into a series of subsets according to tree-depth: $R'_1, R''_1, R'_2, R''_2, ..., R'_T, R''_T$, where $R'_1 = \{a_{\rm s}\}$, R''_t represents the set of power vertices with tree depth $2t - 1$ ($t = 1, 2, 3, ..., T$), and R'_t represents the set of node vertices with tree-depth $2t - 2$. A feasible schedule for FDMECB-int based on the solution of $DDST(G, \mathcal{A}_1, a_{\rm s}, 2T + 1, W)$ is: if $a_i \in R'_t$, then there exists $a_{i,l} \in R''_t$, that is, then v_i uses

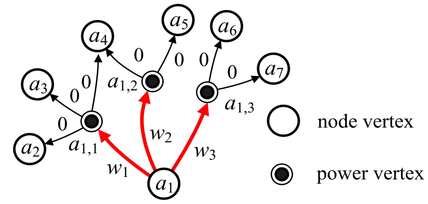


Fig. 3. Auxiliary graph

power level w_l in time slot t for packet relay. Obviously, the total energy consumption of this schedule is the cost of the Steiner tree, which is $\leq W$, and the delay constraint is $\leq T$. Also, $\forall a_i \in \mathcal{A}_1$, the Steiner tree connects to a_i from a node $a_{j,k}$ with $e(a_{j,k}, a_i) = 0$. Then, we can infer that $\forall v_i \in \mathcal{V}$, in this this schedule, there exists a relay v_j that informs v_i using power level w_k. It means that this schedule can inform all the nodes $\in \mathcal{V}$. ■

Property 5.1: The time complexity for constructing FDMECB's auxiliary directed graph is $O((NK)^3)$.

Proof: It requires $O(NK)$ time to build the edges directed from node vertices to their power vertices. As for the edges directed from power vertices to node vertices, for each power level, we need to invoke the function $\mathrm{IsInformed}()$ (based on Equ. (3)) once, which requires $O(N^2K^2)$ time. Because the number of power vertices is $O(NK)$, it takes $O((NK)^3)$ time to build the edges from power vertices to node vertices ■

Property 5.2: The number of nodes, the number of terminals, and the number of edges in FDMECB's auxiliary directed graph are $O(NK)$, $O(N)$, and $O(N^2K)$ respectively.

Proof: Obviously, the number of terminals in the auxiliary graph is $O(N)$. The number of nodes in auxiliary graph is calculated by

$$|\mathcal{A}| = |\mathcal{A}_1| + |\mathcal{A}_2| = O(N) + O(NK) = O(NK) \quad (13)$$

Each node vertex has K edges directing to its power vertices, so $|\mathcal{E}_1| = O(NK)$, and each power vertex has at most N edges directing to node vertices, so $|\mathcal{E}_2| = O(NK \times N) = O(N^2K)$. Thus, the number of edges in FDMECB's auxiliary graph is $O(N^2K)$. ■

We lose a factor of $\log(N)$ to convert FDMECB to FDMECB-int, and the approximation algorithm in [6] can approximate the optimal FDMECB-int within $O(N^\epsilon)$. Therefore, using the auxiliary directed graph introduced above, the approximation algorithm can be used to solve FDMECB within $O(N^\epsilon \log N)$ approximation on the optimal solution, which is asymptotically $O(N^\epsilon)$, and with $O(N^{\frac{\epsilon+1}{\epsilon}} K^{\frac{1}{\epsilon}})$ time complexity. Please refer to paper [6] for the details of this approximation algorithm.

VI. Heuristic Fading-resistant Algorithm

Section V presents an approximation algorithm and proved its for FDMECB. The approximation algorithm does not take advantage of the cooperative communication in broadcasting, resulting in degraded performance. In this section, we propose a computationally efficient heuristic algorithm, named Fading-Resistant Energy-Efficient Broadcast (FREEB). FREEB makes a locally optimal choice based on relays and t to approach the globally optimal solution.

Given a certain energy cost, we hope to use the energy to inform more nodes. Given a certain number of nodes, we hope to use less energy to inform them. Thus, we use these two factors to create a metric called *efficiency* of energy allocation $\mathbf{Z}_t$, denoted by $e(\mathbf{Z}_t)$.

$$e(\mathbf{Z}_t) = \frac{|\mathcal{I}_t^{\text{new}}|}{|\mathbf{Z}_t|}, \tag{14}$$

where $|\mathcal{I}_t^{\text{new}}|$ denotes the number of newly informed nodes in time slot t and $|\mathbf{Z}_t|$ denotes the total energy cost of these relays. The $\mathbf{Z}_t$ with the highest $e(\mathbf{Z}_t)$ is identified as the optimal energy allocation for time slot t.

Considering the broadcast delay constraint T, in each time slot t, we need to set a lower-bound for $|\mathcal{I}_t^{\text{new}}|$ to guarantee that the remaining uninformed nodes can be informed within T (Condition3). We use $\mathcal{I}_t$ to represent the set of nodes that have been informed by time slot t. A larger number of remaining nodes $|\mathcal{V}-\mathcal{I}_t|$ or shorter remaining time $T-t$ entails a larger lower-bound, and vice versa. Thus, we set a lower-bound as:

$$|\mathcal{I}_t^{\text{new}}| \geq |\mathcal{V}-\mathcal{I}_t|/(T-t+1). \tag{15}$$

We then prove that with this lower-bound, Condition4 can be satisfied.

Proposition 6.1: If $|\mathcal{I}_t^{\text{new}}| \geq |\mathcal{V}-\mathcal{I}_t|/(T-t+1)$, all the nodes in $\mathcal{V}/v_{\text{s}}$ can be informed within T time slots.

Proof: We first prove that $|\mathcal{I}_t^{\text{new}}| \geq \frac{N-1}{T}$ by induction. First, it is true for $t=1$ because $|\mathcal{I}_1^{\text{new}}| \geq \frac{|\mathcal{V}-v_{\text{s}}|}{T} = \frac{N-1}{T}$. Assume $|\mathcal{I}_t^{\text{new}}| \geq \frac{N-1}{T}$ for $t \leq k$, then when $t \leq k+1$

$$|\mathcal{I}_{k+1}^{\text{new}}| \geq \frac{N-\sum_{t=1}^{k}|\mathcal{I}_t^{\text{new}}|-1}{T-k} \geq \frac{N-\frac{k(N-1)}{T}-1}{T-k} = \frac{N-1}{T} \tag{16}$$

Thus, $|\mathcal{I}_t^{\text{new}}| \geq \frac{N-1}{T}$ for any t, which implies that the total number of nodes informed within T time slots equals:

$$N' = \left|\bigcap_{t=1}^{T}\mathcal{I}_t^{\text{new}}\right| = \sum_{t=1}^{T}|\mathcal{I}_t^{\text{new}}| \geq N-1 \tag{17}$$

and hence all the nodes in $\mathcal{V}/v_{\text{s}}$ are informed within the T time slots. ■

In conclusion, this optimization problem of selecting relays at each time slot can be formulated as

$$\max e(\mathbf{Z}_t) \tag{18}$$

$$\text{s.t. } |\mathcal{I}_t^{\text{new}}| \geq |\mathcal{V}-\mathcal{I}_t|/(T-t+1) \tag{19}$$

The solution to this problem makes the relays to use their energy as *efficiently* as possible.

Algorithm 2 shows the pseudocode of FREEB, in which function IsInformed$(\mathbf{Z}_t, v_j)$ (Algorithm 1) returns TRUE if v_j can correctly receive the packet with energy allocation $\mathbf{Z}_t$ or FALSE otherwise. Basically, at each time slot t, given a set of informed nodes $\mathcal{I}_t$ and t, all possible energy allocation schedule $\mathbf{Z}_t$ are identified, their newly informed $\mathcal{I}_t^{\text{new}}$ are identified using function IsInformed$(\mathbf{Z}_t, v_j)$, and corresponding

---

Algorithm 1: IsInformed$(\mathbf{Z}_t, v_j)$: indicate if v_j is informed at time slot t.

begin
- $\mathcal{R} \leftarrow$ relay set with $z_{i,k,t}=0$ and $k=1$ in $\mathbf{Z}_t$;
- **for** *each* $v_i \in \mathcal{R}$ **do**
  - $\beta_{i,j,t} \leftarrow \frac{N_0}{\sum_{k=1}^{K} z_{i,k,t} w_k d_{ij}^{\alpha}}$ `// Formulas (1);`
- `// Calculate the Pr that` v_j `is informed based on Formula (3);`
- $Pr \leftarrow 0$;
- **for** *each* $v_i \in \mathcal{R}$ **do**
  - $A \leftarrow 1$;
  - **for** *each* $v_l \in \mathcal{V}$ *and* $l \neq i$ **do**
    - $A \leftarrow A \times \beta_{lj}/(\beta_{lj}-\beta_{ij})$;
  - $Pr \leftarrow Pr + (1-e^{-\beta_{ij}\gamma_{\text{th}}})A$;
- **if** $Pr < \varepsilon$ **then**
  - **return** TRUE;
- **else**
  - **return** FALSE;

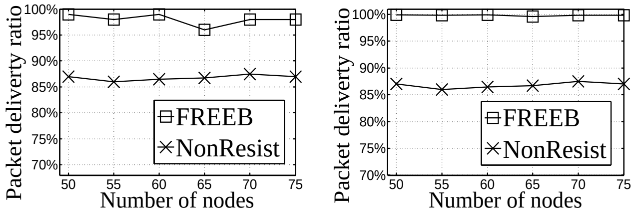
Algorithm 2: FREEB($\mathcal{I}_t$, t): output relay schedule $\mathbf{Z}_t^{\min}$ at time slot t.

begin
- $e \leftarrow 0$ `// Efficiency;`
- **for** *each* $\mathbf{Z}_t$ **do**
  - $n \leftarrow 0$ `// Number of newly informed nodes;`
  - **for** *each* $v_j \in \mathcal{V}/\mathcal{I}_t$ **do**
    - **if** `IsInformed(`$\mathbf{Z}_t$, v_j`) = TRUE` **then**
      - $n \leftarrow n+1$;
  - **if** $n \geq |\mathcal{V}-\mathcal{I}_t|/(T-t+1)$ **and** $n/|\mathrm{Z}_t| > e$ **then**
    - $e \leftarrow n/|\mathbf{Z}_t|$;
    - $\mathbf{Z}_t^{\min} \leftarrow \mathbf{Z}_t$;
- **return** $\mathbf{Z}_t^{\min}$;

$e(\mathbf{Z}_t)$ values are calculated. Among the $\mathbf{Z}_t$ options that satisfy Formula (19), the one with the highest $e(\mathbf{Z}_t)$ is chosen as the relay schedule at time slot t. Finally, the combination of the selected $\mathbf{Z}_t$ at each time slot constitutes the broadcast relay schedule solution of FDMECB.

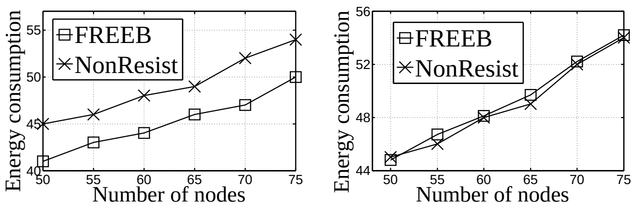
VII. Performance Evaluation

To evaluate the performance of our algorithms, we conduct simulations on MATLAB. We compare our FREEB with the non-fading resistant algorithm [6]. We refer to this algorithm as NonResist. In our discrete-event simulation, each node can finish either sending or receiving a packet but not both in one time slot. Our simulation settings are as follows: all the nodes are randomly placed in a 1000m × 1000m region; path loss exponent $\alpha=4$, maximum transmit power



(a) When $\varepsilon = 0.05$ (b) When $\varepsilon = 0.01$

Fig. 4. Packet delivery ratio of FREEB and NonResist



(a) When $\varepsilon = 0.05$ (b) When $\varepsilon = 0.01$

Fig. 5. Normalized energy consumption of FREEB and NonResist

$P_i = 20\mathrm{dBm}$, decoding threshold $\gamma_{\mathrm{th}} = 25.8\mathrm{dB}$, noise power density $N_0 = 4.32 \times 10^{-18}\mathrm{W/Hz}$ and data rate $R = 1$Mbit/s; the number of adjustable power levels $K = 5$. Each test ran for 10 times and the average result is reported. The metrics used in the evaluation include

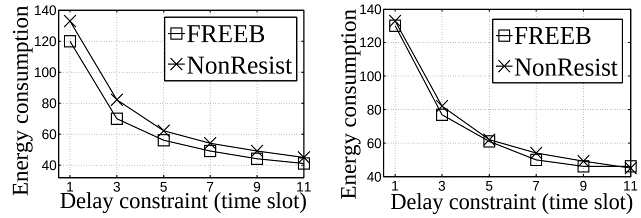
(1) *Packet delivery ratio*, which is defined as the percent of the nodes that successfully received the packet from a source when every node transmits the packet once.

(2) *Normalized energy consumption*, which is defined as the total energy consumption of all the nodes in a broadcast normalized by the threshold γ_{th} at the 11^{th} time slot.

(3) *The average number of relay transmissions per time slot*, which is calculated by the total number of relay transmissions of all nodes divided by the total number of time slots.

Packet delivery ratio. Figure 4 (a) and Figure 4 (b) compare the packet delivery ratio of FREEB and NonResist with acceptable error probabilities $\varepsilon = 0.05$ and $\varepsilon = 0.01$, respectively. We see that, when $\varepsilon = 0.05$ and $\varepsilon = 0.01$, FREEB's packet delivery ratios are about 10% and 15% higher than those of NonResist, respectively. FREEB has higher packet delivery ratio than NonResist because the relays in FREEB always select the power levels that can guarantee successful transmission with high probability $1-\varepsilon$ with fading consideration. NonResist assumes that the channel is non-fading, and hence a packet can be successfully received iff the product of the transmission power and the average value of channel coefficient is higher than γ_{th}. This assumption makes NonResist fading-susceptible since channel coefficient could be smaller than its average value in a fading environment. In addition, comparing Figure 4(a) and Figure 4(b), we find that a lower acceptable error probability ε leads to a higher packet delivery ratio in FREEB since the successful transmission probability equals $1 - \varepsilon$.

Energy consumption. Figure 5 (a) and Figure 5 (b) compare the normalized energy consumption of FREEB and NonResist when the number of nodes ranges from 50 to 75 with acceptable error probabilities $\varepsilon = 0.05$ and $\varepsilon = 0.01$, respectively. From the figures, we have two observations: (1) the normalized energy consumption of FREEB is lower than that of NonResist when $\varepsilon = 0.05$, but it is almost the same as that of NonResist when $\varepsilon = 0.01$, and (2) the normalized energy consumption increases as the number of nodes increases for both algorithms. Recall that NonResist has lower packet delivery ratio than FREEB, so NonResist needs more packet transmissions hence more energy to reach complete distribution when $\varepsilon = 0.05$. Though FREEB requires fewer transmissions than NonResist, each relay in FREEB uses higher transmit power to guarantee higher probability of successful transmission, especially when the acceptable error probability is extremely low. Therefore, when $\varepsilon = 0.01$, FREEB consumes similar energy cost as NonResist. The reason for the observation (2) is obvious since it always requires more transmissions to deliver a packet to a larger number of nodes.

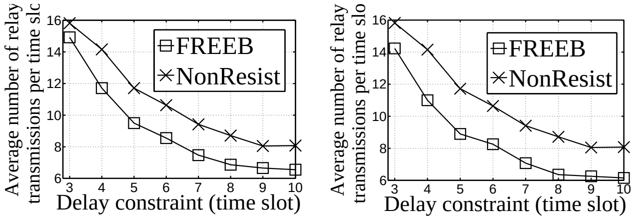


(a) When $\varepsilon = 0.05$ (b) When $\varepsilon = 0.01$

Fig. 6. Energy-delay tradeoff

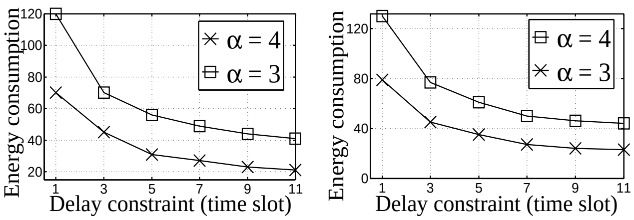
Energy-delay tradeoff. We then study the energy-delay tradeoff of FREEB in comparison with NonResist. Figure 6 (a) and Figure 6 (b) show the normalized energy consumption of FREEB and NonResist versus the delay constraint in time slot when $\varepsilon = 0.05$ and $\varepsilon = 0.01$, respectively. Both figures demonstrate that the energy consumption decreases as the delay constraint increases. The reason is that when the delay constraint is smaller, more nodes need to be informed within the delay constraint and hence more energy is used. In FREEB, Formula (15) shows that when the delay constraint T decreases, the size of the set of the newly informed nodes $|\mathcal{I}_t^{\mathrm{new}}|$ increases. Based on Algorithm 2, we need to find $|\mathbf{Z}_t|$ that produces more $\mathrm{IsInformed}(\mathbf{Z}_t, v_j) = \mathrm{TRUE}$ hence more occurrences of $Pr < \varepsilon$ in Formula (3). Recall that in Formula (3), as Figure 1 shows, increasing the power level and number of relays can increase the Pr. Thus, the selected $|\mathbf{Z}_t|$ uses higher power level and more relays. Consequently, smaller delay constraint increases power level and relays, and hence the total energy consumption in broadcast.

Figure 7 (a) and Figure 7 (b) show the average number of relay transmissions per time slot of FREEB and NonResist versus the delay constraint when $\varepsilon = 0.05$ and $\varepsilon = 0.01$, respectively. We find that the average number of relay transmissions per time slot decreases as the delay constraint increases. As mentioned previously, when delay constraint T is smaller, more nodes need to be successfully informed in each



(a) When $\varepsilon = 0.05$ (b) When $\varepsilon = 0.01$

Fig. 7. Average number of relay transmissions per time slot



(a) When $\varepsilon = 0.05$ (b) When $\varepsilon = 0.01$

Fig. 8. Effect of network channel fading on energy consumption

time slot. As Figure 1 (b) shows, more relays increase the probability of successful delivery. NonResist generates more relays in each time slot because it produces more delivery failures due to neglect of fading, and hence requires more transmissions for complete distribution. By comparing these two figures, we find that in FREEB, a smaller acceptable error probability ε produces smaller average number of relay transmissions per time slot. This is because a smaller ε requires a higher probability of successful transmission, which leads to fewer total relay transmissions to inform all nodes and hence smaller average number of relay transmissions per time slot.

Effect of channel fading. Recall that path loss exponent α represents the degree of network channel fading. α should be set to a larger value in a more severe fading environment. Figure 8 (a) and Figure 8 (b) compare the normalized energy consumption of FREEB with different path loss exponent when $\alpha = 3$ and $\alpha = 4$. We find that the energy consumption is higher when α is larger, i.e., the channel fading is more severe. This is because when fading is more severe, the power strength of signal received decreases more rapidly as the distance increases (according to Formula (1)). Thus, each relay is required to use a higher power level to guarantee successful packet delivery. We also see that the normalized energy consumption decreases as the delay constraint increases due to the same reasons as in Figure 6.

VIII. Conclusions

In this paper, we study the problem of minimizing energy consumption of cooperative broadcast with delay constraint in fading environments in wireless networks. Though channel fading decreases delivery ratio in cooperative broadcast, the previous works did not consider it in energy-delay tradeoff study. Using a Rayleigh fading model, we formulated a Fading-resistant Delay-constrained Minimum Energy Cooperative Broadcast (FDMECB) problem. We proved that this problem is NP-hard and $o(\log(N))$ inapproximable. We identified a polynomial-time Steiner tree based approximation algorithm with $O(n^{\epsilon})$ approximation ratio for FDMECB. However, the approximation algorithm does not use cooperative transmission. To overcome this shortcoming, we further propose a heuristic computationally efficient Fading-Resistant Energy-Efficient Broadcast (FREEB) algorithm. Extensive simulation results demonstrate that FREEB outperforms a previous algorithm without considering fading.

Acknowledgements

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