Link Scheduling In Cooperative Communication With SINR-Based Interference

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***Abstract*—Though intensive research efforts have been devoted to the study of the *link scheduling problem* in wireless networks, no previous work has discussed this problem for *cooperative communication* networks, in which receivers are allowed to combine messages from different senders to combat transmission errors. In this paper, we study the *link scheduling* problem in wireless cooperative communication networks, in which receivers are allowed to combine copies of a message to combat fading. We formulate two problems named *cooperative link scheduling problem (CLS)* and *one-shot cooperative link scheduling problem (OCLS)*. The first problem aims to find a schedule of links that uses the minimum number of time slots to inform all the receivers. The second problem aims to find a set of links that can inform the maximum number of receivers in one time slot. As a solution, we propose an algorithm for both CLS and OCLS with $g(\mathcal{K})$ approximation ratio, where $g(\mathcal{K})$ is so called *diversity of key links*. In addition, we propose a greedy algorithm with $O(1)$ approximation ratio for OCLS when the number of links for each receiver is upper bounded by a constant. Simulation results indicate that our cooperative link scheduling approaches outperform non-cooperative ones.**

1. Introduction

In wireless networks, the problem of scheduling link transmissions, or the *scheduling problem*, has been a subject of much interest over the past years. In the scheduling problem, given a set of links, we need to determine *which links* should be active at *what times* and at *what power levels* should communication take place. The goal of the problem is to optimize one or more of performance objectives, such as *network throughput*, *delay* or *energy consumption*. Though the scheduling problem has been well studied based on various network models [1]–[11], to the best of our knowledge, none of the previous works takes into account cooperative communication (CC) for this problem based on SINR model, in which receivers are allowed to cooperatively combine the received messages to combat transmission errors.

It has been shown that CC has a great potential to increase the capacity of wireless networks [12]–[14]. In wireless networks, before a message reaches the destination (receiver), it may have several copies stored by other nodes. For example, the sender's neighboring nodes can store the unintended message from the sender due to the broadcast nature of wireless transmission; also, in multi-hop transmission, relay nodes can

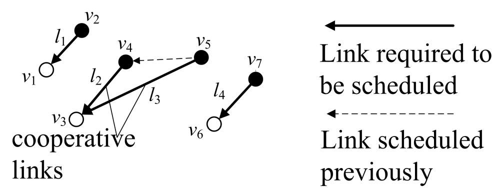


Figure 1. Schedule cooperative links.

store the copies of the original message. In CC, the nodes storing the copies (including the original message) are allowed to send the copies to the receiver simultaneously, and the receiver can combine the signal power of the received copies in an additive fashion using a cooperative diversity technique (*e.g.*, maximal ratio combining (MRC)) [12] to recover the message. Fig. 1 gives a simple example for CC: suppose v_4 has received and stored the messages from v_5, then v_4 and v_5 are able to send the message together to their destination v_3. Because v_3 can combine the messages transmitted from v_4 (in link l_2) and v_5 (in link l_3), the opportunity for v_3 to decode the message is increased. In this paper, we call the links that transmit the same message *cooperative links*. For example, the links l_2 (from v_4 to v_3) and l_3 (from v_5 to v_3) in Fig. 1 are cooperative links.

The objective of our work in this paper is to study the link scheduling problem in wireless cooperative communication networks, namely the *cooperative link scheduling* problem. Similar to the works in [5]–[9], we consider the problem separately from the routing problem and the power control problem, each of which constitutes a topic of their own. Therefore, we concentrate our attention on scheduling single-hop links, assuming all senders transmit at a fixed power level. In summary, our problem has two main differences from the traditional scheduling problem: 1) the received signal power of cooperative links (*e.g.* l_2 and l_3 in Fig. 1) can be combined in an additive fashion at the receiver, and 2) the metric measured in the problem is the number of receivers to be informed, rather than the number of links activated. Notice that the second difference implies that a link will not transmit message once its destination has been informed. Take Fig. 1 for example, l_2 is no longer need to be activated if v_3 has decoded the message from l_3.

When studying the scheduling problem in wireless networks, the choice of the interference model is of fundamental significance. The most commonly used interference model in traditional scheduling problem is so called *graph based* *model* [1], [2], which typically defines a set of *interference edges* to describe the conflicts among nodes, thus modeling interference as a binary measure. Such models cannot describe the interference in CC, because they assume that the signal power of each link is independent, while in CC the signal power of the copies of the same message can be combined. Comparing to graph based model, another interference model, named *physical interference model* (or *SINR model*), offers a more realistic representation of wireless networks. In such a model, a message is received successfully iff the SINR, *i.e.* the ratio of the received signal power to noise plus the sum of the interference caused by all other nodes, is no smaller than a hardware-defined threshold. This definition of a successful transmission, as opposed to the graph-based definition, accounts also for CC. However, the SINR model makes the analysis of algorithms more challenging than the graph-based model.

In this paper, we study the link scheduling problem based on the SINR model. The objective of this work is to optimize *delay* or *throughput* of the network. To achieve these two goals, we formulate two problems, namely *cooperative link scheduling problem (CLS)* and *one-shot cooperative link scheduling problem (OCLS)*. The first problem aims to find a schedule of links to inform all the receivers using the minimum number of time slots. In other words, it tries to minimize the *maximum delay* of all the receivers. The second problem aims to find a set of links to maximize the number of receivers informed in one slot. As a solution, we propose two *link length diversity (LLD)* based algorithms LLD-CLS and LLD-OCLS to solve CLS and OCLS, respectively. The basic idea of these two algorithms is to partition all the links into several classes based on their length (*i.e.*, distance between the link's sender and receiver) and schedule the links in each class separately. We prove that both LLD-CLS and LLD-OCLS have $g(\mathcal{K})$ approximation ratio, where $g(\mathcal{K})$ denotes the diversity of key links (Definition 5.2 and Definition 5.3). In addition, we consider a special case of the OCLS problem, in which the number of links for each receiver is upper bounded by a constant, and propose a simple greedy algorithm for it: in each iteration, the algorithm greedily picks up the "strongest" unpicked links and excludes any link that conflicts with the links we have selected. We prove that this greedy algorithm has $O(1)$ approximation ratio. Simply put, we mainly have two contributions:

1) We formulate two problems: CLS and OCLS.

2) We propose algorithms LLD-CLS and LLD-OCLS for CLS and OCLS, respectively, with $g(\mathcal{K})$ approximation ratio. Furthermore, we propose an algorithm with $O(1)$ approximation guarantee for OCLS when the number of senders in each request is upper bounded by a constant.

The remainder of this paper is organized as follows. Section 3 builds the mathematical model for the link scheduling problem. Section 2 presents related work. Section 4 defines the CLS problem and the OCLS problem and proves the hardness of both problems. In Section 5, we propose one algorithm for CLS and two algorithms for OCLS. Section 6 evaluates the performance of our proposed schemes in comparison with other algorithms. Section 7 concludes this paper with remarks on our future work.

2. Related Work

Recently, intensive research efforts have been devoted to the study of the link scheduling problem. We categorize these works based on the interference model they used: *graph-based model* and *SINR model*.

Graph-based model. Graph based models have been served as the useful abstraction for studying scheduling problems for many years [1], [2]. For example, Sharma *et al.* [1] defined a k-hop interference model for the problem, in which no two links within k-hops can successfully transmit at the same time. The authors showed that when $k > 1$, the problems are NP-hard and cannot be approximated within a factor that grows polynomially with the number of nodes in the network. Hand *et al.* [10] proposed a MAC protocol called RTOB, which can achieve high reliability by making efficient use of radio channels based mobile slotted Aloha. Murakami *et al.* [11] presented an framework where multiple APs working on the same channel concurrently transmit frames to avoid interference and hence increase the throughput. Although these algorithms present extensive theoretical analysis, they are constrained to the limitations of a model that ultimately abstracts away the accumulative nature of wireless signals.

SINR model. There are many works on the link scheduling problem under SINR model [5]–[9]. Goussevskaia *et al.* [5] first formulated this problem in the geometric SINR model, where nodes are arbitrarily distributed in 2D Euclidean space, and showed that the formulated problem is NP-hard. They then proposed a greedy algorithm for the problem with performance guarantee $O(g(L))$, where $O(g(L))$ is so-called link diversity of the networks. Goussevskaia *et al.* also formulated a variation of the problem, in which analog network coding is allowed, and presented NP-hard proof of the problem [7]. Chafekar *et al.* [6] proposed an algorithm for the scheduling problem with SINR constraints, with $O(g(D))$ performance guarantee, where $O(g(D))$ is the ratio between the maximum and the minimum distances between nodes. In addition, some works focus on designing algorithms with lower approximation guarantee [8], [9]. [8] proposed a scheduling algorithm with constant approximation guarantee, which is independent of the topology and the size of the network. Though the link scheduling problem has been well studied in various models, as far as we know, no previous work considers CC.

3. The System Model

In this section, we introduce the mathematical mod- el used throughout this paper. Consider a finite set of nodes V, a set of links $L \subseteq V \times V$, and a set of requests $F = \{f_1, ..., f_N\}$. Each request f_i ($1 \le i \le N$) can be represented by a tuple (I_i, r_i) ($I_i \subset L$ and $r_i \in V$), where I_i and r_i denote the *set of links* and the *receiver* in f_i, respectively. The set of all receivers is denoted by $R = \{r_1, ..., r_N\}$. For each receiver r_i, we call I_i the *desired link set* of r_i. We represent the link from a sender s to its destination r by $l_{s,r}$. The Euclidean distance between any two nodes $u, v \in V$ is denoted by $d_{u,v}$, and the *length* of a link $l_{s,r}$, denoted by $d(l_{s,r})$, is defined as the Euclidean distance between s and r: $d(l_{s,r}) \triangleq d_{s,r}$. We assume a time slotted system with time slots normalized to integral units, so that slot boundaries occur at times $t \in \{0, 1, 2, ...\}$, and slot t refers to the time interval $[t, t+1)$. It is assumed that the length of every link is known at the beginning of each time slot.

Geometric SINR ($\mathrm{SINR_G}$) **model**. In the Signal to Interference plus Noise Ratio (SINR) model, whether a message can be transmitted correctly depends on the ratio of the received signal strength and the sum interference caused by the senders sending simultaneously plus noise level. In this paper, we consider a geometric SINR ($\mathrm{SINR_G}$) model, in which the nodes live in 2D Euclidean space, and the gain (or signal attenuation) between two nodes is determined by the distance between the two nodes. In particular, a signal fades with the distance to the power of α, namely *path loss parameter*. The exact value of α depends on external conditions of the medium (*e.g.*, humidity) and the exact sender-receiver distance. By convention, we assume that $\alpha > 2$. We also assume that all nodes transmit with the same power level P. Then, for any link $l_{s,r}$, the signal power received at the receiver r is

$$P(l_{s,r}) = P \times d(l_{s,r})^{-\alpha}. \tag{1}$$

For any link $l_{s',r'}$ such that $r' \neq r$, the interference of $l_{s',r'}$ on $l_{s,r}$ is calculated by

$$P_{\mathrm{interf}}(l_{s',r'}, l_{s,r}) = P \times d_{s',r}^{-\alpha}. \tag{2}$$

We define: $\mathrm{SINR}(l_{s,r}) \triangleq \frac{P \times d(l_{s,r})^{-\alpha}}{\sum_{l_{s',r'} \in \mathcal{I} \setminus l_{s,r}} P \times d_{s',r}^{-\alpha} + N_0}$, where $\mathcal{I}$ is the set of all active links and N_0 is noise power density. Then, a message from s to r can be decoded correctly iff $\mathrm{SINR}(l_{s,r})$ is no smaller than the decoding threshold γ_{th}.

Cooperative communication. In CC, the reliability of a message can be improved by using several different links to transmit the copies of the message to one receiver, namely *diversity gain*. The multiple copies of a message can be combined at the receiver into a single message to combat fading. For this process, we assume the Maximum Ratio Combining (MRC) filter [12] as commonly used in diversity receivers. It can be modeled by computing the sum of all the received instantaneous SINRs. If this sum is above the decoding threshold, the original message can be successfully decoded from the packet copies. We use $\mathcal{I}_i$ to represent the set of active links in I_i. Let $\mathcal{I} = \cup_{i=1}^{N} \mathcal{I}_i$ denote the set of all active links. Then, the SINR for a receiver r_i is defined by

$$\mathrm{SINR}_{r_i} \triangleq \frac{\sum_{l_{s,r_i} \in \mathcal{I}_i} P d(l_{s,r_i})^{-\alpha}}{\sum_{l_{s,r} \in \mathcal{I} \setminus \mathcal{I}_i} P d_{s,r_i}^{-\alpha} + N_0}. \tag{3}$$

For the sake of simplicity, in the following, we ignore the influence of N_0 in the calculation of SINR since N_0 has no significant effect on the results [5]. Then,

$$\mathrm{SINR}_{r_i} \triangleq \frac{\sum_{l_{s,r_i} \in \mathcal{I}_i} d(l_{s,r_i})^{-\alpha}}{\sum_{l_{s,r} \in \mathcal{I} \setminus \mathcal{I}_i} d_{s,r_i}^{-\alpha}} \tag{4}$$

and r_i can correctly decode the message (or be *informed*) iff $\mathrm{SINR}_{r_i} \geq \gamma_{\mathrm{th}}$.

4. Problem Formulation and Analysis

In this section, we formulate two problems, named CLS (Section 4.1) and OCLS (Section 4.2), and prove that the problems are NP-hard.

4.1. The CLS problem

For the CLS problem, we determine the set of active links at each time slot. Hence, a CLS schedule can be represented by a link set sequence $\mathcal{I}_{\mathrm{cls}} = \{\mathcal{I}^1, ..., \mathcal{I}^T\}$, where $\mathcal{I}^t$ is the set of active links at time slot t and T is the number of time slots the schedule takes. We say a CLS schedule is *feasible* iff this schedule enables every intended receiver to be informed. The objective of the CLS problem is to find a feasible CLS schedule that takes the minimum number of time slots. Formally, the decision version of CLS is defined as follows:

Instance: A finite set of nodes in a geometric plane V, a set of requests $F = \{f_1, ..., f_N\}$ (each request $f_i \in F$ has a set of links $\mathcal{I}_i$ and a receiver r_i), and constants γ_{th} and T.

Question: Existence of a CLS schedule $\mathcal{I}_{\mathrm{cls}}$ *s.t.*

- $\mathcal{I}^t \cap \mathcal{I}^{t'} = \phi \ \ \forall 1 \leq t < t' \leq T$;
- each r_i can be informed by time slot T.

4.2. The OCLS problem

In contrast to the CLS problem, which aims to inform all the receivers using the minimum number of time slots, the objective of the OCLS problem is to pick a subset of links, denoted by $\mathcal{I}_{\mathrm{ocls}}$, such that the number of receivers to be informed is maximized. In other words, we attempt to use one slot to its full capacity. Formally, we define the decision version of the OCLS problem as follows:

Instance: A finite set of nodes in a geometric plane V, a set of requests $F = \{f_1, ..., f_N\}$ (each request $f_i \in F$ has a set of links $\mathcal{I}_i$ and a receiver r_i), and constants γ_{th} and M.

Question: Existence of a subset of links $\mathcal{I}_{\mathrm{ocls}}$ *s.t.* at least M receivers can be informed.

5. Approximation Algorithms

When each flow has only one link, CLS and OCLS are actually the scheduling problem and one-shot scheduling problem in [5], respectively, both of which are proved to be NP-hard. Hence, both CLS and OCLS are also NP-hard. Due to the hardness of CLS and OCLS, no polynomial time algorithm exists for determining the optimal schedule for either problem. In this section, we propose a link length diversity (LLD) based algorithm for both CLS (Section 5.2) and OCLS (Section 5.3), with a bounded performance guarantee $O(g(\mathcal{K}))$. In addition, we propose a constant approximation ratio algorithm for OCLS, when the link set size for each request is upper bounded by a constant (Section 5.4). Before presenting these algorithms, we first introduce some definitions and notations (Section 5.1).

5.1. Definitions and Notations

Definition 5.1. (*Relative interference (RI)*) Given a receiver r_i and its active desired link set $\mathcal{I}_i$, the RI of link $l_{s,r}$ ($r \neq r_i$) on r_i is the increase caused by $l_{s,r}$ in the inverse of the SINR at r_i, scaled by γ_{th}

$$RI_{l_{s,r}}(r_i, \mathcal{I}_i) = \gamma_{\mathrm{th}} \cdot \frac{d_{s,r_i}^{-\alpha}}{\sum_{l \in \mathcal{I}_i} d(l)^{-\alpha}}. \tag{5}$$

Similarly, the RI of a set of links $\mathcal{I}'$ ($\mathcal{I}' \cap \mathcal{I}_i = \phi$) on r_i is the sum RI of the links in $\mathcal{I}'$ on r_i

$$RI_{\mathcal{I}'}(r_i, \mathcal{I}_i) = \sum_{l \in \mathcal{I}'} RI_l(r_i, \mathcal{I}_i). \tag{6}$$

Property 5.1. Suppose all links in a link set $\mathcal{I}'$ are activated simultaneously, then a receiver r_i, with active desired link set $\mathcal{I}_i$, can be informed iff $RI_{\mathcal{I}'}(r_i, \mathcal{I}_i) \leq 1$.

Lemma 5.1. Given a set of disjoint link sets $\mathcal{I}_1$, ..., $\mathcal{I}_n$ and a receiver r_i, which has active desired link set $\mathcal{I}_i$ ($\mathcal{I}_i \cap \mathcal{I}_j = \phi, \forall 1 \leq j \leq n$), the RI of the union $\mathcal{I} = \cup_{j=1}^{n} \mathcal{I}_j$ on a receiver r_i is the sum RI of all link sets $\mathcal{I}_1, ..., \mathcal{I}_n$ on r_i:

$$RI_{\mathcal{I}}(r_i, \mathcal{I}_i) = \sum_{j=1}^{n} RI_{\mathcal{I}_j}(r_i, \mathcal{I}_i). \tag{7}$$

Proof By Definition 5.1,

$$RI_{\mathcal{I}}(r_i, \mathcal{I}_i) = \sum_{j=1}^{n} \sum_{l \in \mathcal{I}_j} RI_l(r_i, \mathcal{I}_i) = \sum_{j=1}^{n} RI_{\mathcal{I}_j}(r_i, \mathcal{I}_i). \tag{8}$$

Definition 5.2. (*Key link*) Given a receiver r_i and its link set $\mathcal{I}_i$. The *key link* of r_i, denoted by $\kappa(r_i)$, is defined as the shortest link in $\mathcal{I}_i$:

$$\kappa(r_i) \triangleq \arg\min\{d(l) | l \in \mathcal{I}_i\}. \tag{9}$$

In the following, we use $\mathcal{K} = \{\kappa(r_1), ..., \kappa(r_N)\}$ to denote the set of all key links.

Definition 5.3. (*Length diversity*) Length diversity [5] of a set of links L, denoted by $g(L)$, indicates the number of magnitudes of link distances of L. We define the *link length set* of L by

$$G(L) \triangleq \{h | \exists l, l' \in L : \lfloor \log(d(l)/d(l')) \rfloor = h\}, \tag{10}$$

and define the link length diversity (LLD) by $g(L) \triangleq |G(L)|$. In reality, $g(L)$ is usually a small constant [5].

Definition 5.4. (*Receiver density*) Given a set of receivers $\mathcal{R}$ and an area A (*e.g.,* a square), the receiver density of $\mathcal{R}$ in A is defined as the number of receivers in $\mathcal{R}$ that reside in A.

5.2. LLD based algorithm for CLS

The LLD based algorithm for CLS (LLD-CLS for short) consists of three steps: 1) Calculate the key link for each receiver; 2) Build disjoint link classes according to the links' length; 3) For each link class, construct a feasible schedule using a greedy strategy. In the following we introduce this algorithm in detail.

As we stated in the introduction part, CC can help each receiver decode the message from its desired link set. However, it also generates more interference to other links that transmit the message simultaneously. Hence, we set a link size constraint Δ for each request in LLD-CLS. The algorithm starts by calculating the key link set $\mathcal{K}$ and its link length set

Algorithm 1: Pseudo code for LLD-CLS.

```
input : {L_1, ..., L_g(K)}, {R_1, ..., R_g(K)};
output: I_cls = {I^1, I^2, ...I^t};
1  t ← 0;
2  for k ← 1 to g(K) do
3      Partition the region into squares A^k = {A^k_{a,b}} of
       size β_k × β_k;
4      Color the squares with {1, 2, 3, 4} s.t. no two
       adjacent squares have the same color (see Fig. 2
       (a));
5      for j ← 1 to 4 do
6          while R_k has receivers located in squares in j
           do
7              t ← t + 1;
8              for each square in j that has receivers in
               R_k do
9                  Pick one receiver r_i in the square;
10                 if |I_i| > Δ then
11                     Add the shortest Δ links in I_i to I^t;
12                 else
13                     Add all the links in I_i to I^t;
14                 Remove r_i from R_k;
15 return I_cls = {I^1, I^2, ...I^t};
```

$G(\mathcal{K}) = \{h_1, ..., h_{g(\mathcal{K})}\}$. Then, we build $g(\mathcal{K})$ disjoint link classes L_1, ..., $L_{g(\mathcal{K})}$ from L, *s.t.*

$$L_k = \{l \in L | 2^{h_k} \cdot \sigma \leq d(l) < 2^{h_k+1} \cdot \sigma\} \tag{11}$$

where σ is the length of the shortest link in L. Next, each link set L_k is scheduled separately (see Algorithm 1). When scheduling L_k, the whole region is partitioned into a set of squares $\mathcal{A}^k = \{A^k_{a,b}\}$, where (a, b) represents the location of the square in the grid and each square has size $\beta_k = 2^{h_k+1} \cdot \sigma\beta$, where

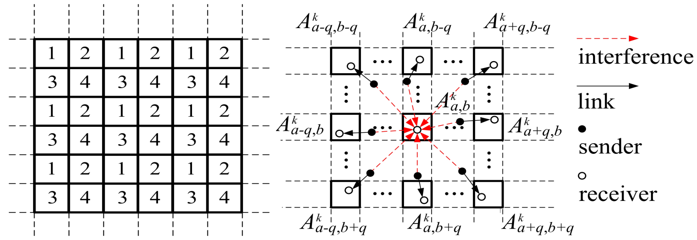
$$\beta = \left(\frac{8\Delta(\alpha-1)\gamma_{\text{th}}}{\alpha-2}\right)^{\frac{1}{\alpha}}. \tag{12}$$

Then, all the squares in $\mathcal{A}^k$ are colored regularly with 4 colors (see Fig. 2 (a)). Links whose receivers belong to different cells of the same color are scheduled simultaneously (lines 6-12).

Notice that each receiver's key link must be in one of these classes. Hence, we can partition the receiver set R into $g(\mathcal{K})$ disjoint receiver classes $R_1, ..., R_{g(\mathcal{K})}$ based on the link classes the receivers' key links belong to, *i.e.*, $R_k = \{r_i | \kappa(r_i) \in L_k, r_i \in R\}$. In Algorithm 1, the goal of scheduling each link class L_k is actually to make all receivers in R_k be informed. In Theorem 5.1, we show that the schedule calculated by LLD-CLS is feasible, *i.e.*, any receiver $r_i \in R_k$ can be informed by the active links in L_k.

Theorem 5.1. LLD-CLS is feasible.

Proof Without loss of generality, we examine any receiver $r_i \in R_k$. Because $\kappa(r_i) \in L_k$, $2^{h_k}\sigma \leq \kappa(r_i) < 2^{h_k+1}\sigma$,



(a) Partition and coloring (b) Proof of Theorem 5.1

Figure 2. LLD based algorithm for CLS.

which implies that the signal power received at r_i from its active desired link set $\mathcal{I}_i$ is at least

$$P_{\mathcal{I}_i,r_i} \geq P/2^{\alpha(h_k+1)}\sigma^{\alpha}. \tag{13}$$

Now, we consider the interference caused by the transmission from other requests. Suppose r_i is located in square $A^k_{a,b}$, since links are scheduled concurrently iff their receivers reside in the square with the same color, the interference can only be caused by the senders whose receivers are in $A^k_{a\pm 2q,b\pm 2q}$, $A^k_{a\pm 2q,b\mp 2q}$, $A^k_{a,b\pm 2q}$, and $A^k_{a\pm 2q,b}$, where $q \in \mathbb{N}$ (see Fig. 2 (b)). We represent the set of all active links whose receivers are in the $8q$ squares by $\mathcal{Q}^k_q$. For any link $l \in \mathcal{Q}^k_q$, because the distance between r_i and l's sender is at least $2q\beta_k - 2^{h_k+1}\sigma$, the RI of l on r_i is at most

$$\begin{aligned} RI_l(r_i,\mathcal{I}_i) &\leq \frac{P \times (2q\beta_k - 2^{h_k+1}\sigma)^{-\alpha}}{P_{\mathcal{I}_i,r_i}} \cdot \gamma_{\text{th}} \\ &\leq \frac{(2q\beta_k - 2^{h_k+1}\sigma)^{-\alpha}}{2^{-\alpha(h_k+1)}\sigma^{-\alpha}} \cdot \gamma_{\text{th}} \\ &= (2q\beta - 1)^{-\alpha} \cdot \gamma_{\text{th}}. \end{aligned} \tag{14}$$

Since there are at most $8q\Delta$ links in $\mathcal{Q}^k_q$, the RI of $\mathcal{Q}^k_q$ on r_i is upper bounded by

$$RI_{\mathcal{Q}^k_q}(r_i,\mathcal{I}_i) = \sum_{l\in\mathcal{Q}^k_q} RI_l(r_i,\mathcal{I}_i) \leq \frac{8q\Delta \cdot \gamma_{\text{th}}}{(2q\beta-1)^{\alpha}}, \tag{15}$$

and the RI of all active links $\mathcal{Q}^k = \cup_q \mathcal{Q}^k_q$ on r_i is upper bounded by (according to Lemma 5.1))

$$\begin{aligned} RI_{\mathcal{Q}^k}(r_i,\mathcal{I}_i) &= \sum_{q=1}^{\infty} RI_{\mathcal{Q}^k_q}(r_i,\mathcal{I}_i) \sum_{q=1}^{\infty} \frac{8q\Delta\gamma_{\text{th}}}{(2q\beta-1)^{\alpha}} \\ \leq \sum_{q=1}^{\infty} \frac{8q\Delta\gamma_{\text{th}}}{q^{\alpha}\beta^{\alpha}} &\leq \frac{8\Delta\gamma_{\text{th}}}{\beta^{\alpha}} \frac{\alpha-1}{\alpha-2} = 1, \end{aligned}$$

which implies that r_i can be informed.

Now, we turn our attention to the approximation ratio of LLD-CLS (Theorem 5.2). To prepare the proof of Theorem 5.2, we first introduce the following two Lemmas. Table 1 lists some notations used in the proofs.

Lemma 5.2. The number of time slots calculated by LLD-CLS, denoted by T_{lld}, is upper bounded by $T_{\text{lld}} \leq 4 \cdot \rho(A^w_{\max}) \cdot g(\mathcal{K})$.

TABLE 1. NOTATIONS.

Notation	Description
$\rho(A^k_{a,b})$	The receiver density of R_k in $A^k_{a,b}$.
$A^k_{\max}$	The square that has the highest $\rho(A^k_{a,b})$ in $\mathcal{A}^k$.
$A^w_{\max}$	The square that has the highest $\rho(A^k_{\max})$ over all $A^1_{\max}$, ..., $A^{g(\mathcal{K})+1}_{\max}$. Without loss of generality we assume that $A^w_{\max}$ is in $\mathcal{A}^w$.

Proof Our first observation is that when the link set L_k are scheduled (the loop in lines 4-8 in Algorithm 1), only the receivers in R_k are newly informed in this loop. Otherwise, the receiver must have been informed in some previous loop. It implies that there are at most $\rho(A^k_{\max})$ receivers required to be informed in each square in this loop. Then, the inner repeat loop (lines 7-10) can be repeated at most $\rho(A^k_{\max})$ times. Given that there are 4 colors and $g(\mathcal{K})$ link classes, the number of time slots T_{lld} in this algorithm is upper bounded by

$$T_{\text{lld}} \leq \sum_{k=1}^{g(\mathcal{K})} 4 \cdot \rho(A^k_{\max}) \leq 4 \cdot \rho(A^w_{\max}) \cdot g(\mathcal{K}). \tag{16}$$

Lemma 5.3. Given a pair of receivers $r_1, r_2 \in R_k$ that are located in a square $A^k_{a,b}$. Represent the active desired link sets of r_1 and r_2 by $\mathcal{I}_1$ and $\mathcal{I}_2$, respectively. The RI of $\mathcal{I}_2$ on r_1 is then lower bounded by: $RI_{\mathcal{I}_2}(r_1,\mathcal{I}_1) \geq \eta\gamma_{\text{th}} \cdot \frac{P_{\mathcal{I}_2,r_2}}{P_{\mathcal{I}_1,r_1}}$, where η is a constant $\eta = \left(1+2\sqrt{2}\beta\right)^{-\alpha}$, and $P_{\mathcal{I}_1,r_1}$ and $P_{\mathcal{I}_2,r_2}$ are the signal powers that r_1 and r_2 receive from their active desired link sets $\mathcal{I}_1$ and $\mathcal{I}_2$, respectively

Proof Because both r_1 and r_2 reside in the same square $A^k_{a,b}$, the distance between r_1 and r_2, denoted by d_{r_1,r_2}, is upper bounded by $\sqrt{2}\beta_k$. For any link $l_{s,r_2} \in \mathcal{I}_2$, we have $d_{s,r_1} \leq d(l_{s,r_2}) + d_{r_1,r_2}$ (triangular inequality) and $d(l_{s,r_2}) \geq 2^{h_k} \cdot \sigma$, then we can derive

$$\begin{aligned} \frac{d(l_{s,r_1})^{-\alpha}}{d(l_{s,r_2})^{-\alpha}} &\geq \left(\frac{d(l_{s,r_2}) + d_{r_1,r_2}}{d(l_{s,r_2})}\right)^{-\alpha} \\ &\geq \left(1+\frac{\sqrt{2}\beta_k}{2^{h_k}\cdot\sigma}\right)^{-\alpha} = \left(1+2\sqrt{2}\beta\right)^{-\alpha}. \end{aligned}$$

Hence, we can get that

$$\frac{P_{\mathcal{I}_2,r_1}}{P_{\mathcal{I}_2,r_2}} = \frac{\sum_{l_{s,r}\in\mathcal{I}_2} P \cdot d^{-\alpha}_{s,r_1}}{\sum_{l_{s,r_2}\in\mathcal{I}_2} P \cdot d(l_{s,r_2})^{-\alpha}} \geq \left(1+2\sqrt{2}\beta\right)^{-\alpha} = \eta. \tag{17}$$

Consequently, we can derive

$$RI_{\mathcal{I}_2}(r_1,\mathcal{I}_1) = \gamma_{\text{th}} \frac{P_{\mathcal{I}_2,r_1}}{P_{\mathcal{I}_2,r_2}} \frac{P_{\mathcal{I}_2,r_2}}{P_{\mathcal{I}_1,r_1}} \geq \eta\gamma_{\text{th}} \frac{P_{\mathcal{I}_2,r_2}}{P_{\mathcal{I}_1,r_1}}. \tag{18}$$

Theorem 5.2. LLD-CLS's approx. ratio is $O(g(\mathcal{K}))$.

Proof We proceed by showing that an optimum solution OPT can inform all the receivers in R_w in $A^w_{\max}$ using at least $T_w = \lceil \rho(A^w_{\max})/m \rceil$ time slots, where m is a constant

$$m = \lceil (\eta\gamma_{\text{th}})^{-1} + 1 \rceil. \tag{19}$$

For the sake of contradiction, assume that OPT informs R_w using less than T_{opt} time slots. Therefore, there must exist a time slot t, $1 \leq t \leq T_w$, such that at least $m+1$ receivers in R_w located in $A^w_{\max}$ are informed simultaneously. Without loss of generality, let r_1, r_2,..., r_{m+1} be the $m+1$ receivers informed at this time slot, which have the active desired link sets $\mathcal{I}_1$, ..., $\mathcal{I}_{m+1}$, respectively, and let $P_{\mathcal{I}_1,r_1} = \min\{P_{\mathcal{I}_i,r_i} | k = 1,2,...,m+1\}$, where $P_{\mathcal{I}_i,r_i}$ represents the signal power r_i receives from $\mathcal{I}_i$ ($i = 1,2,...,m+1$). Hence, the RI of $\mathcal{I} = \cup_{i=2}^{m+1} \mathcal{I}_i$ on r_1 is given by (according to Lemma 5.1 and Lemma 5.3)

$$RI_{\mathcal{I}}(r_1, \mathcal{I}_1) = \sum_{i=2}^{m+1} RI_{\mathcal{I}_i}(r_1, \mathcal{I}_1) \tag{20}$$

$$\geq \sum_{i=2}^{m+1} \eta\gamma_{\text{th}} \cdot \frac{P_{\mathcal{I}_i,r_i}}{P_{\mathcal{I}_1,r_1}} > 1 \tag{21}$$

which implies r_1 cannot be informed. Hence, it needs at least $\lceil \rho(A^w_{\max}/m) \rceil$ time slots for OPT to inform all the receivers in R_w in $A^w_{\max}$. On the other hand, LLD-CLS can inform all receivers within $T_{\text{lld}} \leq 4 \cdot \rho(A^w_{\max}) \cdot g(\mathcal{K})$ time slots (by Lemma 5.2). Therefore the approximation ratio follows

$$T_{\text{lld}}/T_{\text{opt}} \leq T_{\text{lld}}/T_w \leq 4m \cdot g(\mathcal{K}) = O(g(\mathcal{K})), \tag{22}$$

where T_{opt} denotes the number of time slots that the optimal solution OPT needs to inform all the receivers.

5.3. LLD based algorithm for OCLS

Similar to LLD-CLS, in the LLD based algorithm for OCLS (or LLD-OCLS for short), we construct $g(\mathcal{K})$ disjoint link classes $L_1, L_2, ..., L_{g(\mathcal{K})}$ according to Equ. (11) and schedule each link class separately (Algorithm 2). For each link class L_k we partition the whole network region into a set of squares $\mathcal{A}^k = \{A^k_{a,b}\}$ and color these squares with 4 colors $j \in \{1,2,3,4\}$, where each square has size $\beta_k \times \beta_k$. Then, we pick up one receiver for each square of color j (if the square has receivers in R_k) and add the receiver's active desired link set to $\mathcal{I}(k,j)$. Note that if the size of desired link set is larger than Δ, we pick the shortest Δ links from the link set. Consequently, we can get $4g(\mathcal{K})$ feasible schedules: $\mathcal{I}(k,j)$ ($k = 1,...,N, j = 1,2,3,4$). Finally, the schedule with most receivers informed is determined (line 12): $\mathcal{I}_{\text{ocls}} = \arg\max\{U(\mathcal{I}(k,j)) | \mathcal{I}(k,j), \forall k,j\}$, where $U(\mathcal{I}')$ denotes the number of receivers informed by link set $\mathcal{I}'$. Since we pick one receiver per selected square, the feasibility of the schedule constructed by Algorithm 2 has been proved in Theorem 5.1. In the next theorem, we calculate the approximation ratio of this algorithm.

Theorem 5.3. LLD-OCLS has approximation ratio $O(g(\mathcal{K}))$.

Proof We start the proof by defining $\mathcal{I}_{\max}(k) \triangleq \arg\max\{U(\mathcal{I}(k,j)) | \mathcal{I}(k,j), j = 1,2,3,4\}$.. Since Algorithm 2 returns the schedule of the maximum number of informed receivers over all length classes and colorings, the number of receivers informed by LLD-OCLS is given by $U_{\text{lld}} = \max\{U(\mathcal{I}_{\max}(k)), k = 1,2,...,g(\mathcal{K})\}$. We use U_{opt} to represent the number of receivers informed by the optimal solution

Algorithm 2: Pseudo code for LLD-OCLS.

```
input : {L_1, ..., L_g(K)}, {R_1, ..., R_g(K)};
output: I_ocls;
1  for k ← 1 to g(K) do
2      Partition the region into squares A^k = {A^k_{a,b}} of
       size β_k × β_k;
3      Color the squares with {1,2,3,4} s.t. no two
       adjacent squares have the same color (see Fig. 2
       (a));
4      for j ← 1 to 4 do
5          for each square in j that has receivers in R_k do
6              Pick one receiver r_i in the square;
7              if |I_i| > Δ then
8                  Add the shortest Δ links in I_i to
                   I(k, j);
9              else
10                 Add all the links in I_i to I(k, j);
11             Remove r_i from R_k;
12 I_ocls ← arg max{U(I(k, j)) | I(k, j), ∀k, j};
13 return I_ocls;
```

OPT. Also, we use U^k_{opt} to denote the number of receivers in R_k informed by OPT. Then, we have $U_{\text{opt}} = \sum_{k=1}^{g(\mathcal{K})} U^k_{\text{opt}}$. In Theorem 5.2 we have showed that any feasible schedule can inform at most m (defined in Equ. (19)) receivers in each square in $\mathcal{A}^k$ at each time slot. Then, $U^k_{\text{opt}}/U(\mathcal{I}_{\max}(k)) \leq 4m\cdot$ and the approximation ratio follows:

$$\frac{U_{\text{opt}}}{U_{\text{lld}}} = \sum_{k=1}^{g(\mathcal{K})} \frac{U^k_{\text{opt}}}{U_{\text{lld}}} \leq \sum_{k=1}^{g(\mathcal{K})} \frac{U^k_{\text{opt}}}{U(\mathcal{I}_{\max}(k))} \leq 4m \cdot g(\mathcal{K}). \tag{23}$$

5.4. A greedy algorithm for OCLS

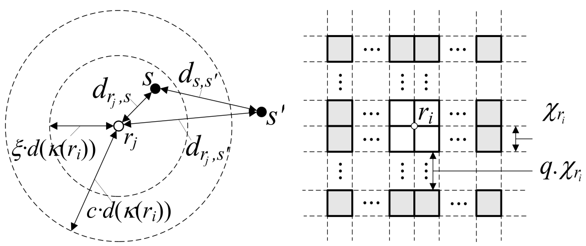
In this section, we present a greedy algorithm (see Algorithm 3) for a special case of OCLS, in which the desired link set of each receiver is upper bounded by a constant Ω ($\Omega \geq 2$). For example, three-node model for CC [15] assumes that there are at most two senders for each receiver. In each iteration, the algorithm greedily selects the uninformed receiver with the shortest key link in $\mathcal{K}$, say r_i, and activates all the links with lengths no larger than $\xi \cdot d(\kappa(r_i))$ in I_i, where ξ is a constant set by the algorithm. To guarantee that r_i is informed, the algorithm deletes the links that may conflict with the selected links. First, all links whose senders are within the radius $c \cdot d(\kappa(r_i))$ of the receiver r_i are removed from L, where c is a constant $c = \sqrt{2} \cdot \left(\frac{10\Omega \cdot (\alpha-1) \cdot \gamma_{\text{th}}}{\alpha-2}\right)^{\frac{1}{\alpha}} + \xi$. Second, for any link set I_j, such that the RI of the selected links on r_j rose above $1/2$, is removed. This process (lines 3-7) is repeated until all links in L have been either active or deleted. Next, we prove that the obtained schedule from the OCLS algorithm is both feasible (Theorem 5.4) and competitive, *i.e.*, is only a constant factor away from the optimum (Theorem 5.5).

Let r_i be any receiver selected in Algorithm 3, which has active desired link set $\mathcal{I}_i$, and let $\mathcal{I}_i^-$ and $\mathcal{I}_i^+$ be the set of links added *after* and *before* $\mathcal{I}_i$, respectively.

Algorithm 3: Pseudo code for the greedy algorithm.

input : $L = \{I_1, ..., I_N\}$
output: $\mathcal{I}_{\text{ocls}}$

$\mathcal{I}_{\text{ocls}} \leftarrow \phi$;
while $L \neq \mathcal{I}_{\text{ocls}}$ **do**
    Pick up the receiver r_i with the shortest link in L;
    Add the link set $\mathcal{I}_i = \{l \in I_i | d(l) < \xi \cdot d(\kappa(r_i))\}$ to $\mathcal{I}_{\text{ocls}}$;
    Remove $I_i \backslash \mathcal{I}_i$ from L;
    Remove all the links $l_{s,r}$, *s.t.* $d_{s,r} < c \cdot d(\kappa(r_i))$ from L;
    Remove any link set I_j, *s.t.* $RI_{\mathcal{I}_{\text{ocls}}}(r_j, I_j) > 1/2$;
return $\mathcal{I}_{\text{ocls}}$;



(a) Proof in Lemma 5.4 (b) Proof in Theorem 5.4

Figure 3. Proof of the approximation ratio of the greedy algorithm.

Lemma 5.4. The distance between the senders for different receivers in $\mathcal{I}_i^+$ is lower bounded by $(c - \xi)d(\kappa(r_i))$.

Proof For any receiver r_j whose active desired links are in $\mathcal{I}_i^+$, there is no sender (in different request with r_j) in $\mathcal{I}_i^+$ that has distance smaller than $c \cdot d(\kappa(r_j))$ from r_j. Using this fact and the triangular inequality (see Fig. 3 (a)), we can lower bound the distance between two senders in different requests in $\mathcal{I}_i^+$

$$\begin{aligned} d_{s,s'} &\geq d_{s',r_j} - d_{s,r_j} \geq d_{s',r_j} - \xi \cdot d(\kappa(r_j)) \\ &\geq c \cdot d(\kappa(r_j)) - \xi \cdot d(\kappa(r_j)) \\ &\geq (c - \xi)d(\kappa(r_i)). \end{aligned} \tag{24}$$

Theorem 5.4. LLD-OCLS is feasible.

Proof When a link set $\mathcal{I}_i$ of r_i is added to the schedule, the RI of $\mathcal{I}_i^-$ on r_i must be no larger than $1/2$; otherwise, it has already been deleted in a previous step. Therefore, the RI on r_i by concurrently active link set $\mathcal{I}_i^-$ is $RI_{\mathcal{I}_i^-}(r_i, \mathcal{I}_i) \leq 1/2$. It remains to show that $RI_{\mathcal{I}_i^+}(r_i, \mathcal{I}_i) \leq 1/2$. The transmission power received at r_i from its active link set $\mathcal{I}_i$ is at least

$$P_{\mathcal{I}_i, r_i} \geq P/d(\kappa(r_i))^\alpha. \tag{25}$$

We partition the whole network region into squares with size $\chi_{r_i} \times \chi_{r_i}$ (see Fig. 3 (b)), where $\chi_{r_i} = \sqrt{2}(c - \xi)d(\kappa(r_i))/2$. According to Lemma 5.4, any two senders for different receivers in $\mathcal{I}_i^+$ cannot be located in the same square. We use $\mathcal{Q}_q^i$ to denote the set of links whose senders are in the squares that are $q \cdot \chi_{r_i}$ away from r_i. Then, there are at most $4(q+1) \cdot \Omega$ links in $\mathcal{Q}_q^i$. The distance between the senders in $\mathcal{Q}_q^i$ and r_i is at least $q \cdot \chi_{r_i}$, so the RI of l on r_i is at most

$$RI_l(r_i, \mathcal{I}_i) \leq \frac{P\chi_{r_i}^{-\alpha}}{P_{\mathcal{I}_i, r_i}} \leq \left(q\sqrt{2}(c - \xi)/2\right)^{-\alpha}. \tag{26}$$

The RI of $\mathcal{Q}_q^i$ on r_i is then upper bounded by

$$RI_{\mathcal{Q}_q^i}(r, \mathcal{I}_i) = \sum_{l \in \mathcal{Q}_q^i} RI_l(r_i, \mathcal{I}_i) \leq \frac{4(q+1)\Omega}{(q\frac{\sqrt{2}}{2}(c - \xi))^\alpha}, \tag{27}$$

and the RI of all active links $\mathcal{I}_i^+ = \cup_q \mathcal{Q}_q^i$ on r_i is upper bounded by (according to Lemma 5.1)

$$\begin{aligned} RI_{\mathcal{I}_i^+}(r_i, \mathcal{I}_i) &= \sum_{q=1}^{\infty} RI_{\mathcal{Q}_q^i}(r_i, \mathcal{I}_i) \\ \leq & \sum_{q=1}^{\infty} \frac{4(q+1)\Omega \cdot \gamma_{\text{th}}}{(q \cdot \frac{\sqrt{2}}{2}(c - \xi))^\alpha} \leq \sum_{q=1}^{\infty} \frac{5q\Omega \cdot \gamma_{\text{th}}}{(q \cdot \frac{\sqrt{2}}{2}(c - \xi))^\alpha} \\ = & \frac{5\Omega \cdot \gamma_{\text{th}}}{(\frac{\sqrt{2}}{2}(c - \xi))^\alpha} \frac{1}{q^{\alpha - 1}} \leq \frac{5\Omega \cdot \gamma_{\text{th}}}{(\frac{\sqrt{2}}{2}(c - \xi))^\alpha} \frac{\alpha - 1}{\alpha - 2} \\ = & \frac{1}{2}. \end{aligned} \tag{28}$$

which implies that r_i can be informed.

Lemma 5.5. Let $\mathcal{I}_{\text{ocls}}$ be a feasible solution and let r_i be an informed receiver, which has key link l_{s,r_i}. Denote the active desired link set of r_i by $\mathcal{I}_i$. The number of senders in $\mathcal{I}_{\text{ocls}} \backslash \mathcal{I}_i$ with distance $k \cdot d(\kappa(r_i))$ from s is at most $(k+1)^\alpha \Omega / \gamma_{\text{th}}$.

Proof The RI of each link $l_{s',r'} \in \mathcal{I}_{\text{ocls}} \backslash \mathcal{I}_i$ on r_i is lower bounded by

$$\begin{aligned} RI_{l_{s',r'}}(r_i, \mathcal{I}_i) &= \frac{d_{s',r_i}^{-\alpha} \cdot \gamma_{\text{th}}}{\sum_{l \in \mathcal{I}_{\text{ocls}}} d(l)^{-\alpha}} \\ \geq & \frac{(d_{s,r_i} + d_{s',s})^{-\alpha} \cdot \gamma_{\text{th}}}{\sum_{l \in \mathcal{I}_{\text{ocls}}} d_{s,r_i}^{-\alpha}} \geq \frac{(d(l_{s,r}) + d_{s',s})^{-\alpha} \cdot \gamma_{\text{th}}}{|\mathcal{I}_i| \cdot d(l_{s,r})^{-\alpha}} \\ = & \frac{\gamma_{\text{th}}}{|\mathcal{I}_i|} \left(1 + \frac{d_{s,s'}}{d(l_{s,r})}\right)^{-\alpha} \geq \frac{(1+k)^{-\alpha}\gamma_{\text{th}}}{\Omega} \end{aligned} \tag{29}$$

Since the RI of $\mathcal{I}_{\text{ocls}} \backslash \mathcal{I}_i$ on r_i cannot exceed one, there are at most $(k+1)^\alpha \cdot \Omega$ such senders senders with distance no larger than $k \cdot d(\kappa(r_i))$ from s.

Definition 5.5. (*Blue and red points* [8]) Let $\mathcal{S}_r$ and $\mathcal{S}_b$ be two disjoint sets of points (*red* and *blue*) in a 2D Euclidean space. For any $z \in \mathbb{N}$, a point $s_b \in \mathcal{S}_b$ is *z-blue-dominant* if every circle $B_\delta(s_b)$ around s_b, comprised by points p such that $d(p, s_b) \leq \delta$, contains z times more blue than red points, or formally

$$|B_\delta(s_b) \cap \mathcal{S}_b| > z \cdot |B_\delta(s_b) \cap \mathcal{S}_r| \quad \forall \delta \in \mathbb{R}^+. \tag{30}$$

Lemma 5.6. (*Blue-dominant centers lemma* [8]) For any $z \in \mathbb{N}$, if $|\mathcal{S}_b| > 5z \cdot |\mathcal{S}_r|$, then there exists at least one z-blue-dominant point s_b in $\mathcal{S}_b$. In addition, given a z-blue-dominant point s_b, for each point s_r in $\mathcal{S}_r$, there exists a subset of $\mathcal{S}_b$ corresponding to s_r, denoted by $G(s_r)$, *s.t.*, 1) any point in $G(s_r)$ is farther from s_r than from s_b: $\forall s \in$

$G(s_r)$, $d_{s_r,s} > d_{s_b,s}$; 2) for any pair of points $s_r, s_r' \in \mathcal{S}_r$, $G(s_r) \cap G(s_r') = \phi$; 3) the number of points in each subset $G(s_r)$ is no smaller than z: $|G(s_r)| \geq z \ \forall s_r \in \mathcal{S}_r$.

Lemma 5.7. Denote the set of all senders in the optimal schedule and the greedy algorithm by $\mathcal{S}_{\text{opt}}$ and $\mathcal{S}_{\text{gre}}$, respectively. Then, $|\mathcal{S}_{\text{opt}} \backslash \mathcal{S}_{\text{gre}}| \leq 3^\alpha \times 5\Omega \cdot |\mathcal{S}_{\text{gre}}|$.

Proof For the sake of contradiction, assume that $|\mathcal{S}_{\text{opt}} \backslash \mathcal{S}_{\text{gre}}| > 3^\alpha \cdot 5\Omega \times |\mathcal{S}_{\text{gre}}|$. Label the set of senders in $\mathcal{S}_{\text{opt}}$ by blue ($\mathcal{S}_b = \mathcal{S}_{\text{opt}}$) and $\mathcal{S}_{\text{gre}}$ by red ($\mathcal{S}_r = \mathcal{S}_{\text{gre}}$). By Lemma 5.6, there is a z-blue-dominant point (sender) $s^* \in \mathcal{S}_b$ with sender set $\mathcal{S}^*$, where $z = 3^\alpha \times \Omega$. We shall argue that the link l_{s^*,r^*} (or l^* for simplicity) would have been picked by our algorithm, which leads to a contradiction.

According to Lemma 5.6, for any red point $s_r \in \mathcal{S}_r$, there exists a subset of blue points $G(s_r)$ such that all the points in $G(s_r)$ are closer to s^* than to s_r and $|G(s_r)| \geq z$ ($z = 3^\alpha \times \Omega$). We can derive that $d_{s^*,s_r} > 2 \cdot d(l^*)$; otherwise, the number of senders within distance $2 \cdot d(l^*)$ from s^* would be larger than $(2+1)^\alpha \cdot \Omega \geq 3^\alpha \cdot |\mathcal{S}^*|$, which contradicts with the conclusion in Lemma 5.5. Based on triangle inequality, $d_{s_r,r^*} \geq d_{s^*,s_r} - d(l^*) > d_{s^*,s_r}/2$. Denote the sum signal power that r^* receives from $\mathcal{S}^*$ by P^*. The RI of the red sender s_r on r^* is then upper bounded by

$$RI_{s_r}(r^*, \mathcal{S}^*) = \frac{d_{s_r,r^*}^{-\alpha} P}{P^*} \gamma_{\text{th}} \leq \frac{d_{s_r,s^*}^{-\alpha} P}{2^{-\alpha} P^*} \gamma_{\text{th}}. \qquad (31)$$

Also, for any point $s_b \in G(s_r)$,

$$\begin{aligned} d_{s_b,r^*} &\leq d_{s_b,s^*} + d_{s^*,r^*} < d_{s_r,s^*} + d_{s^*,r^*} \\ &< d_{s_r,s^*} + d_{s_r,s^*}/2 \\ &= 3d_{s_r,s^*}/2. \end{aligned} \qquad (32)$$

Hence, the sum RI of the blue senders in $G(s_r)$ on r^* is lower bounded

$$\begin{aligned} &\sum_{s_b \in G(s_r)} RI_{s_b}(r^*, \mathcal{S}^*) = \sum_{s_b \in G(s_r)} \frac{d_{s_b,r^*}^{-\alpha} P}{P^*} \cdot \gamma_{\text{th}} \\ &> 3^\alpha \Omega \cdot \left(\frac{3}{2}\right)^{-\alpha} \frac{d_{s_r,s^*}^{-\alpha} P}{P^*} \gamma_{\text{th}} \\ &\geq \Omega \cdot RI_{s_r}(r^*, \mathcal{S}^*). \end{aligned} \qquad (33)$$

This relationship holds for any $s_r \in \mathcal{S}_r$, and $G(s_r)$ and $G(s_r')$ are disjoint $\forall s_r, s_r' \in \mathcal{S}_r$, then the total RI that r^* receives from the senders in OPT (blue points) is at least Ω times as high as the RI it would receive from the senders in the greedy algorithm (red points). Because s^* is in $\mathcal{S}_b$, its RI on r is at most 1. Therefore, we have

$$RI_{\mathcal{S}_r}(r^*, \mathcal{S}^*) < \frac{1}{\Omega} RI_{\mathcal{S}_b}(r^*, \mathcal{S}^*) \leq \frac{1}{2}. \qquad (34)$$

Since $RI_{\mathcal{S}_r}(r^*, \mathcal{S}^*)$ is less than $1/2$, it would not have been deleted by the greedy algorithm, which establishes the contradiction.

Theorem 5.5. Algorithm 3's approx. ratio is $O(1)$.

Proof Denote the number of receivers informed by the greedy algorithm and the optimal schedule by U_{gre} and U_{opt}, respectively. Then, according to Lemma 5.7,

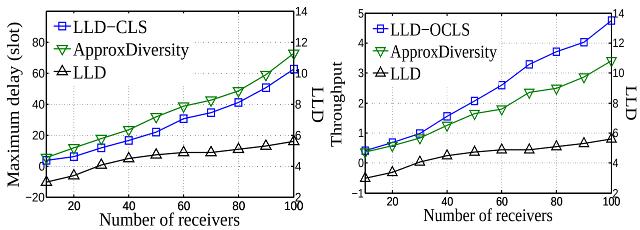
$$\begin{aligned} \frac{U_{\text{opt}}}{U_{\text{gre}}} &\leq \frac{\Omega \cdot |\mathcal{S}_{\text{opt}}|}{|\mathcal{S}_{\text{gre}}|} = \frac{\Omega \cdot (|\mathcal{S}_{\text{opt}} \backslash \mathcal{S}_{\text{gre}}| + |\mathcal{S}_{\text{gre}}|)}{|\mathcal{S}_{\text{gre}}|} \\ &\leq (3^\alpha \times 5\Omega + 1)\, \Omega = O(1). \end{aligned}$$

6. Performance Evaluation

In this section, we present the simulation results of LLD-CLS, LLD-OCLS, and the greedy algorithm (CC-Greed) using MATLAB. All nodes were distributed uniformly at random on a plane field of size 100×100. In the simulation, we measured the following two metrics: (1) *maximum delay*, which is defined as the number of time slots used to inform all receivers, and (2) *throughput*, which is defined as the number of receivers informed in a single time slot. We compared these two metrics of our algorithms with two smart non-cooperative link scheduling algorithms: ApproxDiversity [5] and ApproxLogN [8]. Like ours, both ApproxLogN and ApproxDiversity are polynomial time algorithms for the SINR model. The main difference is that ApproxLogN and ApproxDiversity do not allow CC in transmission. Since ApproxLogN is particularly efficient for the one-shot scheduling problem, we only compare ApproxLogN with our algorithms in terms of throughput.

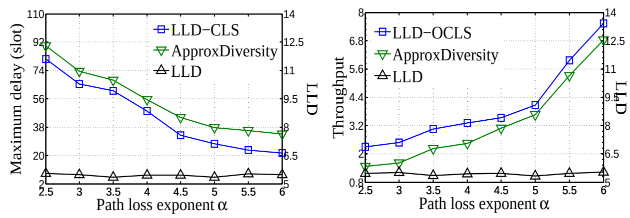
First, we evaluate the performance of three LLD based algorithms: LLD-CLS, LLD-OCLS, and ApproxDiversity. In Fig. 4 (a) and Fig. 4 (b), we vary the number of receivers from 10 to 100 with 10 increase in each step, and compare the maximum delay and throughput, respectively. We set the number of senders to 200. As expected, LLD-CLS outperforms ApproxDiversity in maximum delay and LLD-OCLS outperforms ApproxDiversity in throughput. This is becasue LLD-CLS (LLD-OCLS) allows receivers to combine weak signal powers from senders, which helps increase the opportunities for receivers to recover their messages. In addition, we have two observations from the figures: (1) the maximum delay increases as the LLD increases, and (2) the maximum delay increases as the number of receivers increases. These two observations are caused by the LLD-based algorithms' mechanism, which first partitions the link set into disjoint link classes, and then separately schedules the links in each class in squares. For (1), higher LLD always generates more link classes, leading to more time slots to schedule the whole link set. As for (2), higher receiver density causes more nodes to be in each square, and hence more time slots to schedule each link class.

In Fig. 5 (a) and Fig. 5 (b), we compare different algorithms when the path loss exponent α was varied from 2.5 to 6 with 0.5 increase in each step. The number of senders and receivers are set to be 1000 and 100, respectively. Similar to Fig. 4, both figures demonstrate that LLD-CLS and LLD-OCLS outperform ApproxDiversity in terms of maximum delay and throughput, respectively, because of the benefit of CC. Another interesting observation is that with the increase of α, the maximum delay decreases and the throughput increases for both algorithms. This is because when α is smaller, the size of the squares partitioned by the LLD-based algorithms



(a) Maximum delay (b) Throughput

Figure 4. Different number of receivers.



(a) Maximum delay (b) Throughput

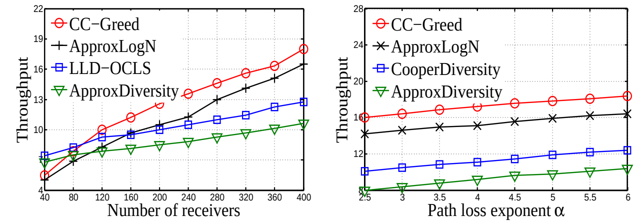
Figure 5. Different pass loss exponent.

is larger (by Equ. (12)), which leads to more receivers located in each square and hence more time slots to schedule each link class.

We then compare the throughput of CC-Greed, LLD-OCLS, ApproxDiversity, and ApproxLogN. In Fig. 6 (a), we varied the number of receivers from 40 to 400 and set α to 3. In Fig. 6 (b), we varied α from 2.5 to 6 and set the number of receivers to 400. In both figures, each request has exactly two links. From both figures, we can find that CC-Greed is always better than ApproxLogN. Furthermore, we observe that when the number of receivers is low, CC-Greed has no significant better performance than LLD-OCLS and Approx-Diversity. However, as the density of receivers increases, CC-Greed presents increasingly better relative performance. This is because that CC-Greed can achieve constant approximation ratio in throughput (according to the analysis in Section 5.4), which enables it to achieve higher throughput than LLD-OCLS and ApproxDiversity when the receiver density of the network is high.

7. Conclusion

In this paper, to study the link scheduling problem in CC networks, we have formulated two problems, namely the CLS problem and the OCLS problem. The goal of CLS is to inform all receivers using as few time slots as possible, while the goal of OCLS is to maximize the number of informed receivers in one time slot. As a solution, we have proposed a *link length diversity* (*LLD*) based algorithm for both CLS and OCLS problems, with $g(\mathcal{K})$ performance guarantee. Further, we have proposed an algorithm with $O(1)$ approximation guarantee for OCLS in the case that the number of senders in each request is upper bounded by a constant. The experimental results indicate that our cooperative link scheduling algorithms outperform non-cooperative algorithms. In our future work, we will take into account probabilistic fading models for this problem.



(a) Different number of receivers (b) Different α values

Figure 6. Throughput of GREEDY, ApproxLogN, CoopDiversity, and ApproxDiversity.

8. Acknowledgements

This research was supported in part by U.S. NSF grants NSF-1404981, IIS-1354123, CNS-1254006, IBM Faculty Award 5501145 and Microsoft Research Faculty Fellowship 8300751.

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