Fading-Resistant Link Scheduling in Wireless Networks

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***Abstract*—In this paper, we study the *link scheduling* problem considering the fluctuating fading effect in transmissions. We extend the previous deterministic physical interference model to the Rayleigh-fading model that uses the stochastic propagation to address fading effects. Based on this model, we formulate a problem called *Fading-Resistant Link Scheduling (Fading-R-LS) problem*, which aims to maximize the throughput of all links in a single time slot. We prove that this problem is NP-hard. Based on the geometric structure of *Fading-R-LS*, we then propose two centralized schemes with $O(g(L))$ and $O(1)$ performance guarantee, respectively, where $g(L)$ is the number of magnitudes of transmission link lengths. Our experimental results show that the superior performance of our proposed schemes compared to previous schemes.**

***Index Terms*—Link Scheduling; NP-hard; approximation algorithm;**

I. Introduction

In wireless networks, the problem of scheduling link transmissions, or the *link scheduling problem*, has been a subject of significant research interest over the past years. Given a set of transmission links, each of which is from a sender to a receiver and associated with a specific data rate, this problem is to determine which links should be active at what times in order to optimize one or multiple performance objectives such as throughput and delay. Due to the broadcast nature of wireless communication, when a sender transmits a packet to a specific receiver, it could become interference to other receivers. Thus, when scheduling links, we need to consider how to select links such that the interference among the links will not fail transmissions. When studying the scheduling problem in wireless networks, the choice of the interference model is of fundamental significance. Though the scheduling problem has been widely studied based on different interference models [1]–[15], none of the previous works has proposed approximation algorithms for this problem based on the physical interference model considering fading effect in transmissions.

One of the most commonly used interference model in the traditional scheduling problem is graph based model [1]–[9]. It only considers the interference on a receiver from other senders within the transmission range. However, although the interference from a single far-away sender can be relatively small, the accumulated interference from several such senders can be sufficiently high to corrupt a transmission. Hence, the scheduling problem solutions based on the graph based model cannot be guaranteed to work in many real scenarios. Another interference model, named physical interference model or Signal-to-Interference-plus-Noise Ratio (SINR) model, offers a more realistic representation of wireless networks [10]–[15]. In this model, a message is received successfully iff the SINR is no smaller than a hardware-defined threshold. This definition of a successful transmission, as opposed to the graph based definition, accounts for interference generated by senders located far away.

However, the SINR model still uses a limited view of signal propagation. Its main assumption is that any signal transmitted at power level P is always received at distance d with strength $Pd^{-\alpha}$, where α is path loss exponent. The real signal propagation is actually not deterministic, e.g., the links may become susceptible to fading fluctuations in signal strength due to mobility in a multi-path propagation environment [16]. Therefore, some advanced models using stochastic approaches to consider fading effects have been proposed [17], [18]. Most prominently, in the Rayleigh-fading model, the signal strength is modeled by an exponentially distributed random variable with mean $Pd^{-\alpha}$ [16], [19]. This however also makes the SINR non-deterministic, and hence causes the judgement of successful transmission in analyzing the link scheduling problem much more complicated. As a result, finding solutions for the link scheduling problem with the Rayleigh-fading model is a non-trivial task.

To address this problem, in this paper, we formulate a link scheduling problem called *Fading-Resistant Link Scheduling problem* (Fading-R-LS), in which the interferences among links are modeled by the Rayleigh-fading channel model. Given a set of links L, *Fading-R-LS* is to determine which subset of L should be activated such that the total throughput is maximized in one time slot. We first prove that this problem is NP-hard, and then propose three solutions and analyze their performance guarantees. In summary, this paper mainly has four contributions:

1) **Fading-R-LS formulation and analysis**. We formulate the *Fading-R-LS* problem that takes into account the fading effect, which is not considered in previous scheduling problems. In addition, we give an integer linear programming (ILP) formulation of *Fading-R-LS* and prove it is NP-hard.
2) **Link diversity partition algorithm (LDP)**. According

to the geometric structure of *Fading-R-LS*, we propose the LDP centralized method. It builds several link classes based on link lengths and schedule the links in each class separately. We prove that LDP has the performance guarantee of $O(g(L))$, where $g(L)$ denotes the link diversity, i.e., the number of length magnitudes of link set L.

3) **Recursive link elimination algorithm (RLE)**. We then consider a special case of *Fading-R-LS*, in which the data rate of each link is the same, and propose the RLE algorithm accordingly. RLE iteratively picks up the unpicked link with the shortest link length and eliminates other links that interfere with the picked link. We prove RLE has the performance guarantee of $O(\Delta^{\alpha})$, where Δ is the ratio between the maximum and the minimum distances between nodes.

The remainder of this paper is organized as follows. Section II builds the mathematical system model. Section III defines *Fading-R-LS* and proves its hardness. To solve this problem, we propose two centralized algorithms in Section IV. Section V evaluates the performance of our proposed algorithms in comparison with a pervious solution. Section VI presents related work. Section VII concludes this paper with remarks on our future work.

II. System Model

In this section, we introduce the mathematical model to calculate SINR with the consideration of the fading effect, which will be used throughout this paper. Consider a wireless network with N communication links

$$L = \{(s_1, r_1), ..., (s_N, r_N)\}, \tag{1}$$

where (s_i, r_i) represents a transmission link from sender s_i to receiver r_i with transmission rate λ_i. We do not consider the scenario in which either a sender transmits to multiple receivers or multiple senders transmit to a receiver, so we assume that $s_i \neq s_j$ and $r_i \neq r_j$ $\forall i \neq j$. The set of receivers and the set of senders are denoted by

$$R = \{r_1, ..., r_N\} \tag{2}$$

and

$$S = \{s_1, ..., s_N\}, \tag{3}$$

respectively. For each receiver r_j, we call s_i the *desired sender* of r_j if $j = i$; otherwise an *interfering sender* of r_j. The Euclidean distance between sender s_i and receiver r_j is denoted by $d_{i,j}$, and that two senders s_i and s_j is denoted by d_{s_i,s_j}. We call $d_{i,i}$ the *length* of link (s_i, r_i).

We consider time-varying and frequency-flat fading wireless channels. The channel effects from sender s_i to receiver r_j can be modeled by a single, complex and random channel coefficient $h_{i,j}$. We consider the Rayleigh fading channel model [16]–[18], in which all $|h_{i,j}|^2$ are independent and exponentially distributed with a mean value

$$\sigma_{i,j}^2 = P d_{i,i}^{-\alpha} \tag{4}$$

where α is path loss exponent. By convention, we assume that $\alpha > 2$. We use $Z_{i,j}$ to represent the instantaneous signal power received by r_j from s_i. $Z_{i,j}$ is a random variable with Cumulative Distribution Function (CDF) of

$$F_{Z_{i,j}} = \Pr\{Z_{i,j} \leq x\} = 1 - e^{-x/Pd_{i,j}^{-\alpha}}. \tag{5}$$

When multiple users transmit simultaneously, they interfere with each other. We model interference by regarding all competing transmissions as additive noise and denote it by SINR. We denote $Z_{\mathcal{P},j}$ as the sum signal that r_j receives from sender set $\mathcal{P}$ ($\mathcal{P} \subset S$), i.e.,

$$Z_{\mathcal{P},j} = \sum_{s_i \in \mathcal{P}} Z_{i,j}. \tag{6}$$

We use a non-negative random variable X_j to represent the SINR received by r_j:

$$X_j = \frac{Z_{j,j}}{N_0 + Z_{\mathcal{P}\setminus s_j,j}}. \tag{7}$$

As N_0 has negligible effect on the results [14], [15], [19], we then ignore the influence of N_0, hence

$$X_j = \frac{Z_{j,j}}{Z_{\mathcal{P}\setminus s_j,j}}. \tag{8}$$

Receiver r_j can correctly decode the message (or informed) iff $X_j \geq \gamma_{\text{th}}$, where γ_{th} is decoding threshold. In fading channel models, the probability of successful transmission never can be 0, so we assume an acceptable error probability ε for transmission. That is, for any receiver r_j, we say r_j can be *informed* by its desired sender s_j if the probability of $X_j < \gamma_{\text{th}}$ is no larger than ε.

III. Problem Analysis

In this section, we first formulate the *Fading-R-LS* problem. Its objective is to identify a subset of senders, denoted by $\mathcal{P}$ ($\mathcal{P} \subset S$), such that the throughput (i.e., the total data rate successfully received by receivers) is maximized in one time slot. In other words, we attempt to use one time slot to its full capacity. Formally, we define the *Fading-R-LS* as follows:

Instance: A finite set of senders S and their respective receivers R in a geometric plane, decoding threshold γ_{th}, acceptable error rate ε, and a constant Λ.

Question: Existence of a subset of senders $\mathcal{P}$, namely a *schedule*, such that the total successful transmission rate is no smaller than Λ, *i.e.*,

1) $\Pr(X_j < \gamma_{\text{th}}) < \varepsilon,\ \forall s_j \in \mathcal{P}$ and
2) $\sum_{s_j \in \mathcal{P}} \lambda_j \geq \Lambda$.

We say a schedule $\mathcal{P}$ is *feasible* if all the senders in $\mathcal{P}$ can successfully transmit the message with probability at least $1-\varepsilon$. Below, we first derive the closed-form expression for the probability of successful transmission $\Pr(X_j \geq \gamma_{\text{th}})$ for any receiver r_j (Theorem 3.1). Then, we prove that *Fading-R-LS* is NP-hard (Theorem 3.2).

Theorem 3.1: Given an active link (s_j, r_j) and active sender set $\mathcal{P}$, the probability of successful transmission from s_j to r_j is:

$$\Pr(X_j \geq \gamma_{\text{th}}) = \prod_{s_i \in \mathcal{P} \setminus s_j} \frac{1}{1 + d_{i,j}^{-\alpha} \gamma_{\text{th}} / d_{j,j}^{-\alpha}}. \tag{9}$$

Proof The CDF of the quotient $X_j = Z_{i,j}/Z_{\mathcal{P}\setminus s_j, j}$ can be computed as follows:

$$\begin{aligned} F_{X_j}(x) &= \mathrm{P}\left(Z_{j,j}/Z_{\mathcal{P}\setminus s_j,j} \leq x\right) \\ &= P(Z_{j,j} \leq x Z_{\mathcal{P}\setminus s_j,j}) \\ &= \int_0^\infty \int_0^{xz} f_{Z_{j,j}}(y)\mathrm{d}y \cdot f_{Z_{\mathcal{P}\setminus s_j,j}}(z)\mathrm{d}z. \end{aligned} \tag{10}$$

By differentiating, we can obtain

$$\begin{aligned} f_{X_j}(x) &= \frac{\mathrm{d}}{\mathrm{d}x} F_{X_j}(x) \\ &= \int_0^\infty z f_{Z_{j,j}}(xz) f_{Z_{\mathcal{P}\setminus s_j,j}}(z)\mathrm{d}z \\ &= \int_0^\infty \frac{z}{P d_{i,j}^{-\alpha}} e^{-\frac{xz}{P d_{i,j}^{-\alpha}}} f_{Z_{\mathcal{P}\setminus s_j,j}}(z)\mathrm{d}z. \end{aligned} \tag{11}$$

Then, the probability of successful transmission from s_i to r_j equals

$$\begin{aligned} \Pr(X_j \geq \gamma_{\text{th}}) &= \int_0^\infty \int_{\gamma_{\text{th}}}^\infty \frac{z}{P d_{j,j}^{-\alpha}} e^{-\frac{xz}{P d_{j,j}^{-\alpha}}} f_{Z_{\mathcal{P}\setminus s_j,j}}(z)\mathrm{d}x\mathrm{d}z \\ &= \int_0^\infty e^{-\frac{\gamma_{\text{th}} z}{P d_{j,j}^{-\alpha}}} f_{Z_{\mathcal{P}\setminus s_j,j}}(z)\mathrm{d}z \\ &= \mathcal{L}_{Z_{\mathcal{P}\setminus s_j,j}}\left(\gamma_{\text{th}}/(P d_{j,j}^{-\alpha})\right) \end{aligned} \tag{12}$$

where $\mathcal{L}_{Z_{\mathcal{P}\setminus s_j,j}}(\nu)$ represents the Laplace transform of $f_{Z_{\mathcal{P}\setminus s_j,j}}(x)$. Because the Laplace transform of the exponential distribution with mean $1/\mu$ equals $\mu/(\mu+\nu)$,

$$\mathcal{L}_{Z_{\mathcal{P}\setminus s_j,j}}(\nu) = \prod_{s_i \in \mathcal{P}\setminus s_j} \frac{1}{1 + P d_{i,j}^{-\alpha} \nu}. \tag{13}$$

Consequently,

$$\begin{aligned} \Pr(X_j \geq \gamma_{\text{th}}) &= \mathcal{L}_{Z_{\mathcal{P}\setminus s_j,j}}\left(\gamma_{\text{th}}/P d_{j,j}^{-\alpha}\right) & (14) \\ &= \prod_{s_i \in \mathcal{P}\setminus s_j} \frac{1}{1 + d_{i,j}^{-\alpha} \gamma_{\text{th}} / d_{j,j}^{-\alpha}}. & (15) \end{aligned}$$

According to Theorem 3.1, in the following, we formulate the ILP form of the *Fading-R-LS* problem and prove that this problem is NP-hard. First, we take the logarithm on both sides of Equ. (9):

$$\ln \Pr(X_j \geq \gamma_{\text{th}}) = -\sum_{s_i \in \mathcal{P}\setminus s_j} f_{i,j}, \tag{16}$$

where

$$f_{i,j} = \begin{cases} \ln\left(1 + (d_{i,j}/d_{j,j})^{-\alpha} \gamma_{\text{th}}\right) & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases} \tag{17}$$

We call $f_{i,j}$ the *interference factor* of s_i on r_j. Accordingly, we use $f_{\mathcal{P}\setminus r_j, r_j}$ to denote the interference factor of $\mathcal{P}\setminus r_j$ on r_j, where

$$f_{\mathcal{P}\setminus r_j, r_j} = \sum_{s_i \in \mathcal{P}\setminus r_j} f_{i,j}. \tag{18}$$

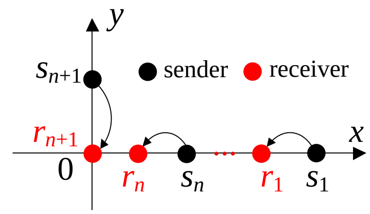


Fig. 1. Mapping *Fading-R-LS* to Knapsack.

Corollary 3.1: Given an active link (s_j, r_j) and the active sender set $\mathcal{P}$, r_j can be informed iff

$$\sum_{s_i \in \mathcal{P}\setminus s_j} f_{i,j} \leq \gamma_\varepsilon, \tag{19}$$

where $\gamma_\varepsilon = \ln\left(\frac{1}{1-\varepsilon}\right)$ is a constant.

By Corollary 3.1, we formulate the ILP form of *Fading-R-LS* as follows:

$$\max \sum_{i=1}^N \lambda_i x_i \tag{20}$$

$$\text{s.t.} \quad \sum_{i=1}^N f_{i,j} x_i \leq \gamma_\varepsilon + M(1 - x_j), \tag{21}$$

$$x_i \in \{0, 1\}, i, j = 1, ..., N, \tag{22}$$

where M is a constant with a very large value.

Theorem 3.2: The *Fading-R-LS* problem is NP-hard.

Proof We construct a polynomial time reduction from the well-known Knapsack NP-hard problem [20] to *Fading-R-LS*. The Knapsack problem can be formulated as follows: given n kinds of items, $x_1, ..., x_n$; each item x_j has a value p_j and a weight w_j, and a bag that can carry weight W maximally, the goal is to choose the items to put into the bag such that the sum of the items' values is no smaller than a constant C. For any instance in the Knapsack problem, we construct a *Fading-R-LS* instance that can be mapped to the Knapsack instance (see Fig. 1). We position a sender node s_i in the plane for each x_i, such that the received signal power from s_i at $(0, 0)$ is w_i, i.e.,

$$\text{Loc}(s_i) = \left(\left(\frac{e^{\frac{\gamma_\varepsilon w_i}{W}} - 1}{\gamma_{\text{th}}} \right)^{-\frac{1}{\alpha}}, 0 \right), \ \forall 1 \leq j \leq n. \tag{23}$$

Then, we set r_i close enough to s_i to guarantee successful reception regardless of other links:

$$\text{Loc}(r_i) = \text{Loc}(s_i) + (\delta, 0), \ \forall 1 \leq i \leq n, \tag{24}$$

where

$$\delta = d_{\min} / \left(\left((e^{\gamma_\varepsilon/(n+1)} - 1)/\gamma_{\text{th}} \right)^{-\frac{1}{\alpha}} + 1 \right) \tag{25}$$

and $d_{\min}$ is the minimum distance between any pair of senders. After that, we place one more link l_{n+1}, s.t.

$$\begin{aligned} \text{Loc}(s_{n+1}) &= (0, 1), & (26) \\ \text{Loc}(r_{n+1}) &= (0, 0). & (27) \end{aligned}$$

Thereafter, we assign a weight to each link:

$$\lambda_i = p_i, \ \forall 1 \le i \le n \ \lambda_{n+1} = 2\sum_{j=1}^{n} p_j. \tag{28}$$

The question is whether there exists a schedule to make total data rate (i.e., throughput) at least $2\sum_{j=1}^{n} p_j + C$ for *Fading-R-LS*. Now, we need to prove that the solution of the *Fading-R-LS* instance exists iff the solution of the Knapsack problem exists.
$\Rightarrow$: Suppose that $\exists \mathcal{X}$ *s.t.*

$$\sum_{x_j \in \mathcal{X}} p_j \ge C \text{ and } \sum_{x_j \in \mathcal{X}} w_j \le W. \tag{29}$$

We activate each sender s_i if $x_i \in \mathcal{X}$. Also, s_{n+1} must be active; otherwise the total data can never reach $2\sum_{j=1}^{n} p_j + C$. First, r_{n+1} can successfully receive the packet because

$$\begin{aligned}\sum_{s_i \in \mathcal{P} \setminus s_{n+1}} f_{j,n+1} &= \sum_{s_i \in \mathcal{P} \setminus s_{n+1}} \ln\left(1 + \frac{d_{i,j}^{-\alpha}\gamma_{\text{th}}}{d_{n+1,n+1}^{-\alpha}}\right) \\ &= \sum_{s_i \in \mathcal{P} \setminus s_{n+1}} \ln\left(1 + \left(\frac{e^{\frac{\gamma_\varepsilon w_i}{W}} - 1}{\gamma_{\text{th}}}\right)\gamma_{\text{th}}\right) \\ &\le \gamma_\varepsilon \end{aligned} \tag{30}$$

Then, for each receiver r_j s.t. $x_j \in \mathcal{X}$, we have

$$\begin{aligned}\sum_{s_i \in \mathcal{P} \setminus s_j} f_{i,j} &\le \sum_{s_i \in \mathcal{P} \setminus s_j} \ln\left(1 + \left(\frac{d_{\min} - \delta}{\delta}\right)^{-\alpha}\gamma_{\text{th}}\right) \\ &\le \gamma_\varepsilon. \end{aligned} \tag{31}$$

Hence, the total data rate is

$$\lambda_{\text{total}} = \sum_{x_i \in \mathcal{X}} p_i x_i + 2W \ge C + 2W, \tag{32}$$

which implies that exists a schedule such that total data rate is at least $2\sum_{j=1}^{n} p_j + C$ for *Fading-R-LS*.
$\Leftarrow$: Suppose there exists a *Fading-R-LS* schedule $\mathcal{P}$ such that the total data rate is at least $2\sum_{j=1}^{n} p_j + C$, then r_{n+1} must successfully receive the message, and hence

$$\sum_{s_j \in \mathcal{P} \setminus s_{n+1}} f_{j,n+1} \le \gamma_\varepsilon, \tag{33}$$

which implies that $\sum_{s_i \in \mathcal{P} \ s_j} w_j \le W$ (by Equ. (30)) and $\sum_{s_i \in \mathcal{P} \ s_j} p_j \ge C$. Let $\mathcal{X} = \{x_i | s_i \in \mathcal{P}/s_{n+1}\}$, then

$$\sum_{x_i \in \mathcal{X}} p_i \ge C \text{ and } \sum_{x_i \in \mathcal{X}} w_i \le W. \tag{34}$$

IV. Centralized Algorithms

Since *Fading-R-LS* is a NP-hard problem, there are no polynomial time solutions for determining the optimal schedule. To solve this problem, in this section, we propose the Link Diversity Partition algorithm (LDP) (Section IV-A). We further propose a constant approximation ratio algorithm, namely Recursive Link Elimination algorithm (RLE) (Section IV-B) for the case when the data rate of each link is the same.

A. Link Diversity Partition Algorithm

The LDP algorithm is developed based on the approximation algorithm using the SINR model proposed in [14]. As previously explained, the deterministic SINR model does not consider the fluctuating fading in transmissions, which makes the algorithm susceptible to fading environment. Instead, LDP is advantageous by taking into account Rayleigh fading, which however is a non-trivial task. Below, we first briefly introduce the algorithm in [14], explain the faced challenge and advantages of the LDP design, and then present LDP.

The algorithm in [14] builds disjoint link classes by classifying the links with similar lengths to one class. For each link class, it partitions the entire network region into squares and set neighboring squares to different colors (Figure 2(a)). Such color setting makes the transmissions in the same-color squares always have a certain distance between each other. The size of the squares is calculated based on the SINR model to ensure the successful transmissions of a selected link from each same-color square when all these selected links transmit simultaneously. Then, all the selected links from the same-color squares form a feasible schedule. This algorithm selects the schedule with the highest data rate among all the feasible schedules.

To extend this algorithm for the Rayleigh fading model is challenging because calculating the closed form of successful transmission probability in Rayleigh fading model is much more complex than that in the deterministic SINR model, which makes the size of each square in the grid difficult to estimate. Fortunately, in Corollary 3.1, we have derived a linear formula (Formula (19)) to judge a successful transmission under Rayleigh fading model. Also, this previous algorithm sets both upper and lower bounds for the link length of each class when building link classes. We further improve this algorithm by only upper bounding the link length of each class, since the transmission of a shorter-length link will be successful if the transmission of a longer-length link in the same square area is successful. This improvement enhances the throughput as there are more link candidates possibly with higher data rates for a schedule.

Definition 4.1: Length diversity set of a set of links L, denoted by $G(L)$, is defined by

$$G(L) = \{h | \exists l, l' \in L : \lfloor \log(d(l)/d(l')) \rfloor = h\} \tag{35}$$

and the link length diversity $g(L)$ is defined by $g(L) = |G(L)|$, where $|G(L)|$ denotes the size of $G(L)$. $G(L)$ lists the magnitudes of transmission link lengths, and $g(L)$ represent the number of these magnitudes. In real applications, $g(L)$ is usually a small constant [14].

In the following, we introduce LDP in detail. This algorithm starts by building $g(L)$ disjoint link classes L_1, ..., $L_{g(L)}$ from L, *s.t.*

$$L_k = \{(s,r) \in L | d_{s,r} < 2^{h_k+1}\delta\} \tag{36}$$

where δ is the length of the shortest link in L. That is, each class includes the links with lengths no larger than a specific magnitude. Thus, for each link class, the length of

Algorithm 1: Pseudo-code for LDP.

input : $\{L_1, ..., L_{g(L)}\}$, $\{R_1, ..., R_{g(L)}\}$;
output: $\mathcal{P}$;
for $k \leftarrow 1$ *to* $g(L)$ **do**
  Partition the network region into squares $\mathcal{A}^k = \{A^k_{a,b}\}$ of size $\beta_k \times \beta_k$;
  Color the squares with $\{1, 2, 3, 4\}$ *s.t.* no two adjacent squares have the same color (see Fig. 2 (a));
  for $j \leftarrow$ *1 to 4* **do**
    for *each square in* j *that has receivers in* R_k **do**
      Pick the receiver r_i with the maximum data rate in the square and put it in $\mathcal{P}^{(k,j)}_{\text{ldp}}$;
      Remove r_i from R_k;
$\mathcal{P} \leftarrow \arg\max\{U(\mathcal{P}^{(k,j)}_{\text{ldp}})|\mathcal{P}^{(k,j)}_{\text{ldp}}, \forall k, j\}$;
return $\mathcal{P}$;

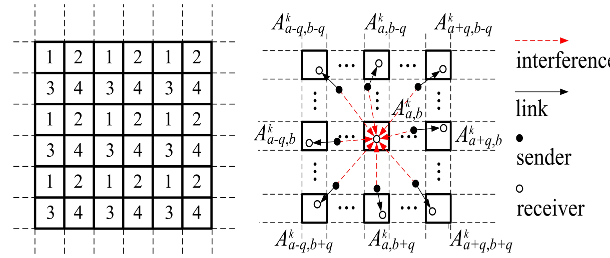
links is upper bounded. Then, the desired signal power for each link class has a lower bound. Next, for each link class L_k, LDP constructs feasible schedules. The pseudocode of this procedure is shown in Algorithm 1. When scheduling L_k, the entire network region is partitioned into a set of squares $\mathcal{A}^k = \{A^k_{a,b}\}$ $(a, b = 1, 2, ...)$, where (a, b) represents the location of the square in the grid. Each square has size $\beta_k = 2^{h_k+1}\beta\delta$, where

$$\beta = \left(\frac{8\zeta(\alpha-1)\gamma_{\text{th}}}{\gamma_\varepsilon}\right)^{\frac{1}{\alpha}}, \tag{37}$$

in which $\zeta(\alpha - 1)$ is the Riemann zeta function and it is a constant for $\alpha > 2$. We will show how the square size is calculated with the consideration of the Rayleigh fading model in the Proof of Theorem 4.1. We set these squares with four colors $j \in \{1, 2, 3, 4\}$ as shown in Figure 2(a). Then, each pair of the same-color squares are kept far away from each other. By picking senders in the same-color squares, we can guarantee that the distances between the active senders are large enough, and hence the interference is upper bounded. In scheduling L_k, for each square of a specific color j, if the square contains receivers in R_k which denotes the set of receivers whose links are in L_k, we pick up the receiver with the highest data rate and add its desired sender to $\mathcal{P}^{(k,j)}_{\text{ldp}}$, which denotes the schedule built for link class L_k on color j. Because there are $g(L)$ link classes and each class has four schedules for four colors, we can finally get $4g(L)$ feasible schedules: $\mathcal{P}^{(k,j)}_{\text{ldp}}$ $(k = 1, ..., g(L), j = 1, 2, 3, 4)$. The purpose of the above steps is to ensure that the SINR for each transmission in the Rayleigh fading model is large enough to reach the decoding threshold. As the objective of this link scheduling problem is to maximize the throughput, the schedule with the largest data rate is chosen from these schedules finally:

$$\mathcal{P}_{\text{ldp}} = \arg\max\{U(\mathcal{P}^{(k,j)}_{\text{ldp}})|\mathcal{P}^{(k,j)}_{\text{ldp}}, \forall k, j\}, \tag{38}$$

where $U(\mathcal{P})$ denotes the data rate transmitted by sender set $\mathcal{P}$.



(a) Partition (b) Proof of Theorem 4.1

Fig. 2. Link diversity partition algorithm (LDP).

Theorem 4.1: LDP provides a feasible schedule.

Proof Without loss of generality, we examine whether any receiver $r_j \in R_k$ can successfully receive the packet from s_j. Consider the interference caused by the transmission from other requests. Suppose r_j is located in square $A^k_{a,b}$, since links are scheduled concurrently iff their receivers reside in different squares with the same color, and the distance between same-color squares is $2q\beta_k$ $(q \in \mathbb{N})$, the interference can only be caused by the senders whose receivers are in $A^k_{a\pm 2q,b\pm 2q}$, $A^k_{a\pm 2q,b\mp 2q}$, $A^k_{a,b\pm 2q}$, and $A^k_{a\pm 2q,b}$ (see Fig. 2 (b)). We use $\mathcal{Q}^k_q$ to denote the set of all active senders whose receivers are in the $8q$ squares. For any interference link $(s_i, r_j) \in \mathcal{Q}^k_q$, because the distance between r_i and r_j is at least $2q\beta_k$ and the distance between r_i and s_i is at most $2^{h_k+1}\delta$. By triangle inequality,

$$\begin{aligned} d_{s_i,r_j} &\geq d_{r_i,r_j} - d_{s_i,r_i} && (39)\\ &\geq 2q\beta_k - 2^{h_k+1}\delta && (40)\\ &= 2^{h_k+2}q\delta\beta - 2^{h_k+1}\delta. && (41)\end{aligned}$$

Then, the interference factor of s_j on r_i is at most

$$\begin{aligned} f_{i,j} &= \ln\left(1 + \frac{d^{-\alpha}_{i,j}\gamma_{\text{th}}}{d^{-\alpha}_{j,j}}\right) && (42)\\ &\leq \frac{d^{-\alpha}_{i,j}\gamma_{\text{th}}}{d^{-\alpha}_{j,j}} && (43)\\ &= (2q\beta - 1)^{-\alpha}\gamma_{\text{th}}. && (44)\end{aligned}$$

Since there are at most $8q$ links in $\mathcal{Q}^k_q$, the interference factor of $\mathcal{Q}^k_q$ on r_j is upper bounded by $f_{\mathcal{Q}^k_q,r_j} = \sum_{s_i \in \mathcal{Q}^k_q} f_{s_i,r_j} \leq \frac{8q\cdot\gamma_{\text{th}}}{(2q\beta-1)^\alpha}$, and the interference factor of all active links $\mathcal{Q}^k = \cup_q \mathcal{Q}^k_q$ on r_i is upper bounded by

$$\begin{aligned} f_{\mathcal{P}_{\text{ldp}},r_j} &= \sum_{q=1}^{\infty} f_{\mathcal{Q}^k_q,r_j} && (45)\\ &\leq \sum_{q=1}^{\infty} \frac{8q\gamma_{\text{th}}}{(2q\beta-1)^\alpha} && (46)\\ &\leq \sum_{q=1}^{\infty} \frac{8q\gamma_{\text{th}}}{q^\alpha\beta^\alpha} \leq \gamma_\varepsilon, && (47)\end{aligned}$$

which implies that r_j can be informed.

Theorem 4.2: The approximation ratio of the link diversity partition algorithm (LDP) is $O(g(L))$.

Proof We start by defining $\mathcal{P}_{\text{opt}}^k$ to be the optimum schedule comprised by the links (s_i, r_i) such that

$$2^{h_k}\delta \le d_{i,i} < 2^{h_k+1}\delta. \tag{48}$$

Then, $U(\mathcal{P}_{\text{opt}}) = \sum_{k=1}^{g(L)} U(\mathcal{P}_{\text{opt}}^k)$. Now we show that any optimal scheme can schedule at most u receivers within each square in $\mathcal{A}^k$ in $\mathcal{P}_{\text{opt}}^k$, where u is a constant:

$$u = \left\lceil \frac{\gamma_\varepsilon}{\ln\left(1 + \frac{1}{2^\alpha \beta^\alpha \gamma_{\text{th}}}\right)} \right\rceil. \tag{49}$$

For the sake of contradiction, assume that there exists a square containing more than u receivers. We pick any link receiver r_i and calculate its interference factor:

$$\begin{aligned} f_{\mathcal{P}_{\text{opt}},r_j} &\ge \sum_{s_i \in \mathcal{P}_{\text{opt}}^k} f_{i,j} && (50)\\ &= \sum_{s_i \in \mathcal{P}_{\text{opt}}^k} \ln\left(1 + \frac{d_{i,j}^{-\alpha}\gamma_{\text{th}}}{d_{j,j}^{-\alpha}}\right) && (51)\\ &\ge \sum_{s_i \in \mathcal{P}_{\text{opt}}^k} \ln\left(1 + \frac{2^{\alpha h_k}\delta^\alpha}{(2^{h_k+1}\beta\delta)^\alpha \gamma_{\text{th}}}\right) && (52)\\ &\ge \gamma_\varepsilon. && (53) \end{aligned}$$

Therefore, given that every feasible schedule $\mathcal{P}_{\text{ldp}}$ computed by LDP contains the heaviest link in every square, the following bound holds:

$$U(\mathcal{P}_{\text{ldp}}^k) \ge \frac{U(\mathcal{P}_{\text{opt}}^k)}{4q},\ j \in \{1, ..., 4\},\ k \in \{1, 2, ..., g(L)\} \tag{54}$$

where $\mathcal{P}_{\text{ldp}}^k = \arg\max\{U(\mathcal{P}_{\text{ldp}}^{(k,j)}) | \mathcal{P}_{\text{ldp}}^{(k,j)}, \forall j\}$. Since LDP returns the schedule of the maximum data rate over all length classes and colorings, the approximation ratio follows:

$$\begin{aligned} U(\mathcal{P}_{\text{ldp}}) &\ge \frac{\sum_{k=1}^{g(L)} U(\mathcal{P}_{\text{ldp}}^k)}{4g(L)} && (55)\\ &\ge \frac{\sum_{k=1}^{g(L)} U(\mathcal{P}_{\text{opt}}^k)}{16g(L)} \\ &= \frac{U(\mathcal{P}_{\text{opt}})}{16g(L)} && (56) \end{aligned}$$

which implies that $\frac{U(\mathcal{P}_{\text{opt}})}{U(\mathcal{P}_{\text{ldp}})} \le 16g(L)$.

B. Recursive Link Elimination Algorithm

In this section, we consider a special case of *Fading-R-LS*, in which the transmission rate of each link is the same, i.e., $\lambda_i = \lambda\ \forall 1 \le i \le N$, which is true in many applications. For example, sensors need to periodically report their collected data to sink node, in which all the sensors have the same data rate [21]. The link scheduling problem with the same data rate for all the senders has been discussed in many link scheduling works, like [22] [23]. We propose a greedy algorithm, namely recursive link elimination algorithm (RLE), for this special case. Algorithm 2 shows the pseudocode for RLE. In each iteration, the algorithm first greedily selects the unpicked sender with the shortest link length, say s_i. The rationale of this strategy is that the signal power received by the receiver with a shorter link is always stronger, and hence the receiver is more likely to successfully receive the packet. Then, all links whose senders are within the radius $c_1 d_{s_i,r_i}$ of the receiver r_i are removed from L, where c_1 is a constant to be set later on (in Formula (59)). Second, all senders whose receivers have interference factors above c_2 from the selected senders are removed, where c_2 is a constant smaller than 1. This process is repeated until all links in L have been either active or deleted. Note that though this algorithm has a number of iterations, all identified active links conduct transmissions in one time slot simultaneously.

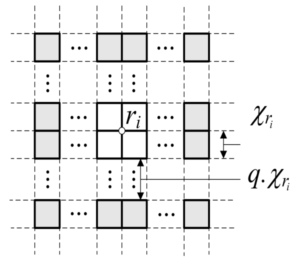


Fig. 3. Proof of Theorem 4.3.

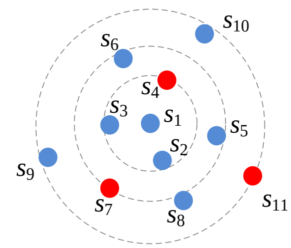


Fig. 4. z-blue-dominant.

Algorithm 2: Pseudo-code for RLE.

input : $S = \{s_1, ..., s_N\}$
output: $\mathcal{P}$
$\mathcal{P} \leftarrow \phi$;
while $S \ne \mathcal{P}$ **do**
  Pick up the sender s_i that has the shortest link length in L and add it to $\mathcal{P}$;
  Remove each sender s_j, *s.t.* $d_{s_j,r_i} < c_1 d_{s_i,r_i}$ from S;
  Remove each sender s_j, *s.t.* $f_{\mathcal{P},r_j} > c_2\gamma_\varepsilon$ from S;
return $\mathcal{P}$;

Below, we prove that the schedule obtained by RLE is both feasible (Theorem 4.3) and efficient, i.e., only a constant factor away from the optimal (Theorem 4.4). We use r_i to represent a receiver whose desired sender is selected in Algorithm 2, and use $\mathcal{P}_i^-$ and $\mathcal{P}_i^+$ to denote the set of senders added after and before s_i is selected, respectively.

Lemma 4.1: The distance between any two senders in $\mathcal{P}_i^+$ is no smaller than $(c_1 - 1)d_{s_i,r_i}$.

Proof The detailed proof can be found in Appendix.

Theorem 4.3: RLE provides a feasible solution.

Proof When sender s_i is added to the schedule, the interference factor of $\mathcal{P}_i^-$ on r_i must be no larger than $c_2\gamma_\varepsilon\gamma_{\text{th}}$; otherwise it has been deleted in a previous step. Therefore, the interference factor on r_i from concurrent active link set

$\mathcal{P}_i^-$ is $f_{\mathcal{P}_i^-,r_i} \le c_2\gamma_\varepsilon\gamma_{\text{th}}$. It remains to show that $f_{\mathcal{P}_i^+,r_i} \le (1-c_2)\gamma_\varepsilon\gamma_{\text{th}}$.

We partition the entire network region into squares with size $\chi_i \times \chi_i$ (see Fig. 3), where $\chi_i = (c_1-1)d_{s_i,r_i}/\sqrt{2}$. According to Lemma 4.1, any two senders in $\mathcal{P}_i^+$ cannot be located in the same square because the distance between the senders in $\mathcal{P}_i^+$ is at least $(c_1-1)d_{s_i,r_i}$, and the size of square is $(c_1-1)d_{s_i,r_i}/\sqrt{2}$. We use $\mathcal{Q}_q^i$ to denote the set of senders in the squares that is $q\chi_i$ away from r_i. Then, there are at most $4(2q+1)$ senders in $\mathcal{Q}_q^i$. The distance between the senders in $\mathcal{Q}_q^i$ and r_i is at least $q\chi_i$, hence the interference factor of any sender $s_j \in \mathcal{Q}_q^i$ on r_i is at most

$$\begin{aligned} f_{j,i} &= \ln\left(1+\frac{d_{j,i}^{-\alpha}\gamma_{\text{th}}}{d_{i,i}^{-\alpha}}\right) \\ &\le \frac{d_{j,i}^{-\alpha}\gamma_{\text{th}}}{d_{i,i}^{-\alpha}} \\ &\le \frac{(q\chi_i)^{-\alpha}\gamma_{\text{th}}}{d_{i,i}^{-\alpha}}. \end{aligned}$$

The interference factor of $\mathcal{Q}_q^i$ on r_i is then upper bounded by

$$f_{\mathcal{Q}_q^i,r_i} = \sum_{s_j\in\mathcal{Q}_q^i} f_{j,i} \le \frac{4(2q+1)(q\chi_i)^{-\alpha}\gamma_{\text{th}}}{d_{i,i}^{-\alpha}}, \tag{57}$$

and the interference factor of all active links $\mathcal{P}_i^+ = \cup_q \mathcal{Q}_q^i$ on r_i is upper bounded by

$$\begin{aligned} f_{\mathcal{P}_i^+,r_i} &= \sum_{q=1}^{\infty} f_{\mathcal{Q}_q^i,r_i} \le \sum_{q=1}^{\infty} \frac{4(2q+1)q^{-\alpha}\chi_i^{-\alpha}\gamma_{\text{th}}}{d_{i,i}^{-\alpha}} \\ &\le \sum_{q=1}^{\infty} \frac{12q\cdot q^{-\alpha}\chi_i^{-\alpha}\cdot\gamma_{\text{th}}}{d_{i,i}^{-\alpha}} \\ &\le \frac{12\chi_i^{-\alpha}\cdot\gamma_{\text{th}}}{d_{i,i}^{-\alpha}}\zeta(\alpha-1). \end{aligned} \tag{58}$$

We set c_1 by (to make $f_{\mathcal{P}_i^+,r_i} \le (1-c_2)\gamma_\varepsilon$)

$$c_1 = \sqrt{2}\left(12\zeta(\alpha-1)\gamma_{\text{th}}/(\gamma_\varepsilon(1-c_2))\right)^{\frac{1}{\alpha}} + 1, \tag{59}$$

we can get that

$$\begin{aligned} f_{\mathcal{P}_i^+,r_i} &\le \frac{12\chi_i^{-\alpha}\zeta(\alpha-1)\gamma_{\text{th}}}{d_{s_i,r_i}^{-\alpha}} && (60) \\ &= \frac{12(\sqrt{2}(c_1-1)d_{i,i}/2)^{-\alpha}\zeta(\alpha-1)\gamma_{\text{th}}}{d_{s_i,r_i}^{-\alpha}} \\ &= (1-c_2)\gamma_\varepsilon, && (61) \end{aligned}$$

In the following, we then analyze the efficiency of RLE. We first derive Lemmas 4.2 - 4.4, based on which we prove Theorem 4.4.

Lemma 4.2: Let $\mathcal{P}$ be a feasible solution and let $s_i \in \mathcal{P}$. The number of senders in $\mathcal{P}\backslash s_i$ with distance kd_{s_i,r_i} from s_i is at most $\frac{e^{\gamma_\varepsilon}-1}{\gamma_{\text{th}}}(1+k)^\alpha$.

Proof The detailed proof can be found in Appendix.

Definition 4.2: *(z-blue-dominant [15])* Let $\mathcal{N}_r$ and $\mathcal{N}_b$ be two disjoint sets of points in a 2D Euclidean space, namely red and blue points, respectively. Let circle $B_d(s_b)$ be the set of points p such that $d(p,s_b) \le d$. Then, for any positive integer z, a point $s_b \in \mathcal{N}_b$ is *z-blue-dominant* if every circle $B_d(s_b)$ contains z times more blue than red points, or formally

$$|B_d(s_b)\cap\mathcal{N}_b| > z|B_d(s_b)\cap\mathcal{N}_r| \quad \forall d \in \mathbb{R}^+. \tag{62}$$

Fig. 4 gives an example for this definition: $\mathcal{N}_r = \{s_4, s_7, s_{11}\}$ and $\mathcal{N}_b = \{s_1, s_2, s_3, s_5, s_6, s_8, s_9, s_{10}\}$. Because every circle centered at s_1 contains 2 times more blue points than red points, s_1 is a 2-blue-dominant.

Lemma 4.3: *(Blue-dominant centers lemma [15])* For any positive integer z, if $|\mathcal{N}_b| > 5z|\mathcal{N}_r|$ then there exists at least one z-blue-dominant point s_b in $\mathcal{N}_b$. In addition, given a z-blue-dominant point s_b, for each point s_r in $\mathcal{N}_r$, there exists a subset of $\mathcal{N}_b$ corresponding to s_r, denoted by $G(s_r)$, *s.t.*, 1) any point in $G(s_r)$ is farther from s_r than from s_b; 2) for any pair of points $s_r, s_r' \in \mathcal{N}_r$, $G(s_r)$ and $G(s_r')$ are disjoint; and 3) the number of points in each subset $G(s_r)$ is no smaller than z (See the proof in Lemma 4.4 in [15]).

Proof The detailed proof can be found in Appendix.

Lemma 4.4: Denote the set of all senders in the optimal schedule and RLE by $\mathcal{P}_{\text{opt}}$ and $\mathcal{P}_{\text{rle}}$, respectively. Then, $|\mathcal{P}_{\text{opt}}\backslash\mathcal{P}_{\text{rle}}| \le \frac{3^\alpha\times 5\varepsilon}{c_2(1-\varepsilon)\gamma_{\text{th}}}|\mathcal{P}_{\text{rle}}|$.

Theorem 4.4: The approximation ratio of the link elimination algorithm (RLE) is a constat: $\frac{3^\alpha\times 5\varepsilon}{c_2(1-\varepsilon)\gamma_{\text{th}}} + 1$.

Proof Denote the number of receivers informed by RLE and the optimal schedule by U_{rle} and U_{opt}, respectively. Then, according to Lemma 4.4,

$$\begin{aligned} \frac{U_{\text{opt}}}{U_{\text{rle}}} &\le \frac{|\mathcal{P}_{\text{opt}}|}{|\mathcal{P}_{\text{rle}}|} = \frac{|\mathcal{P}_{\text{opt}}\backslash\mathcal{P}_{\text{rle}}|}{|\mathcal{P}_{\text{rle}}|} + 1 \\ &\le \frac{3^\alpha\times 5\varepsilon}{c_2(1-\varepsilon)\gamma_{\text{th}}} + 1. \end{aligned}$$

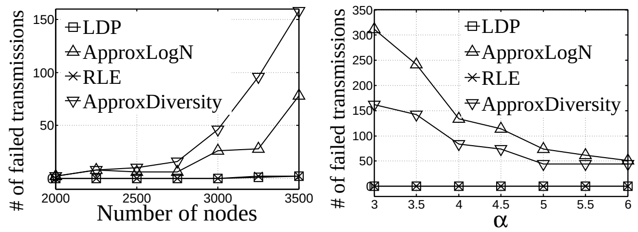
V. Performance Evaluation

In this section, we present experimental results to better illustrate the practical appeal of our scheduling algorithms. In the experiment, each sender was given a random location in a 500×500 square, and each receiver was located from its sender with a distance randomly selected from $[5,20]$ in a random direction. The accepted error rate was set to 0.01, the decoding threshold was set to 1, and the data rate of every link was set to 1. We measured the following two metrics:

1) *Throughput* (or the total data rate successfully received by receivers)
2) *The number of failed transmissions.*

We compare our algorithms with two other link scheduling algorithms: ApproxLogN [14] and ApproxDiversity [15].

1) ApproxLogN partitions the link set into disjoint link classes and schedules the links in each class separately.



(a) Number of links (b) Path loss exponent

Fig. 5. Fading-resistant vs. fading-susceptible: # of failed transmissions

2) ApproxDiversity always picks up the shortest link and excludes links conflicted with the picked links in each iteration.

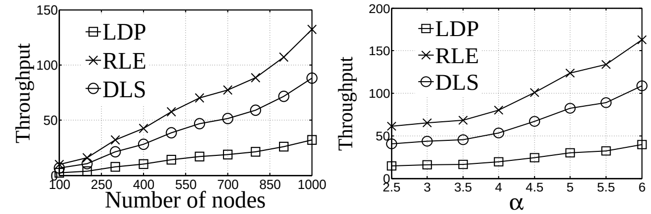
Unlike our algorithms, ApproxLogN and ApproxDiversity are not fading-resistant although they are also polynomial time algorithms based on the SINR model.

Fig. 5(a) and Fig. 5(b) show the number of failed transmissions of different algorithms versus the number of links and path loss exponent (α), respectively. We see that LDP and RLE have almost no failed transmissions, because they always select the links that can guarantee successful transmissions with high probability $1 - \varepsilon$ with fading consideration. ApproxLogN and ApproxDiversity assume that the channel is non-fading, which makes them fading-susceptible. Fig. 5(a) shows that the number of failed transmissions increases as the number of nodes increases. This is because more nodes cause more transmissions hence severer interference, thus increasing the probability of a transmission failure. An interesting observation from Fig. 5(b) is that the number of failed transmissions decreases as α increases. This is because when fading is more severe, the interference factors from all undesired remote nodes are smaller (by Formula (17)), which reduces the probability of a transmission failure.

We then measure the throughput of LDP and RLE. Fig. 6 shows that the throughput follows RLE $>$ LDP with different number of links or different α values. This is because LDP only allows the links in the same class with the same color being scheduled at the same time. Though such a mechanism can prevent the conflict among the links, it reduces the number of links that can be scheduled simultaneously. Fig, 6(a) shows that the throughput increases as the number of links increases since more transmissions lead to higher throughput. From Fig. 6(b), we find that the throughput increases as α increases. For LDP, it is because when α increases, the partitioned square size decreases (by Formula (37)), which leads to more partitioned squares and hence more links to be scheduled. For RLE, it is because smaller α makes few nodes eliminated in each iteration (by Formula (59)). Fig. 5(b) indicates that higher α value leads to higher transmission success probability, and hence more links are likely to be scheduled in DLS.

VI. Related Work

Based on the choice of interference models, the previous works can be classified to two groups: *graph based scheduling* [1]–[9] and *SINR based scheduling* [11]–[15].



(a) Number of links (b) Path loss exponent

Fig. 6. Centralized vs. decentralized: # of links scheduled

A. Graph based scheduling

Graph models have been served as the useful abstraction for studying scheduling problems for many years. For example, Sharma *et al.* [1] defined a k-hop interference model, in which no two links within k-hops can successfully transmit simultaneously. They proved that the scheduling problem is NP-hard when $k > 1$ and cannot be approximated within a factor that grows polynomially with the number of nodes in the network. Lin and Shroff [2] proposed *Greedy Maximal Scheduling* (GMS), which can be implemented in a distributed manner. Joo *et al.* [3] further provided numerous analytic results to characterize the performance limits of GMS. Wang *et al.* [4] studied the link scheduling problem for a multi-hop wireless network to maximize throughput. They assumed each node has different transmission range and interference range, and the methods they presented can achieve a constant factor of the optimum. Cheng *et al.* [5] studied the problem in multi-radio multi-channel wireless networks, and proved that the problem is NP-hard in this scenario, in both the k-hop interference model and unit disc model. Wang *et al.* [6] developed joint TCP congestion control and carrier sense multiple access (CSMA) scheduling schemes for Internet traffic over distributed multi-hop wireless links, in which the interference among the links is modeled by a conflict graph and they proved the global convergence of their schemes to optimal network equilibrium using the Lyapunov method. Kar *et al.* [7] considered the question of obtaining tight delay guarantees for throughout-optimal link scheduling in arbitrary topology wireless ad-hoc networks. Jiang *et al.* [8] presented a distributed randomized scheme for scheduling and congestion control, which achieves near-optimal resource allocation when nodes have concave utilities. Krifa and Barakat [9] investigated both the problems of scheduling and buffer management in delay tolerant networks. Although these algorithms present extensive theoretical analysis, they are constrained to the limitations of the graph interference model that omits the accumulative nature of wireless signals (for both desired signals and undesired interference). Comparing to graph model, SINR model offers a more realistic representation of wireless networks. As proved by Gronkvist *et al.* [10] using both theoretical analysis and experiments, the graph based scheduling protocols are inefficient in the SINR model.

B. SINR based scheduling

There have been many works studying the problem of joint link scheduling and power control in the SINR model [24]–[26]. For example, Kozat *et al.* [24] addressed the joint problem to minimize the total transmit power subject to the end-to-end bandwidth guarantees and the bit error rate requirements of each transmission. The problem is proved NP-hard by constructing a reduction from integer programming. Leung and Wang [25] proved that the problem of maximizing throughput by adaptive modulation and power control while meeting packet error constraints is NP-hard. In [26], Pei and Kumar set the goal of the problem as maximizing capacity region of the network, *i.e.* the maximum attainable network throughput. They also proposed a low complexity distributed algorithm for this problem. In addition, Hong and Scaglione [10] showed that the graph based scheduling protocols are inefficient in SINR model using both theoretical analysis and experiments. Huang *et al.* [11] presented an optimization-based formulation for joint scheduling and resource allocation in the uplink OFDM access network and proposed heuristic solutions. Xu *et al.* [12] studied periodic scheduling for data aggregation with minimum delay under various interference models.

In addition, some works focus on designing algorithms with lower approximation guarantee [13]–[15]. Brar *et al.* [13] proposed a polynomial time algorithm and proved an approximation ratio for their method under uniform random node distribution. Goussevskaia *et al.* [14] formulated the scheduling problem in the geometric SINR model, proved its NP-hardness, and proposed a greedy solution with performance guarantee $O(g(L))$, where nodes are arbitrarily distributed in 2D Euclidean space, and showed that the formulated problem is NP-hard. As a solution, they proposed a greedy algorithm for the problem with performance guarantee $O(g(L))$. Goussevskaia *et al.* also proposed a scheduling algorithm with constant approximation guarantee, which is independent of the network topology and size [15]. They further formulated a variation of the problem, in which analog network coding is allowed, and presented NP-hard proof of the problem [22]. However, the SINR model still uses a limited view of signal propagation since it does not consider the fading fluctuations in received signal strength (e.g., caused by the mobility in a multi-path propagation environment). Our work is the first that analyzes the hardness of the link scheduling problem under Rayleigh-fading model and proposes approximation algorithms for the problem.

VII. Conclusions

Previous link scheduling works did not consider the fluctuating fading in transmissions, which makes the proposed scheduling algorithms vulnerable to real wireless network environment. In this paper, by incorporating Rayleigh fading model into the link scheduling problem, we formulated a *Fading-Resistant Link Scheduling problem* (Fading-R-LS) with the objective to maximize the network throughput. The challenge for this problem is its complicated judgement for a successful transmission. As a solution, we derived the closed form of the probability distribution of the SINR received by each receiver, and found that checking transmission success is equivalent to checking whether the sum interference factor from all the senders to this receiver is lower than a threshold. Based on this finding, we proved *Fading-R-LS* to be NP-hard and proposed two centralized algorithms (LDP and RLE) and one decentralized algorithm (DLS). Both theoretical analysis and experimental results demonstrate that LDP and RLE can substantially improve packet delivery ratio in fading environments compared to previous algorithms, and DLS performs even better than DLS in terms of throughput. In our future work, we will further consider how to schedule all the links with the minimum number of time slots, not just to maximize the throughput in one time slot. We will also consider the SINR link scheduling problem in a general case, not limited to geometric model.

VIII. Acknowledgements

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References

[1] G. Sharma, N. B. Shroff, and R. R. Mazumdar, "On the complexity of scheduling in wireless networks.," in *Proc. of ACM Mobicom*, 2006.

[2] X. Lin and N. B. Shroff, "The impact of imperfect scheduling on cross-layer congestion control in wireless networks.," *ACM/IEEE Tran. on Networking*, 2006.

[3] C. Joo, X. Lin, and N. Shroff, "Understanding the capacity region of the greedy maximal scheduling algorithm in multi-hop wireless networks," in *Proc. of IEEE Infocom*, 2008.

[4] W. Wang, Y. Wang, X. Li, W. Song, and O. Frieder, "Efficient interference aware TDMA link scheduling for static wireless networks," in *Proc. of ACM Mobicom*, 2006.

[5] W. Cheng, X. Cheng, T. Znati, X. Lu, and Z. Lu, "Joint TCP congestion control and CSMA scheduling without message passing," in *Proc. of IEEE Infocom*, 2009.

[6] X. Wang, Z. Li, and J. Wu, "Joint TCP congestion control and csma scheduling without message passing," *IEEE TWC*, 2013.

[7] K. Kar, S. Sarkar, A. Ghavami, and X. Luo, "Delay guarantees for throughput-optimal wireless link scheduling.," *IEEE Tran. on Automatic Control*, 2012.

[8] L. Jiang, D. Shah, J. Shin, and J. Walrand, "Distributed random access algorithm: Scheduling and congestion control.," *IEEE Tran. on Information Theory*, 2010.

[9] A. Krifa and C. Barakat, "An optimal joint scheduling and drop policy for delay tolerant networks.," *IEEE WoWMoM Workshop on Autonomic and Opportunistic Communications*, 2008.

[10] J. Gronkvist and A. Hansson, "Comparison between graph-based and interference-based STDMA scheduling.," in *Proc. of ACM Mobihoc*, 2001.

[11] J. Huang, V. G. Subramanian, R. Agrawal, and R. Berry, "Joint scheduling and resource allocation in uplink OFDM systems for broadband wireless access networks.," *IEEE JSAC*, 2009.

[12] X. Xu, X. Li, P. Wan, and S. Tang, "Efficient scheduling for periodic aggregation queries in multihop sensor networks.," *ACM/IEEE Tran. on Networking*, 2012.

[13] G. Brar, D. M. Blough, and P. Santi, "Computationally efficient scheduling with the physical interference model for throughput improvement in wireless mesh networks.," in *Proc. of ACM Mobicom*, 2006.

[14] O. Goussevskaia, Y. A. Osfwald, and R. Wattenhofer, "Complexity in geometric SINR," in *Proc. of ACM Mobihoc*, 2007.

[15] O. Goussevskaia, R. Wattenhofer, M. M. H. orsson, and E. Welzl., "Capacity of arbitrary wireless networks.," in *Proc. of IEEE Infocom*, 2009.

[16] H. S. Lichte, H. Frey, and H. Karl, “Fading-resistant low-latency broadcasts in wireless multihop networks: The probabilistic cooperation diversity approach,” in *Proc. of ACM Mobihoc*, 2010.

[17] E. Biglieri, J. Proakis, and S. Shamai, “Fading channels: Information-theoretic and communications aspects,” *IEEE Tran. on Information Theory*, 1998.

[18] S. Haykin and M. Moher, *Modern Wireless Communications*. Prentice Hall, 2005.

[19] J. Dams, M. Hoefer, and T. Kesselheimy, “Scheduling in wireless networks with rayleigh-fading interference,” in *Proc. of ACM SPAA*, 2012.

[20] B. Korte and J. Vygen, *Combinatorial Optimization Theory and Algorithms, Third Edition.* Springer, 2005.

[21] T. Halford and K. Chugg, “Barrage relay networks,” in *UCSD ITA Workshop*, 2010.

[22] O. Goussevskaia and R. Wattenhofer, “Complexity of scheduling with analog network coding.,” in *Proc. of FOWANC*, 2008.

[23] S. Guha and S. Khuller, “Approximation algorithms for connected dominating sets,” *Algorithmica*, 1998.

[24] U. C. Kozat, I. Koutsopoulos, and L. Tassiulas, “Cross-layer design for power efficiency and qos provisioning in multi-hop wireless networks.,” *Wireless Communications*, 2006.

[25] K. Leung and L. Wang, “Integrated link adaptation and power control for wireless ip networks.,” in *Proc. of VTC*, 2000.

[26] G. Pei and V. A. Kumar, “Low-complexity scheduling for wireless networks.,” in *Proc. of ACM Mobihoc*, 2012.

APPENDIX

A. Proof of Lemma 4.1

Proof For any receiver r_j whose desired sender s_j is in $\mathcal{P}_i^+$, there is no other sender, say s_l, in $\mathcal{P}_i^+$ that has distance smaller than $c_1 d_{s_j,r_j}$ from r_j. Based on this and the triangular inequality, we can calculate the lower bound of the distance between any two senders in $\mathcal{P}_i^+$:

$$\begin{aligned} d_{s_l,s_j} &\geq d_{s_l,r_j} - d_{s_j,r_j} && (63)\\ &\geq c_1 d_{s_j,r_j} - d_{s_j,r_j} && (64)\\ &\geq (c_1-1) d_{s_j,r_j} && (65)\\ &\geq (c_1-1) d_{s_i,r_i}. && (66)\end{aligned}$$

B. Proof of Lemma 4.2

Proof For each sender $s_j \in \mathcal{P}\backslash s_i$, its interference factor on r_i ($f_{j,i}$) cannot be larger than γ_ε. Accordingly, $\left(\frac{d_{j,i}}{d_{i,i}}\right)^{-\alpha} \leq e^{\gamma_\varepsilon} - 1$. Since $\ln(1+x) \geq \frac{\gamma_\varepsilon x}{e^{\gamma_\varepsilon}-1}$ when $x \in [0, e^{\gamma_\varepsilon}-1]$, $f_{j,i}$ is lower bounded by

$$\begin{aligned} f_{j,i} &= \ln\left(1 + \frac{d_{j,i}^{-\alpha}\gamma_{\rm th}}{d_{i,i}^{-\alpha}}\right) \geq \frac{\gamma_\varepsilon}{e^{\gamma_\varepsilon}-1}\frac{d_{j,i}^{-\alpha}\gamma_{\rm th}}{d_{i,i}^{-\alpha}} && (67)\\ &\geq \frac{\gamma_\varepsilon}{e^{\gamma_\varepsilon}-1}\frac{(d_{i,i}+d_{j,i})^{-\alpha}}{d_{i,i}^{-\alpha}}\gamma_{\rm th} && (68)\\ &\geq \frac{\gamma_\varepsilon (1+k)^{-\alpha}}{e^{\gamma_\varepsilon}-1}\gamma_{\rm th}. && (69)\end{aligned}$$

Since the interference factor of $\mathcal{P}\backslash s_i$ on r_i cannot exceed γ_ε, there are at most $\frac{e^{\gamma_\varepsilon}-1}{\gamma_{\rm th}}(1+k)^\alpha = \frac{\varepsilon}{(1-\varepsilon)\gamma_{\rm th}}(1+k)^\alpha$ such senders.

C. Proof of Lemma 4.4

Proof For the sake of contradiction, we assume that

$$|\mathcal{P}_{\rm opt}\backslash\mathcal{P}_{\rm rle}| > \frac{3^\alpha \times 5\varepsilon}{c_2(1-\varepsilon)\gamma_{\rm th}}|\mathcal{P}_{\rm rle}|. \qquad (70)$$

We label the set of senders in $\mathcal{P}_{\rm opt}\backslash\mathcal{P}_{\rm rle}$ by blue ($\mathcal{N}_b = \mathcal{P}_{\rm opt}\backslash\mathcal{P}_{\rm rle}$) and those in $\mathcal{P}_{\rm rle}$ by red ($\mathcal{N}_r = \mathcal{P}_{\rm rle}$). By Lemma 4.3, there is a z-blue-dominant point (sender) $s_i \in \mathcal{N}_b$, where

$$z = \frac{3^\alpha \varepsilon}{c_2(1-\varepsilon)\gamma_{\rm th}}. \qquad (71)$$

We shall argue that the sender s_i would have been picked by RLE, leading to a contradiction.

According to Lemma 4.3, for any red point $s_j \in \mathcal{N}_r$, there exists a subset of blue points $G(s_j)$ such that all the points in $G(s_j)$ are closer to s_i than to s_j and $|G(s_j)| \geq z$ ($z = \frac{3^\alpha\varepsilon}{(1-\varepsilon)\gamma_{\rm th}}$). We can derive that

$$d_{s_i,s_j} > 2d_{s_i,r_i}; \qquad (72)$$

otherwise the number of senders within distance $2d_{s_i,r_i}$ from s_i would be larger than $\frac{(2+1)^\alpha\varepsilon}{c_2(1-\varepsilon)\gamma_{\rm th}} \geq \frac{3^\alpha\varepsilon}{(1-\varepsilon)\gamma_{\rm th}}$, which contradicts with the conclusion in Lemma 4.2. Based on the triangle inequality,

$$d_{s_j,r_i} \geq d_{s_i,s_j} - d_{s_i,r_i} > d_{s_i,s_j}/2. \qquad (73)$$

For any point $s_l \in G(s_j)$,

$$\begin{aligned} d_{s_l,r_i} &\leq d_{s_l,s_i} + d_{s_i,r_i} && (74)\\ &< d_{s_j,s_i} + d_{s_i,r_i} && (75)\\ &< d_{s_j,s_i} + d_{s_j,s_i}/2 && (76)\\ &= 3d_{s_j,s_i}/2. && (77)\end{aligned}$$

Hence, the sum interference factor of the blue senders in $G(s_j)$ on r_i is lower bounded

$$\begin{aligned} \sum_{s_l\in G(s_j)} f_{l,i} &= \sum_{s_l\in G(s_j)} \ln\left(1+\frac{d_{l,i}^{-\alpha}\gamma_{\rm th}}{d_{i,i}^{-\alpha}}\right) && (78)\\ &\geq \sum_{s_l\in G(s_j)} \frac{\gamma_\varepsilon d_{l,i}^{-\alpha}\gamma_{\rm th}}{(e^{\gamma_\varepsilon}-1)d_{i,i}^{-\alpha}} && (79)\\ &> \frac{\gamma_\varepsilon}{e^{\gamma_\varepsilon}-1}\frac{e^{\gamma_\varepsilon}-1}{c_2\gamma_{\rm th}}3^\alpha\frac{3^{-\alpha}}{2^{-\alpha}}\frac{d_{j,i}^{-\alpha}\gamma_{\rm th}}{d_{i,i}^{-\alpha}} && (80)\\ &= \frac{d_{j,i}^{-\alpha}}{2^{-\alpha}d_{i,i}^{-\alpha}}\frac{\gamma_\varepsilon}{c_2} && (81)\\ &> \frac{d_{j,i}^{-\alpha}}{d_{i,i}^{-\alpha}}\frac{\gamma_\varepsilon}{c_2} \geq \frac{\gamma_\varepsilon}{c_2}\ln\left(1+\frac{d_{j,i}^{-\alpha}\gamma_{\rm th}}{d_{i,i}^{-\alpha}}\right) && (82)\\ &= \frac{\gamma_\varepsilon f_{j,i}}{c_2}. && (83)\end{aligned}$$

This relationship holds for any $s_j \in \mathcal{N}_r$, and $G(s_j)$ and $G(s_l)$ are disjoint $\forall s_j, s_l \in \mathcal{N}_r$, then the total interference factor that r_i receives from the senders in $\mathcal{P}_{\rm opt}\backslash\mathcal{P}_{\rm rle}$ (blue points) is at least $\frac{\gamma_\varepsilon}{c_2}$ times as high as the interference factor it would receive from the senders in RLE (red points). Because the interference factor of $\mathcal{N}_b$ on r_i is at most γ_ε. Therefore, we have $f_{\mathcal{N}_r,r_i} < \frac{f_{\mathcal{N}_b,r_i}}{\frac{\gamma_\varepsilon}{c_2}} \leq \frac{\gamma_\varepsilon}{\frac{\gamma_\varepsilon}{c_2}} = c_2$. which implies that s_i should not have been deleted by RLE, which establishes the contradiction.