Power Distribution Scheduling for Electric Vehicles in Wireless Power Transfer Systems

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Abstract*—Electric vehicles (EVs) will become a component of the future generation intelligent transportation system. Because of EVs' limited battery power, the wireless power transfer (WPT) system has drawn much attention in recent years. The WPT system charges EVs in motion when they pass the charging lanes installed in roads without requiring physical contact between utility power supply and vehicle battery. A charging lane has limited power that can be transferred to EVs on the charging lane. A challenge here is how to allocate the limited power to the EVs so that they have sufficient power to arrive at the next charging lane or their destinations (when there are no charging lanes ahead). In this paper, we study this power distribution scheduling problem. We provide solutions to handle this challenge and also achieve each of the following goals as much as possible: i) balancing the state of charge (SOC) of the EVs, ii) balancing the amount of stored power of the EVs, and iii) minimizing the total power charged. This paper is the first work that handles such a power distribution scheduling problem in WPT systems. Our extensive experiments on *MatLab* and *Simulation for Urban MObility* (SUMO) show the effectiveness of our scheduling solutions in achieving the different goals compared with other scheduling methods including first-come-first-serve and equal share.

I. INTRODUCTION

An intelligent transportation system, as a form of the future generation transportation system, realizes real-time sensing, computing, and intelligence capabilities to increase traffic efficiency and provide better reliability and safety [8], [12], [15]. As a component of the intelligent transportation system, electric vehicles (EVs) have drawn much attention in recent years and will be widely used. For example, Tesla Motors will launch its new "Model 3" EVs on March 31, 2016 [3], which is a new and less-expensive of its kinds to make EVs affordable for people. Several works already demonstrate the possible impact of EVs on the road transportation system [11], [16] such as petroleum consumption reduction and less environmental pollution. As the total number of vehicles keeps rising worldwide and nationwide, the world and the nation are urged to switch from gas-driven vehicles to EVs in order to decrease the consumption of oil and petroleum.

The onboard energy storage of an EV supplies the power (power and energy are interchangeable terms in this paper) demands of the vehicle. EVs used for personal transportation can be charged by plugging them to standard power outlet sockets. However, EVs inherit some battery-related drawbacks such as heavy weight, long charging time, large size, and short driving range. To alleviate the battery-related problems, roadway-powered EVs that rely on the inductive wireless power transfer (WPT) systems for power charging have been developed. WPT allows charging procedure to take place on vehicles in motion automatically without having any physical contact between utility power supply and electric battery. That is, power is transferred from the power grid to a vehicle's battery while the vehicle passes charging lanes installed in roads. As an example, researchers from KAIST [5] successfully deployed a 24km long WPT public transportation service in the city of Gumi, Korea and have been developing the fifth generation roadway-powered EVs to increase the efficiency and functionality of the WPT system.

Fig. 1 shows an example of a WPT system architecture. Each EV sends a charging request to the global charging controller (GCC) using vehicle-to-infrastructure (V2I) communication. GCC conducts the power distribution scheduling that allocates power to each EV when it passes each charging section of a charging lane. It then sends the schedule to grid side controllers (GSCs), each of which manages a charging lane. When an EV approaches a charging lane, it contacts GSC and then receives power as scheduled when it passes each charging section of the charging lane.

In this paper, we consider a WPT system in a highway scenario where vehicles follow a similar velocity. An EV spends very little time on top of a charging lane. The shorter duration of time requires a higher power level (i.e., power transmission rate) in the charging infrastructure so that a certain amount of power can be transferred to an EV in motion. Since usually there are a number of vehicles on a charging lane at a time, the charging infrastructure needs to have a higher power level to meet the needs of all the vehicles at the same time. However, a higher power level requires higher investment on the charging infrastructure. For example, according to a recent study [14], the infrastructure setup cost is \$700, \$5000 and \$50,000 for the 1.44kW, 6kW and 90KW power transmission rate to one vehicle, respectively. Therefore, in order to constrain the investment cost, the WPT designers tend to put limits on the power level of the charging infrastructure.

A challenge here is when the infrastructure is not able to fulfill the demands from all EVs on a charging lane, how to al-

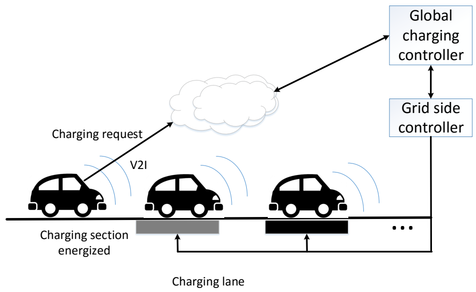


Fig. 1. The wireless power transfer system.

locate the limited power to the EVs so that they have sufficient power to arrive at the next charging lane or their destinations (when there are no charging lanes ahead). In spite of several existing works that try to realize WPT systems considering different perspectives [18], [23], no previous works have been dedicated to handling this challenge. In this paper, we study this power distribution scheduling problem. We provide solutions to handle this challenge and also achieve each of the following goals as much as possible: i) balancing the state of charge (SOC) of the EVs, ii) balancing the amount of stored power of the EVs, iii) minimizing the total power charged, and iv) minimizing the number of charges to increase the lifetime of batteries. We first formulate each power distribution scheduling problem with each of the above-stated objectives. We prove that the problems with goals i)-iii) are convex, and use the subgradient approach or greedy approach to solve the problems. We also prove that the problem with goal iv) is an NP-hard problem, and provide a greedy algorithm to solve the problem. Our extensive experiments on *MatLab* and *Simulation for Urban MObility* (SUMO) show the effectiveness of our scheduling solutions in achieving the different goals compared with other scheduling methods including first-come-first-serve and equal share. This paper is the first that handles such a power distribution scheduling problem in a WPT system.

The rest of this paper is organized as follows. Section II presents the background of the WPT system and our motivation. Section III presents the problem formulation and solution for different power distribution scheduling problems for the WPT system. Section IV presents the performance evaluation of the proposed power distribution scheduling solutions for the WPT system. Section V presents an overview of related work. Finally, Section VI concludes this paper with remarks on our future work.

II. Background and Motivation

We first introduce the architecture of the WPT system. As shown in Fig. 2, a charging lane is a portion of a road that consists of a number of charging sections. A charging section has charging coils embedded which are used for charging EVs. The maximum amount of energy that a charging section can provide to an EV is determined by the power transmission rate of the charging coil and the time that the EV spends on top of the charging section. The charging sections in each charging lane are controlled by a GSC. The GSC receives the schedule from the global charging controller (GCC) in the cloud. Based on the schedule, the GSC delivers power to each charging section at every point of time so that the EVs on the charging lane can be charged accordingly. The GSC can communicate with the vehicles and it is aware of the vehicles that are on top of its respective charging section. For a given vehicle speed, the charging lane can supply a maximum amount of energy which is the sum of the maximum amount of energy that each charging section provides for that particular velocity.

At every given time t, there are a number of vehicles within the charging lane, which form a charging vehicle set. This set changes every time that a vehicle checks in and/or checks out from the charging lane. Each vehicle sends a charging request along with its own information (e.g., SOC, location, velocity) to the GCC. Accordingly, the GCC identifies the vehicles that are on the top of each charging section at each particular time and groups them into a charging vehicle set. After deciding the charging vehicle set at each time point, the GCC uses the information from the set to decide the amount of power allocated to each vehicle at each particular time. The GCC then sends the schedule to the GSC. When a vehicle enters the charging lane, it communicates with the GSC. Thus the GSC is aware of the vehicles entering its domain and transfers power to each charging section to charge each vehicle based on the schedule.

The power capacity of a charging lane (or the GSC) is limited because a charging infrastructure with a higher power transmission rate needs a higher investment cost. The aggregated value of the maximum power transmission rates of all charging sections in a charging lance is equal to the capacity of the charging lane. We consider the WPT system state as overload when the demands of all EVs on a charging lane is higher than the power capacity of the charging lane at a particular time. That means the demands of all EVs on the charging lane cannot be satisfied at the same time. When an overload occurs, it is important for the GCC to use a power distribution scheduling method to determine the power allocated to each charging section to charge each EV at a time in order to achieve certain goals. The scheduling method can be first-come-first-serve, in which the vehicles that arrive at the charging lane earlier have higher priorities to receive their demanded power. It can also be equal share method, in which the vehicles in a charging vehicle set receive an equal portion of the total power that can be supplied by the GSC. When there is no overload, the GCC assigns the amount of power demanded by each vehicle in the charging vehicle set, regardless of the power distribution scheduling strategy selected.

In the following sections, we present different power distribution scheduling problems and their solutions with different goals.

III. Power Distribution Scheduling

WPT enables EVs to increase their driving range. However, a charging lane has limited power that can be transferred to EVs on the charging lane. The power distribution scheduling schedules how to distribute limited power to EVs in the same charging lane. In this section, we formulate different power

TABLE I
SYMBOLS AND DEFINITIONS.

Symbol	Definition
m	The number of EVs
t_i^s	The start time of EV i on the charging lane
t_i^e	The end time of EV i on the charging lane
n	The number of charging sections
c_j	The jth charging section
a_j	The maximum power that can be provided by charging section j
A	The maximum power that can be provided by the GSC
$y_i(t)$	The SOC of EV i at time t
$z_i(t)$	The amount of stored power of EV i at time t

TABLE II
EVS' PARAMETERS AND DEFINITIONS.

Parameter	Definition
$Q_{\text{batt},i}$	The battery capacity of EV i
$V_{\text{oc},i}$	The voltage source of EV i
$R_{\text{int},i}$	The internal battery resistance of EV i
$P_{\text{batt},i}$	EV i's battery power
$P_{\text{batt},i}^{\max}$	The maximum power of battery of EV i
$P_{\text{batt},i}^{\min}$	The minimum power of battery of EV i

distribution scheduling problems for different goals. First, each EV must receive sufficient power to enable it to drive to the next charging lane or its destination (when there are no charging lanes ahead). Besides this goal, we also consider each of the following goals based on different practical needs in the system design: i) balancing the state of charge (SOC) of the EVs, ii) balancing the amount of stored power of the EVs, iii) minimizing the total power charged, and iv) minimizing the number of charges to increase the lifetime of batteries.

The SOC is measured by the percentage of energy stored in the battery. The SOC for a vehicle will change when it passes along a charging lane. It will increase when the power received from a charging section is higher than the power used for vehicle propulsion. A vehicle's SOC indicates the remaining battery charge and it must be maintained within a certain limit to ensure that the vehicle will not be out of power when it runs on the road. The above-stated goal i) aims to balance the SOC of the EVs in a charging vehicle set when they leave the charging lane to achieve a certain fairness. Since different EVs have different battery capacities (i.e., sizes), rather than considering the relative percentage, we consider the absolute amount of stored power for another type of fairness. Thus, goal ii) aims to balance the absolute amount of stored power of the EVs. Because an EV does not have to receive more power than what is needed to drive to the next charging lane or its destination (when there are no charging lanes ahead), goal iii) aims to minimize the total power charged to the EVs in a charging vehicle set. More battery charges lead to shorter lifetime of the battery. Therefore, goal iv) aims to minimize the number of charges in order to increase the lifetime of batteries.

However, these problems are not trivial to solve because of the heterogeneous statuses of the EVs. That is, different EVs have different destinations, different SOCs, different batteries,

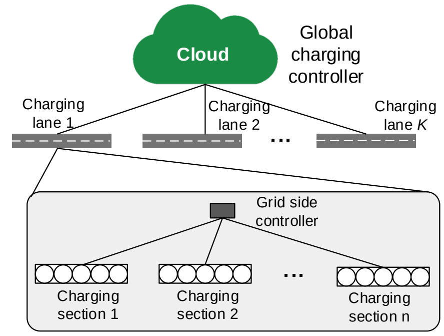


Fig. 2. The WPT system architecture.

voltage sources, and internal battery resistances when they enter a charging lane, which affect their states in terms of the above goals when they leave the charging lane. We formally formulate each problem and then find its solution. In the following, Section III-A introduces our scenario and some definitions. Then, we present each of the above problems and its solution in the subsequent sections.

A. EV Traffic Model

In this paper, we consider a WPT system in a highway scenario where vehicles follow a similar velocity. We focus on scheduling power distribution to the charging sections within one charging lane, so that the EVs in a charging vehicle set can receive power when they pass each charging section for different goals when they leave the charging lane. Suppose there are n charging sections $c_1, c_2, ..., c_n$ in a charging lane that charge m heterogenous EVs $\{1, 2, ..., m\}$ based on the EVs' current stored energy in the batteries. Here, by "heterogenous", we mean that different EVs have different EV parameters as listed in Table II. Also, we assume that the m EVs have similar velocity when traveling through this charging lane. We assume that each charging section can have at most one EV to be located at.

We consider a discrete time system where time $t = 1, 2, ...$ The start time and the end time for each EV i ($i = 1, 2, \dots, m$) on this charging lane are denoted by t_i^{s} and t_i^{e}, respectively. The maximum capacity of the GSC is A and the maximum power that each charging section j can provide is a_j, $\sum_{j=1}^{n} a_j = A$. We represent the time period that EV i is on charging section c_j by $\mathbf{t}_{i,j} = [t_{i,j}^{\text{s}}, t_{i,j}^{\text{e}}]$. We use $y_i(t)$ to represent the SOC of EV i at time t, and let $\mathbf{y}(t) = [y_1(t)\ y_2(t)\ ...\ y_m(t)]^\top$. Then, $y_i(t_i^{\text{s}})$ and $y_i(t_i^{\text{e}})$ represent the SOC of EV i when it enters and leaves the charging lane, respectively. To guarantee that each EV i can drive to the next charging lane, or its destination when there are no charging lanes ahead, EV i needs to guarantee the its SOC is higher than a value $p_{\text{req},i}$ when it leaves the current charging lane, where $p_{\text{req},i}$ is calculated based on inter-distance between the adjacent charging sections [27].

In the following sections, we formulate the problem for the power distribution scheduling with each of the aforementioned goals and then provide problem solution.

B. Balancing the SOCs of the EVs

Given the EV traffic model, in the following, we formulate a new problem, called the *SOC-Balanced Power Distribution Scheduling* problem (SOC-B). It aims to distribute the power to each charging section j in each time slot t, $x_j(t)$, to guarantee all the EVs can finish their trips and the SOCs of all the EVs are balanced when they leave the charging lane. To balance the SOCs of the EVs when they leave the charging lane, we try to minimize the variance of the SOCs [25] denoted by $y_1(t_1^{\mathrm{e}})$, $y_2(t_2^{\mathrm{e}})$, ..., $y_m(t_m^{\mathrm{e}})$. That is,

$$\min \sum_{i=1}^{m} \left(y_i(t_i^{\mathrm{e}}) - \frac{\sum_{j=1}^{m} y_j(t_j^{\mathrm{e}})}{m} \right)^2 \tag{1}$$

where each $y_i(t_i^{\mathrm{e}}) - \frac{\sum_{j=1}^{m} y_j(t_j^{\mathrm{e}})}{m}$ $(i = 1, ..., m)$ measures how far $y_i(t_i^{\mathrm{e}})$ is different from $\frac{\sum_{j=1}^{m} y_j(t_j^{\mathrm{e}})}{m}$, i.e., the mean value of $y_1(t_1^{\mathrm{e}})$, $y_2(t_2^{\mathrm{e}})$, ..., $y_m(t_m^{\mathrm{e}})$.

Equ. (1) can be also written as

$$\min\ \mathbf{y}^{\top}(t_i^{\mathrm{e}})\mathbf{N}\mathbf{N}^{\top}\mathbf{y}(t_i^{\mathrm{e}}) \tag{2}$$

where $\mathbf{N} \in \mathbb{R}^{m\times m}$ is defined by

$$\mathbf{N} = \begin{bmatrix} 1 & -\frac{1}{n} & \cdots & -\frac{1}{n} \\ -\frac{1}{n} & 1 & \cdots & -\frac{1}{n} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{n} & -\frac{1}{n} & \cdots & 1 \end{bmatrix}. \tag{3}$$

In the following, we first introduce the notations we use and then list the constraints for this scheduling problem. $P_{\mathrm{trac},i}$ means the energy consumption of EV i at each time slot. $\mathcal{T}_i$ means the set of time slots that EV i is located at a charging section. $t_{i,j}$ represents the time slot that EV i is located at charging section j. $x_j(t)$ represents the amount of power allocated to charging section j at time t. $p_{\mathrm{th},i}$ is the amount of stored power of EV i to guarantee that EV i can move to the next charging section, and $p_{\mathrm{req},i}$ is the amount of stored power of EV i to guarantee that EV i can finish its trip; that is, it can move 1) to the next charging lane if the current charging lane is not the last charging lane or 2) to the destination if the current charging lane is the last charging lane. Here, the formulas to calculate $p_{\mathrm{th},i}$ and $p_{\mathrm{req},i}$ are given in [24].

Then, the following constraints must be satisfied for the SOC-B power distribution scheduling problem among the charging sections:

$$\sum_{j=1} x_j(t) \leq A,\ \forall j, t \tag{4}$$

$$x_j(t) \leq a_j,\ \forall j, t \tag{5}$$

$$y_i(t) \geq p_{\mathrm{th},i},\ \forall t, i \tag{6}$$

$$y_i(t_i^{\mathrm{e}}) \geq p_{\mathrm{req},i},\ \forall i \tag{7}$$

$$y_i(t) \leq 1,\ \forall t, i \tag{8}$$

$$z_i(t+1) = \begin{cases} z_i(t) - P_{\mathrm{trac},i} & \text{if } t \notin \mathcal{T}_i \\ z_i(t) + x_j(t) - P_{\mathrm{trac},i} & \text{if } t = t_{i,j} \end{cases} \tag{9}$$

- Equ. (4) means that the sum of the allocated power of all the charging sections cannot exceed the maximum power provided by the GSC;
- Equ. (5) means that the power allocated to each charging section j cannot exceed the maximum power provided by charging section j;
- Equ. (6) means that the SOC of each EV i should be at least $p_{\mathrm{th},i}$ in each time point in order to guarantee that EV i can move to the next charging section;
- Equ. (7) means that when EV i leaves the charging lane at time point t_i^{e}, its SOC should be at least $p_{\mathrm{req},i}$ to guarantee that EV i can finish its trip; that is, it can move 1) to the next charging lane if the current charging lane is not the last charging lane, or 2) to the destination if the current charging lane is the last charging lane;
- Equ. (8) means that the SOC of each EV i cannot exceed 1 because the maximum SOC value is 1;
- Equ. (9) means that at time t, if EV i is not located at any charging section, then its power is reduced by $P_{\mathrm{trac},i}$, which is the energy consumption at each time slot; otherwise its power is added by $x_j(t) - P_{\mathrm{trac},i}$, which means that the power gained by EV i at time slot t equals the charged power from charging section j at time slot t minus the power consumed by EV i at time slot t.

The input of the SOC-B problem includes all the parameters of the EVs (listed in Table II) in a charging vehicle set, and the output records how much power is allocated to each charging section to charge the EV on it at each time slot from the time when the first EV enters the charging lane to the time when the last EV leaves the charging lane. In the following, we will prove that the SOC-B problem defined by Equ. (2)- (9) is a convex problem, i.e., the Hessian matrices of Equ. (2)-(9) are all nonnegative definite [21].

First, the Hessian matrix of the objective function (Equ. (2)) is given by:

$$\begin{aligned} \mathbf{H}_{f(\mathbf{y}^{\top}(t_i^{\mathrm{e}}))} &= \begin{bmatrix} \frac{\partial^2 f}{\partial y_1(t_i^{\mathrm{e}})^2} & \cdots & \frac{\partial^2 f}{\partial y_1(t_i^{\mathrm{e}})\partial y_n(t_i^{\mathrm{e}})} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial y_1(t_i^{\mathrm{e}})\partial y_n(t_i^{\mathrm{e}})} & \cdots & \frac{\partial^2 f}{\partial y_n(t_i^{\mathrm{e}})^2} \end{bmatrix} \\ &= \mathbf{N}\mathbf{N}^{\top}, \end{aligned} \tag{10}$$

which is nonnegative definite since for each $\mathbf{x} \in \mathbb{R}^m$, $\mathbf{x}^{\top}\mathbf{N}\mathbf{N}^{\top}\mathbf{x} \geq 0$ [21]. Then, we consider the constraint functions Equ. (4)-(8). They are all linear functions, which indicate that their Hessian matrices are all zero matrices and hence nonnegative definite. Now, we need to prove the convexity of Equ. (9). For this purpose, we first need to derive the quadratic form of Equ. (9), where the detailed process of the derivation is shown as follows.

According to [24], the relationship between EV i's SOC at time $t+1$ and time t is given by

$$\begin{aligned} & y_i(t+1)2R_{\mathrm{int},i}Q_{\mathrm{batt},i} \\ =\ & y_i(t)2R_{\mathrm{int},i}Q_{\mathrm{batt},i} + \sqrt{V_{\mathrm{oc},i}^2 - 4R_{\mathrm{int},i}z_i(t)} - V_{\mathrm{oc},i}, \end{aligned} \tag{11}$$

from which we can derive that

$$z_i(t) = \mathbf{w}_i^{\top}(t)J\mathbf{w}_i(t) + \mathbf{b}^{\top}\mathbf{w}_i(t), \tag{12}$$

where

$$J = \begin{bmatrix} -R_{\text{int},i}Q_{\text{batt},i}^2 & R_{\text{int},i}Q_{\text{batt},i}^2 \\ R_{\text{int},i}Q_{\text{batt},i}^2 & -R_{\text{int},i}Q_{\text{batt},i}^2 \end{bmatrix},$$

$$\mathbf{b} = \begin{bmatrix} Q_{\text{batt},i}V_{\text{oc},i} \\ -Q_{\text{batt},i}V_{\text{oc},i} \end{bmatrix} \text{ and } \mathbf{w}_i(t) = \begin{bmatrix} y_i(t) \\ y_i(t+1) \end{bmatrix}.$$

Then, based on Equ. (12), we can write Equ. (9) in its quadratic form, as shown by Equ. (13) and Equ. (15):
If $t \notin \mathcal{T}_i$,

$$\mathbf{w}_i^\top(t+1)J\mathbf{w}_i(t+1) + \mathbf{b}^\top\mathbf{w}_i(t+1) \tag{13}$$
$$= \mathbf{w}_i^\top(t)J\mathbf{w}_i(t) + \mathbf{b}^\top\mathbf{w}_i(t) - P_\text{c} \tag{14}$$

If $t \in \mathcal{T}_i$ and $t = t_{i,j}$

$$\mathbf{w}_i^\top(t+1)J\mathbf{w}_i(t+1) + \mathbf{b}^\top\mathbf{w}_i(t+1) \tag{15}$$
$$= \mathbf{w}_i^\top(t)J\mathbf{w}_i(t) + \mathbf{b}^\top\mathbf{w}_i(t) + x_j(t) - P_\text{c}. \tag{16}$$

The Hessian matrices of both Equ. (13) and Equ. (15) are J, which is nonnegative definite since for any vector $[x_1, x_2]^\top$ ($x_1, x_2 \in \mathbb{R}$), we have

$$\begin{aligned} & [x_1, x_2]J[x_1, x_2]^\top \\ = \; & R_{\text{int},i}Q_{\text{batt},i}^2x_1^2 - 2R_{\text{int},i}Q_{\text{batt},i}^2x_1x_2 + R_{\text{int},i}Q_{\text{batt},i}^2x_2^2 \\ = \; & R_{\text{int},i}Q_{\text{batt},i}^2\,(x_1 - x_2)^2 \geq 0. \end{aligned} \tag{17}$$

Algorithm 1: The subgradient method for the SOC-B and the Power-B problems.

```
input  : Parameters of all the EVs;
output: v*;
// Power allocation of all charging
   sections
1  Select a starting solution: v^(1) = 0;
2  Put k = 1;
3  // Main step
4  repeat
5      ξ^(k)(v^(k)) = ∇f(v^(k));
6      if ξ^(k)(v^(k)) = 0 then
7          Break;
8      else
9          Select a step size α^(k) > 0 and compute
           v^(k+1) ← v^(k) + α^(k) ξ^(k)(v^(k));
10     k = k + 1;
11 until ξ^(k)(v^(k)) < ε;
12 v* ← v^(k);
13 return v*;
```

Accordingly, the SOC-B problem defined by Equ. (2)-(9) is a convex problem, which can be directly solved by the subgradient method [21]. Algorithm 1 shows the pseudocode of the subgradient method. Here, $\mathbf{v}$ is equivalent to $\mathbf{w}$ in SOC-B. $\mathbf{v}^{(k)}$ is the kth iterate, $\xi^{(k)}(\mathbf{v}^{(k)})$ is the subgradient of f at $\mathbf{v}^{(k)}$, and $\alpha^{(k)} > 0$ is the kth step size. At the beginning, we initiate $\mathbf{v}^{(1)}$ by $\mathbf{0}$ and set k by 1 (lines 1–2). Then, in each iteration, we first derive the subgradient of $f(\mathbf{v}^{(k)})$ (line 4). If $\xi^{(k)}(\mathbf{v}^{(k)}) = 0$, which indicates that $\mathbf{v}^{(k)}$ is the global optimal solution, the algorithm is finished; otherwise, we take a step in the direction of a negative subgradient $\alpha^{(k)}\xi^{(k)}(\mathbf{v}^{(k)})$. The above process is repeated until $\xi^{(k)}(\mathbf{v}^{(k)}) < \epsilon$, i.e., the subgradient is close to 0.

C. Balancing the Amount of the Stored Power of the EVs

In Section III-B, we mainly consider the SOC of EVs for fair power distribution. Since SOC reflects the percentage of power stored in the battery while the EVs' batteries have different capacities (i.e., sizes), balancing the absolute amount of stored power becomes an alternative for fairer power distribution. As a solution, we formulate the power distribution scheduling problem by taking into account the absolute stored power of each EV. In particular, we formulate another new problem, called the *Power-Balanced Power Distribution Scheduling* problem (Power-B). In this problem, we still need to guarantee that each EV can finish its trip. That is, each EV 1) has enough power to move to the next charging section within the charging lane, and 2) has enough power to reach the next charging lane or its destination if there are no charging lanes ahead when it leaves the charging lane. The difference between SOC-B and Power-B is that, in Power-B, we aim to balance the absolute amount of stored power of all the EVs when the EVs leave the charging lane.

Before introducing the problem, we first modify the three constraints for each EV to finish its trip in SOC-B, i.e., Equ. (6)-(8), to:

$$\text{s.t.} \qquad z_i(t) \geq p'_{\text{th},i}, \; \forall t, i \tag{18}$$
$$z_i(t_i^\text{e}) \geq p'_{\text{req},i}, \; \forall i \tag{19}$$
$$z_i(t_i^\text{e}) \leq P_{\text{batt},i}, \; \forall i. \tag{20}$$

Here, $p'_{\text{th},i}$ is the amount of EV i's stored power in each time slot to guarantee that EV can move to the next charging section, and it is calculated by:

$$p'_{\text{th},i} = 0.5\rho_{\text{air},i}C_{\text{d},i}v_i^2\Delta_{\max} \tag{21}$$

where v_i, $\rho_{\text{air},i}$, $C_{\text{d},i}$, and $\Delta_{\max}$ represent the *velocity*, the *air density*, *the drag coefficient* of EV i, and the maximum inter-distance between adjacent charging sections, respectively [24]. $p'_{\text{req},i}$ is the amount of EV i's stored power in each time slot to guarantee that EV i can move 1) to the next charging lane if the current charging lane is not the last charging lane, or 2) to the destination if the current charging lane is the last charging lane. $p'_{\text{req},i}$ is calculated by:

$$p'_{\text{th},i} = 0.5\rho_{\text{air},i}C_{\text{d},i}v_i^2\Delta_{\text{next}} \tag{22}$$

where Δ_{next} represents the distance to the next charging section or the destination [24].

In Power-B, the objective function is to minimize the variance of the absolute amount of stored power of all the EVs when the EVs leave the charging lane. That is,

$$\min \; \sum_{i=1}^{m}\left(z_i(t_i^\text{e}) - \frac{\sum_{j=1}^{m} z_j(t_j^\text{e})}{m}\right)^2 \tag{23}$$

where each $z_i(t_i^{\mathrm{e}}) - \frac{\sum_{j=1}^{m} z_j(t_j^{\mathrm{e}})}{m}$ $(i = 1, ..., m)$ measures how far $z_i(t_i^{\mathrm{e}})$ is different from $\frac{\sum_{j=1}^{m} z_j(t_j^{\mathrm{e}})}{m}$, i.e., the mean value of $z_1(t_1^{\mathrm{e}})$, $z_2(t_2^{\mathrm{e}})$, ..., $z_m(t_m^{\mathrm{e}})$. Equ. (23) can be also written in a quadratic form:

$$\min\ \mathbf{z}^{\top}(t_i^{\mathrm{e}})\mathbf{N}\mathbf{N}^{\top}\mathbf{z}(t_i^{\mathrm{e}}) \tag{24}$$

where $\mathbf{N}$ is given by Equ. (3). In addition, the constraints Equ. (4), (5), (9), (18)-(20) need to be satisfied. Since the objective function (Equ. (24)) is a quadratic function and all the constraints are linear functions, Power-B is a quadratic programming problem, which is a convex problem and hence can be directly solved using the subgradient method [21].

D. Minimizing the Total Power Charged

In the above, we focus on balancing the SOCs or the amount of stored power of EVs for fair power distribution. In this section, we discuss another alternative of power distribution scheduling. Since the power demands of charging EVs impose a high load on the power grid during peak hours, it is desirable to minimize the amount of energy received by each EV so that the power grid can satisfy the energy demands of as more EVs as possible, especially during the peak hours. As an alternative solution, we formulate the power distribution scheduling problem by taking into account the minimization of the amount of charged power of each EV while enabling the EVs to arrive at the next charging lane or their destinations. We consider the problem as the *Power Minimization Power Distribution Scheduling* problem (Power-M). We again use the three constraints for each EV to finish its trip in SOC-B, i.e., Equ. (18)-(20). The difference is that besides satisfying these constraints in Power-M, we aim to minimize the total power charged by all the charging sections in the charging lane, which is represented by:

$$\min \sum_{i=1}^{m}\sum_{t=1}^{T} x_i(t) \tag{25}$$

where $\sum_{t=1}^{T} x_i(t)$ means the amount of charged power of EV i in the whole process and hence $\sum_{i=1}^{m}\sum_{t=1}^{T} x_i(t)$ represents the amount of the total charged power of all the EVs.

Similar to Power-B, the constraints Equ. (4), (5), (9), (18)-(20) need to be satisfied. Since the objective function and all the constraints are linear, Power-M can be directly solved by the simplex method [21], which is a standard method of minimizing problem with a linear objective function and constraints. The basic idea of the simplex method is to explore the extreme points of the feasible region constructed by the linear constraints to find the optimal extreme point that minimizes the objective function. To further increase the time efficiency, in the following part, we will devise a greedy algorithm that has lower time complexity than the simplex method.

The basic idea of the greedy algorithm is to minimize the power charged for each EV in each time slot. Algorithm 2 shows the pseudocode of this greedy algorithm. More specifically, given a charging section j and an EV on it, say EV i, if charging section j is the last charging section in the current charging lane, then the power that charging section j provides to EV i should enable it to move to the next charging lane or its destination. That is, we should guarantee that the power of EV i is at least $p'_{\mathrm{req},i}$ when it leaves the charging lane (line 3). Otherwise, charging section j only needs to provide EV i with the power to enable EV i to move to the next charging section. That is, we need to guarantee that EV i's power is at least $p'_{\mathrm{th},i}$ after the EV is charged (line 5). Here, we assume that GSC has enough power to enable each EV i to leave the charging lane with power $p'_{\mathrm{req},i}$.

Algorithm 2: Greedy algorithm for the Power-M problem.

1 **for** *each charging section j at time slot t* **do**
2   **if** *charging section j is the last charging section* **then**
3   Charge each EV i with power
$$x_j(t) = \max\{p'_{\mathrm{req},i} + P^{j}_{\mathrm{trac},i} - z_i(t), 0\}$$
4   **else**
5   Charge each EV i with power
$$x_j(t) = \max\{p'_{\mathrm{th},i} + P^{j}_{\mathrm{trac},i} - z_i(t), 0\}$$

In the following, we will prove the optimality of the solution obtained from the greedy algorithm in Theorem 3.1. We first give Lemma 3.1 for the aid of the proof of Theorem 3.1.

Lemma 3.1: The total energy charged will not be lower than $\sum_i \left(T \cdot P_{\mathrm{trac},i} + p'_{\mathrm{req},i} - z_i(t_i^{\mathrm{s}})\right)$.

Proof: When each EV i enters the charging lane, the total energy required to reach the destination can be easily calculated: $P_{\mathrm{total},i} = T \cdot P_{\mathrm{trac},i} + p'_{\mathrm{req},i}$. Then, to guarantee that each EV i can reach its destination, the following condition needs to be satisfied:

$$\sum_{t=1}^{T} x_i(t) + z_i(t_i^{\mathrm{s}}) \geq P_{\mathrm{total},i} \tag{26}$$

where $z_i(t_i^{\mathrm{s}})$ means the energy stored in EV i enters the charging lane and $\sum_{t=1}^{T} x_i(t)$ means the total energy charged on the charging lane. Accordingly, we can derive that the total power charged by all the charging sections in the charging lane is lower bounded:

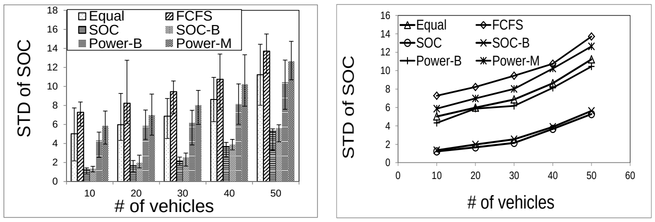
$$\sum_{t=1}^{T}\sum_{i=1}^{m} x_i(t) \geq \sum_{i=1}^{m} \left(P_{\mathrm{total},i} - z_i(t_i^{\mathrm{s}})\right) \tag{27}$$

$$= \sum_{i=1}^{m} \left(T P_{\mathrm{trac},i} + p'_{\mathrm{req},i} - z_i(t_i^{\mathrm{s}})\right). \tag{28}$$

The proof is completed. ■

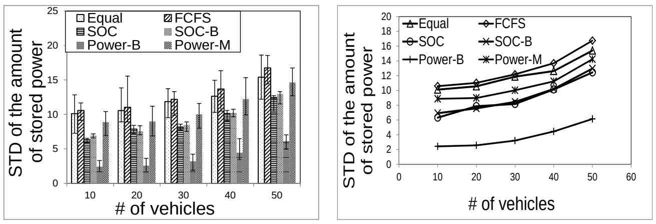
Theorem 3.1: The greedy algorithm can achieve the optimal solution.

Proof: We sum the amount of power charged by all the



(a) Standard deviation with error bars (b) The average standard deviation

Fig. 3. Comparison performance of balancing the SOCs of the EVs.



(a) Standard deviation of the amount of stored power with error bars (b) The average standard deviation of the amount of stored power

Fig. 4. Comparison performance of balancing the amount of the stored power of the EVs.

TABLE III
EV PARAMETERS - NISSAN LEAF

Parameters	Value
Air density ρ_{air}	1.225 kg/m^3
Drag Coefficient C_d	1
EV frontal area A_f	0.725 m^2
EV Mass M	1521 kg
Rolling Ressistace Coefficient C_r	0.02
Transmission Efficiency η_{tx}	0.98
Gearbox Efficiency η_{GB}	0.98
Electric Motor Efficiency η_{EM}	0.99
Open circuit voltage $V_{\rm oc}$	364.8 V
Battery Capacity $Q_{\rm batt}$	66.2 Ah
Battery internal resistance $R_{\rm int}$	0.01 Ω
Battery Nominal Power $P_{\rm batt}^{\rm max}$	24 kW

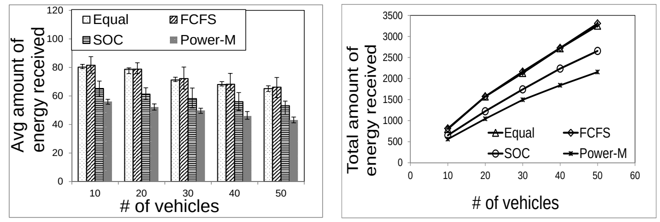
charging sections in the charging lane:

$$\sum_{j=1}^{n}\sum_{t=1}^{T} x_j(t) = \sum_{i=1}^{m} \left(T \cdot P_{\mathrm{trac},i} + p'_{\mathrm{req},i} - z_i(t_i^{\mathrm{s}})\right). \qquad (29)$$

According to Lemma 3.1, there is no solution with lower total amount of charged power than the greedy algorithm. Consequently, we can derive that the solution derived by the greedy algorithm is the optimal solution. ■

IV. PERFORMANCE EVALUATION

Experimental Settings. In this section, we evaluate the performance of our proposed power distribution scheduling solutions. Here, we used MatLab to get the solution of our power scheduling optimization problem, and then used SUMO to apply the solution in the realistic traffic scenario. For the simulation, we varied the number of EVs from 10 to 50 and set the number of charging sections to 10. We randomly selected a value in [0.4,0.8] as the SOC for each EV when it enters the charging lane. We randomly generated the SOC value required for each EV to arrive at its destination. Three types of EVs were considered (Nissan Leaf, Toyota Prius, and Chevy Volt) in our experiment [1], [2], [4]. Table III shows the parameters of EVs and We set the charging section length L, the maximum capacity A, and the coil maximum power C by 200m, 600kW, and 100kW, respectively. We repeat each experiment for 20 times. In each experiment, the power capacity of the GSC was randomly chosen from [40-100]Kw. Unless otherwise specified, the experimental result is the average of the 20 experiments. We compared our solutions with the following power distribution methods:



(a) The average amount of energy received per EV with error bars (b) Total amount of energy received by all EVs

Fig. 5. Comparison performance of minimizing the total power charged.

Equal sharing method. In the equal sharing method (denoted by Equal), suppose there are m vehicles on top of a charging lane at a particular time, the amount of power that each EV is scheduled to receive is A/m where A is the power capacity of that charging lane.

First come first serve method. In the First Come First Serve power distribution method (denoted by FCFS), the EVs are assigned to receive power in the order that they arrive at each charging section. Each EV's battery is fully charged until all power in the charging lane is transferred.

State of charge method. The state of charge method (denoted by SOC) only tries to balance the SOCs of the EVs when they leave the charging lane without considering any other factors (e.g., enough power to arrive at destinations, priorities, etc).

A. Experimental Results

Balancing the SOCs of the EVs. In this experiment, we calculated the standard deviation of SOCs of the EVs for the six power distribution methods: Equal, FCFS, SOC, SOC-B, Power-B, and Power-M. Fig. 3(a) shows the median, the 5th and the 95th percentiles of the standard deviation of SOCs when different number of EVs are considered in the power distribution methods. We see that the median and the variance of the standard deviation of SOC follow SOC≈SOC-B<Power-B<Equal<Power-M<FCFS. The result means that SOC-B and SOC can balance EVs' SOC better than other methods. SOC-B aims to balance the SOC levels of EVs and also guarantee that the EVs can arrive at their destinations when they leave the charging lane. As a result, the deviation of SOC-B is always small. The SOC power distribution method only considers balancing SOC without considering the EVs'

destinations. Thus, though its resultant SOC variance is also small, this method cannot guarantee that each EV can arrive at its destination with sufficient power. Our Power-B method considers balancing EVs' absolute stored power and also their destinations. As a result, its deviation of SOC is larger than SOC and SOC-B but smaller than other methods. The Equal Share power distribution method only considers the equal distribution of energy and it does not pay attention to SOC. Thus, its deviation is moderate compared to other method. Our proposed Power-M power distribution method aims to minimize the power distribution which causes larger deviation than our proposed SOC-B and Power-B methods. Finally, the FCFS power distribution method distributes power based on EV's arrival time and its SOC deviation is large. Fig. 3(b) shows the standard deviations of the six methods. We can also make the same observations as Fig. 3(a) due to the same reasons. The results confirm that our SOC-B solution can achieve its goal of balancing the SOCs and also guarantee that the EVs can arrive at their destinations with enough power.

Balancing the Amount of the Stored Power of the EVs. In this experiment, we calculated the standard deviation of the amount of the stored power for the six power distribution methods: Equal, FCFS, SOC, SOC-B, Power-B, and Power-M. Fig. 4(a) shows the median, the 5th, and the 95th percentiles of standard deviation of the amount of the stored power when different number of EVs are considered in the power distribution scheme. We see that the median and the variance of the standard deviation of the stored power follow Power-B<SOC≈SOC-B<Power-M<Equal<FCFS. It means that our proposed Power-B power distribution method can balance EVs' stored power better than other methods. The Power-B power distribution method considers balancing the stored power levels of EVs and their destinations when they leave the charging lane. Thus, the deviation of the stored power in Power-B is always small. Since the Equal share power distribution method and FCFS power distribution method do not consider to balance the stored power, their variance are large comparing with other methods. The SOC method and SOC-B method consider balancing the SOCs of EVs. Thus, their deviations of stored power are moderate compared to other methods. Fig. 4(b) shows the standard deviations of the six methods. We can find that the standard deviation of our Power-B is lower than the standard deviations of other methods because of the reasons described above. We can also make the same observations as Fig. 4(a) due to the same reasons.

Minimizing the Total Power Charged. In order to show the effectiveness of our proposed Power-M solution in minimizing the total power charged of the EVs, we compared the performance of Power-M with Equal share, FCFS, SOC. Fig. 5(a) shows the median, the 5th, and the 95th percentiles of the average energy received per EV when different number of EVs are considered in the power distribution methods. We can see that the median and the variance of energy received per EV follow Power-M<SOC<Equal≈FCFS. The result means that our proposed Power-M solution can minimize the energy received per EV than other three methods. Power-M considers minimizing total power charged and the constraints to guarantee that the EVs can finish their trips at the same time. Thus, all EVs receive less amount of energy. The SOC power distribution method only considers balancing SOC without taking into account of the amount of received energy for EVs. In SOC method, the amount of energy received per EV is higher than the energy received in Power-M. The Equal share and FCFS power distribution methods distribute all available energy among EVs. Thus, the levels of energy received by EVs in Equal share and FCFS are higher than other methods. Fig. 5(b) shows the total energy received by all the EVs of the four methods. We also find that the total energy received follows Power-M<SOC<Equal≈FCFS due to the same reasons mentioned above. The results confirm the effectiveness of our Power-M in minimizing the total power charged.

V. Related Work

The implementation of efficient WPT systems and EVs techniques are critical to improve energy efficiency and safety of in-motion EVs. Several works have studied on the WPT systems and EVs techniques. For example, Li *et al.* [20] presented an analytic study of the existing technologies in the WPT system applicable to EV wireless charging. Similarly, Lukic *et al.* [22] presented the background study, motivations, an overview of different charging components of EVs. Hori *et al.* [17] discussed different types of EVs with their future impacts, and briefly discussed different EVs' components with several technical and research challenges based on the existing technologies. Onar *et al.* [23] examined the technical aspects and charging topology of in-motion wireless power charging of EVs. The authors discussed several factors of power transfer procedures by considering highway surfacing materials and presented an overview of WPT magnetic field measurements.

There are also many efforts devoted to the design and implementation of the WPT systems for EVs in recent years. For example, Shin *et al.* [26] presented the design of an optimized core structure and electric components to implement the WPT system for moving EVs [6], [9], [10]. Yilmaz *et al.* [29] presented the general design requirements and analysis of WPT systems for online EVs. They presented and analyzed three different generic roadbed structures: 1) based on a long wire loop, 2) based on sectioned wire loops, and 3) based on spaced loops. Ahn *et al.* [7] presented the design methodologies and the reduction of electromagnetic fields for high-efficiency WPT systems. The authors also suggested power pickup coils with optimized design parameters. They also proposed passive and active plate shields to minimize the leakage electromagnetic field from the WPT system in online EVs. Lee *et al.* [19] presented a dynamic model for identifying the maximum pickup in the WPT system of the online EVs. In their model, a simple "second-order inductive circuit" is obtained by applying the Laplace phasor transformation to the first order WPT system. The discovered pickup current during the transient state is found to be relatively unchanged for various load resistances. Power pulsations is another major technical challenge for the WPT systems of online EVs.

The battery size and the positions of power transmitters on the road are another major issues in the WPT system design for online EVs. The study by Ko and Jang [18] presents an online charging EV system and discuss these major issues. It tries to minimize the infrastructure setup cost by using an optimization model, where the battery size and the number of power coils with their allocations are used as decision variables. Then, the solution is achieved using particle swamp optimization technique. However, this work only considers static number of vehicles. Instead, we consider how a power grid controller distributes power so that the heterogeneous EVs have enough power to reach their destinations with different goals.

VI. Conclusion

In a WPT system, a number of in-motion EVs on a charging lane are simultaneously charged by a GSC. Because the power capacity of the GSC is limited, the power demands from all these EVs may not be fully satisfied. In this paper, we studied the power distribution scheduling problem about how a GSC distributes power to enable the EVs to receive enough power to reach their destinations and meanwhile achieve a goal. In particular, considering the fairness among EVs, we formally formulated two problems, called SOC-B and Power-B, with the goals to balance the EVs' SOC and stored power, respectively. We showed that SOC-B and Power-B are convex problems, which can be directly solved using an existing method, e.g., the subgradient method. Also, we formulated a problem, called Power-M, to minimize the total power charged to all the EVs, and also designed a greedy algorithm that achieves the optimal solution for this problem. We have conducted extensive experimental study on our problem solutions. Our experimental results confirm that our solutions are effective in achieving their respective goals while enabling EVs to reach their destinations. Currently, we assume that the EVs follow similar velocity in the highway scenario. In our future work, we will consider different velocities and velocity variation of vehicles in general roads.

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