

Homework #4: Decidability and Turing Machines
Due Date: Tuesday, November 2, 2006

Problem 1. [15 points]

Show that there is no program for solving the following problem: Given a program Q and an input y , during the execution of Q on y do two variables A and B ever take on the same value?

Problem 2. [15 points]

Consider the deterministic Turing machine M given by:

$$M = (\{q_0, q_1, q_2\}, \{a, b\}, \{a, b, B\}, \delta, q_0, B, \{q_2\})$$

which has exactly four transitions defined:

1. $\delta(q_0, a) = (q_0, B, R)$
2. $\delta(q_0, b) = (q_1, B, R)$
3. $\delta(q_1, b) = (q_1, B, R)$
4. $\delta(q_1, B) = (q_2, B, R)$

- (a) specify the execution trace of m on the two input strings abb and bb . give the complete sequence of id's for both inputs.
- (b) provide a regular expression for $L(m)$.
- (c) suppose we added the following transition to the above machine:

$$\delta(q_1, a) = (q_0, b, r).$$

provide a regular expression for the language of the resulting turing machine.

Problem 3. [20 points]

Design a Turing machine that takes as input a number N and adds 1 to it in binary. To be precise, the tape initially contains a \$ followed by N in binary. The tape head is initially scanning the \$ in state q_0 . Your TM should halt with $N+1$, in binary, on its tape, scanning the leftmost symbol of $N+1$, in state q_f . You may destroy the \$ in creating $N+1$, if necessary. For instance, $q_0\$10011 \vdash^* q_f10100$, and $q_0\$11111 \vdash^* q_f100000$.

- (a) Give the transitions of your Turing machine, and explain the purpose of each state.
- (b) Show the sequence of ID's of your TM when given input \$111.

Problem 4. [20 points]

Consider the nondeterministic Turing machine

$$M = (\{q_0, q_1, q_2, q_f\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, \{q_f\}).$$

Informally but clearly describe the language $L(M)$ if δ consists of the following sets of rules:

1. $\delta(q_0, 0) = \{(q_0, 1, R), (q_1, 1, R)\}$
2. $\delta(q_1, 1) = \{(q_2, 0, L)\}$
3. $\delta(q_2, 1) = \{(q_0, 1, R)\}$
4. $\delta(q_1, B) = \{(q_f, B, R)\}$