Optical properties (bidirectional reflectance distribution function) of shot fabric

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To study the optical properties of materials, one needs a complete set of the angular distribution functions of surface scattering from the materials. Here we present a convenient method for collecting a large set of bidirectional reflectance distribution function (BRDF) samples in the hemispherical scattering space. Material samples are wrapped around a right-circular cylinder and irradiated by a parallel light source, and the scattered radiance is collected by a digital camera. We tilted the cylinder around its center to collect the BRDF samples outside the plane of incidence. This method can be used with materials that have isotropic and anisotropic scattering properties. We demonstrate this method in a detailed investigation of shot fabrics. The warps and the fillings of shot fabrics are dyed different colors so that the fabric appears to change color at different viewing angles. These color-changing characteristics are found to be related to the physical and geometrical structure of shot fabric. Our study reveals that the color-changing property of shot fabrics is due mainly to an occlusion effect. © 2000 Optical Society of America

OCIS codes: 100.2960, 160.1190, 160.4760, 290.1350, 290.5820, 290.5880.

1. Introduction

An accepted method for investigating the optical properties of materials is to analyze the bidirectional reflectance distribution function (BRDF). The material sample is illuminated from a known source, and the radiance of the scattered beam is measured in different directions. In most BRDF measurements where the sample is attached to a flat surface [see Fig. 1(a)], one data point at a time is obtained for every position and direction arrangement of sample, source, and detector [see Fig. 1(b)]. Collecting sufficient BRDF samplings for analysis is a tedious process. Automation has been introduced to facilitate the data collection, but we still obtain only one sampling from one configuration of the setup. In our previous study of velvet we introduced a novel, to our knowledge, technique that employed a cylindrical setup to collect BRDF data in the plane of incidence in a single measurement. Samples of velvet were wrapped around a right-circular cylinder [see Fig. 1(c)]. The curvature of the cylindrical surface ensures continuous orientation of the surface normal at points of interest. One scan of samples on a cylinder produces a stream of BRDF data points, the number of which is inversely proportional to the angular spacing of points of interest [see Fig. 1(d)]. For an investigation of anisotropic materials, BRDF’s for different orientations of the sample are needed. The common method is to rotate the sample on the sample holder. We cut strips of samples in different directions and wrapped them around a cylinder [see Fig. 1(e)]. In this way BRDF’s on planes of different orientations can be measured simultaneously [see Fig. 1(f)]. The efficiency of data acquisition is remarkably improved in comparison with the setup of samples on a flat surface.

In our previous study of the optical properties of velvet the BRDF was investigated only in the plane of incidence, the cylinder being placed upright in front of the camera and the light source. In this study we tilt the cylinder about the midpoint of its axis. In this way BRDF’s outside the plane of incidence can be measured in the scattering space. A full BRDF measurement not only contributes to the accurate description of the scattering properties of material surfaces but also allows analytical scattering models to be evaluated. In BRDF studies of different materials this setup provides a simple and convenient method for dealing with any sample that is flexible enough to be wrapped around a cylinder. One might find more scattering directions from a sphere, but it is...
The shot fabrics are usually produced by a plain weave, a type of weave that shows the cangiante effect clearly, and occasionally by twill or satin weave styles.

Shot fabrics have been depicted by many artists since the beginning of the Italian Renaissance, particularly by the Florentines. In paintings of draped clothing on human bodies the artists traditionally used patches of light and shadow to convey the contour and shape of the wearer. If the fabrics are made of shot silk, then they can be created with numerous varieties of contrasting colors of constituent yarns ranging from quite subdued and harmonious to rather garish cangiante effects. Even without extreme chiaroscuro the paintings are much brighter. This technique seemed to be innovative and soon gained in popularity. The garments of personages are painted with a multitude of different color patches.

Color-changeable fabrics are an artificial human creation. There are a number of natural phenomena that have been compared with shot fabrics, e.g., a dove’s neck and a mallard’s neck, a peacock’s tail, and wings of certain butterflies and insects. However, the color-changing mechanism of natural phenomena is due mainly to diffraction; the mechanism of shot fabrics is quite different. We hope that our study will contribute to a deeper understanding of reflectance properties of anisotropic materials such as shot fabrics.

The light scattered from the surface of a shot fabric made of silk is systematically investigated here as a function of several parameters, including the angle of incidence of the irradiating beam, the direction of the scattered beam, and the radiance and color of the scattered beam. We measured the BRDF in the RGB (red–green–blue) channels of a CCD camera. BRDF’s in the plane of incidence are evaluated, and the color-changing property is related to the geometrical structure of shot fabrics. The sampling of the reflectance (BRDF) of the shot fabric in the hemispherical scattering space is irregular, owing to the method. However, this is not a major problem, since an interpolation scheme can be used to represent the data in a coherent manner.


A. Apparatus Schematic and Coordinate Systems

In this study we use a cylindrical setup to collect BRDF data. A schematic diagram of the experimental apparatus is shown in Fig. 2(a). Samples of the shot fabric are wrapped around a right-circular cylinder and irradiated by a wide, uniform, and parallel beam directed toward the center of the cylinder, and the scattered light is detected by a digital camera located at a preselected position. When the cylinder is placed perpendicular to the plane of the incident and the scattered beams, BRDF’s in the plane of incidence are collected. When the cylinder is tilted around its center, BRDF’s outside the plane of incidence can be measured. During the course of the
measurement the digital camera is fixed, whereas the position of the source is rotated clockwise around the cylinder, and the cylinder is tilted around its center. The parameter \( \alpha \) denotes how much the \( Z' \) has tilted from the \( Z \) axis in the \( Y-Z \) plane. The parameter \( \beta \) is the angle between scattered and incident beams in the \( X-Y \) plane. The parameter \( \gamma \) is the angle of the point of interest on the cylindrical surface measured in the cross-sectional plane perpendicular to the \( Z' \) axis. The angle of incidence, \( \theta_i \), and the viewing angle, \( \theta_r \), are expressed as functions of \( \alpha \), \( \beta \), and \( \gamma \). Proportions are not drawn to scale.

The orientations of source and detector with regard to the sample are described in Fig. 2(b). The parameter \( \alpha \) is the angle by which the axis of the cylinder, denoted by \( Z' \), is tilted from the \( Z \) axis in the \( Y-Z \) plane; \( \alpha \) ranges from \(-60^\circ\) to \(60^\circ\), each increment being \( 30^\circ \). Angles measured from the \( Z \) axis toward the \( Y \) axis are positive. The parameter \( \beta \) denotes the angle between the incident beam from the light source and the scattered beam to the camera in the \( X-Y \) plane; \( \beta \) is varied from \( 10^\circ \) to \( 160^\circ \), each increment being \( 10^\circ \). The \( Y \) axis is positive. The parameter \( \gamma \) denotes the angle of the point of interest on the cylindrical surface measured in the cross-sectional plane perpendicular to the \( Z' \) axis. The origin of the angle \( \gamma \) lies on the \( Z' \) axis and extends from a designated \( 0^\circ \) reference point (parallel to the direction of negative \( X \) axis) to the point of interest on the cylindrical surface. As the light source is rotated clockwise around the cylinder, the illuminated part of the cylindrical surface seen from the position of the camera becomes narrower, and its angular range varies between \( \beta \) and \( 180^\circ \). In the analysis that is needed for extracting BRDF information, a limited range of \([\beta + 10^\circ, 160^\circ]\) is chosen for \( \gamma \), and the angular spacing between the points of interest is \( 5^\circ \). At each configuration the digital camera
scans a picture of the cylindrical surface. Consequently, in all we obtain \(16 \times 5\) images of shot fabric. The BRDF can be obtained from the images by means of data processing.

For every point of interest on the cylindrical surface, the angles of incidence, \(\theta_i\) and \(\phi_i\), and the viewing angles, \(\theta_v\) and \(\phi_v\), can be found as a function of \(\alpha\), \(\beta\), and \(\gamma\) as follows:

\[
\cos \theta_i = \cos \alpha \cos \beta \sin \gamma - \sin \beta \cos \gamma, \quad (1)
\]

\[
\cos \phi_i = \frac{\cos \alpha \cos \beta \cos \gamma + \sin \beta \sin \gamma}{\sin \theta_i}, \quad (2)
\]

\[
\sin \phi_i = \frac{\sin \beta \sin \alpha}{\sin \theta_i}, \quad (2)
\]

\[
\cos \theta_v = \frac{\cos \alpha \cos \gamma}{(1 - \cos^2 \alpha \sin^2 \gamma)^{1/2}}, \quad (3)
\]

\[
\sin \phi_v = \frac{\sin \alpha}{(1 - \cos^2 \alpha \sin^2 \gamma)^{1/2}}. \quad (4)
\]

The range of variable \(\alpha\) is \([-60^\circ, 60^\circ]\), \(\beta\) is \([10^\circ, 160^\circ]\), and that of \(\gamma\) is \([\beta + 10^\circ, 160^\circ]\).

To study the changeable optical effect of shot fabric, we cut six strips from the fabric, with a rotational angle separation of \(30^\circ\), and measured the radiance [see Fig. 3(a)]. This introduces six variations into the \(\phi_i\) and \(\phi_v\) values. Figure 3(b) shows an image of six strips of shot fabric wrapped around a cylinder. Strips are placed in reverse order from top to bottom, starting with strip 6. In this example the angle \(\beta\) between irradiating and scattering beams is \(20^\circ\), and the cylinder is rotated \(\alpha = 30^\circ\) around its axis toward the camera. The six strips are visibly distinct in the image, and they show different patterns of light and shadow. This demonstrates that the orientation does indeed influence our visual perception of the shot fabric.

B. Details of Apparatus and Data Processing

The setup for this study is as shown in Fig. 2(a). The source, the CCD camera, and the sample are kept at the same height and placed in a studio that has dark coverings on the floor, walls, and ceiling to keep reflection to a minimum. The sample of shot fabric is wrapped around a right-circular cylinder with a diameter of \(6.3\) cm. The cylinder is painted in black, so it does not affect the reflectance properties of the shot fabric. The cylinder can be tilted around a fixed position. The source is a profile spotlight containing a 12-V 75-W tungsten halogen bulb. Its beam is focused by a lens into parallel rays. Light scattered from the shot fabric is collected by a 1:2.8/180-mm Nikon objective and focused on the CCD chip of the digital camera. The images of shot fabrics around the cylinder are recorded by the digital camera in both the red and the green channels with the same exposure time.

In the radiance measurement a Lumina digital scanning camera, a model from Leaf Systems, Inc., is used as the detector device. The camera has an infrared-blocking filter that covers the focal plane. The linear CCD consists of three columns of light-sensitive cells, each 28.5 mm long and covered with a different color filter intended to capture a different portion of the visible light spectrum. The image data, consisting of three filtered columns (red, green, and blue), are then transferred to the computer by the Lumina software. The Lumina camera has 12 bits of resolution for each color plane. However, the 12-bit values are mapped to a single 8-bit representation in the radiance measurement, namely, the numerical values (red, green, and blue) for each pixel range from 0 (black) to 255 (white). The spatial uniformity of the response of the CCD camera is rather stable.

After the radiance measurement the pixel values are converted to radiance values with a postprocessing calibration and segmentation scheme. During data processing, each pixel value is taken to be an average of three color values. The calibration scheme requires several radiance measurements with a spectroradiometer (Model PR-704/PR-714 Photo Research) to convert the pixel values of the digitized images to radiance values. The luminosity of the source fluctuates slightly with the temperature of the light bulb; thus it is necessary to perform the calibration for every acquired image. In addition, a white reflection standard is used for absolute calibration.

In the segmentation scheme the image of the shot fabric wrapped around the cylinder is read in Mathematica. Streams of points of interest, shown as white dots in Fig. 3(b), are selected on the side of the cylinder for all strips. Subsequently, the radiance values of a sample window of \(3 \times 20\) pixels inside an individual dot are averaged; in this way a radiance value for every point of interest on the cylindrical surface is obtained. The sample area is parallel to the axis of the cylinder, so all the pixels in the window have the same angles of incidence and reflectance. Obviously, as the point of interest on the cylindrical surface shifts, the sample area changes in accordance with the cosine law. The BRDF is defined as the ratio of radiance of the scattered beam to irradiance that is due to the incident beam. In our experimental setup the cylinder is placed in the parallel incident beam. The intensity of the reflected beam is linearly proportional to BRDF values.

The accuracy of the experimental data is influenced by the fluctuation of the light source, the spatial uniformity of the CCD camera, the conversion of the digitized images to pixel values, and the calibration operations. The uncertainties are not presented here, since they are rather insignificant in the attempt to understand the light interaction with this color-changing material from the study of the reflectance distribution.
C. Shot Fabric Specimen

The shot fabric used in this study consists of opaque silk filament with green warps and red fillings. Close inspection reveals that the fabric is woven in the plain style with threads crossing one another. Each thread is made up of a tight bundle of silk fibers. The red threads are 0.11 mm thick, and the green ones are 0.06 mm thick. There are fewer red threads than green ones. Within 1 mm there are four red threads and five green threads; the separation distance between two parallel threads is 0.14 mm wide. If we simplify the surface structure of the shot fabric as perpendicularly interwoven sine waves, then the amplitude-to-wavelength ratio of the green thread is 0.23, and that of the red thread is 0.35.

There are obvious visual changes in radiance and color as the shot fabric is rotated under a directed beam of white light. At different angles of viewing, one color or the other predominates, giving rise to the changeable effect. The dominant color of this shot fabric is green, the color of the warp. We examined the fabric under a microscope and found the green warp threads to be only half as thick as those of the red threads. The red threads are made of tightly spun silk fibers, each thread being visually distinct from the other. The green threads are formed with loose fibers, the separation between two threads being hardly distinguishable. The red yarns are stretched straight in the fabric; the thinner green threads wrap around the red ones. From the line of sight the green threads cover the red ones, so the fabric looks more green than red.

The fabric is woven on a loom with green yarns extended lengthwise and crossed by red filling yarns. The geometrical structure is symmetric; green threads are parallel to one another and perpendicular to red threads. There is exact correspondence of form and constituent configuration on opposite sides of a dividing green thread or a red thread on the fabric. Symmetry is expected in visual observation as well. The fabric has exactly the same color appearance even if the angles of incidence and reflectance are switched to their symmetric direction about a dividing thread or a point.

When the fabric is viewed along the direction of the green warp yarn, at acute angles it is affected mainly by the green color, changing to a dullish mixed color near the perpendicular. If the fabric is viewed along the direction of the red filling yarn, its color changes from red at acute angles to a mixed color near the perpendicular. The effects are much clearer in the backscattering lobe, that is to say, when the light source is behind the observer. This color-changing property of the shot fabric can be explained by the mutually obscuring effect of the two threads of different colors. When we observe the fabric, the threads in front in the three-dimensional space obscure those behind. The threads that are cut off from sight contribute little to our color perception of the fabric. The color of the thread in front appears to be dominant, and the fabric takes on this color.

When the source is behind the observer, effects of cast shadows are minimized, and one observes the pure effects of occlusion. For different positions of the source, shadowing effects partly or wholly mask the effect of occlusion.

3. Evaluation of the Bidirectional Reflectance Distribution Function

A. Evaluation of the Bidirectional Reflectance Distribution Function in the Plane of Incidence

The BRDF measurements in both the red and the green channels of six strips in the plane of incidence are presented in Fig. 4. In each row the BRDF is given as a function of the viewing angle \( \theta_v \), when the incident angles \( \theta_i \) are 10°, 30°, 50°, and 70°. The black curves are measurements in the red channel, and the gray curves are those in the green channel. The vertical black lines specify where \( \theta_i = \theta_v \), and the vertical gray lines denote where \( \theta_i = \theta_v \). The Helmholtz reciprocity relation is used to extend the data to the full angle span of the planes.

An overall inspection of Fig. 4 reveals that the black curves (red BRDF) intersect and intertwine with the gray curves (green BRDF). Furthermore, all the figures exhibit multiple peaks in the directional reflectance distribution. One peak appears in the vicinity of \( \theta_i = -\theta_v \); this is apparently related to specular reflection, where the scattered radiance peaks at the mirror angle of the incident light. One peak, whose magnitude is smaller than the previous one, lies in the area around \( \theta_v = \theta_i \); this can be attributed to the backscattering phenomenon, in which the scattered radiance reaches a maximum at the same angle as the incident light. The third peak, whose magnitude is comparable with the first mentioned peak, occurs near \( \theta_v = -70° \); this is apparently a peculiar feature of the geometrical structure of the shot fabric. In the following the characteristics found in the BRDF in the plane of incidence are discussed in detail.

In Fig. 5 the color ratios of red to green are given for strips 1 and 4. A general impression reveals that the color ratio changes at different incident and viewing angles. In strip 1, where the fabric is viewed in the plane perpendicular to the orientation of the green threads, the ratios of red to green are mainly higher than 1. The red-to-green ratio increases in magnitude as the viewing angles approach \( \theta_v = 70° \). There are spikes in the distribution of the ratios around \( \theta_v = 70° \), a sign that the green BRDF falls off faster than the red BRDF in the viewing angles around that region. In strip 4, where the fabric is viewed in the plane perpendicular to the orientation of the red threads, green BRDF's are higher than red, except around angles of \( \theta_v = -\theta_i \). This exception is discussed below in relation to specular reflection. Note that the red-to-green ratio decreases as the viewing angles approach \( \theta_v = 70° \).

The color-changing property of the shot fabric can be partially explained by the occlusion effect. The fabric is woven on a loom with a series of yarns ex-
tended lengthwise and crossed by a filling yarn. The threads in front in the three-dimensional space not only occlude those behind in the line of sight but also block them from illumination, so the front threads contribute mainly to the color perception of the fabric. In Fig. 6 we have simulated the fabric with two interwoven sine waves, the dark color representing the red threads, and the light denoting the green ones. Next to each graph a ratio of red to green is given. In column \( a \) the fabric is viewed in the plane along the direction of green threads. As the viewing direction changes from position 1 to 5, the simulated fabric gradually becomes dominated by red. This explains why the fabric looks predominantly red when the fabric is viewed in the plane perpendicular to the surface along the green threads. This phenomenon is shown in strip 1 in Fig. 5 where the red-to-green ratio increases as the viewing angles become large, approaching \( \theta_v = 70^\circ \). In column \( b \) the fabric is rotated, and there is color change if the simulated fabric is viewed from different directions. This color change can be observed in Fig. 4, where, on the right-hand side of the plots in a column, the red BRDF’s are found above the green BRDF’s in strip 1, gradually shifting downward under the green BRDF’s in strip 4, then rising up to slightly above the green BRDF’s again in strip 6. Here we assume that the direction of incidence coincides with the viewing direction such as to minimize effects of vignetting. In Fig. 4 we have marked the places with vertical gray lines where backscattering can occur, namely, \( \theta_v = \theta_i \). Note that the backscattering never peaks at exactly \( \theta_v = \theta_i \) in our data, the reason being that the BRDF measurement at \( \theta_v = \theta_i \) is not available. In our experimental setup the light source and the camera have specific physical sizes, so they cannot be placed on the same spot. The backscattering maxima are observed in all strips, although they possibly...
merge with specular peaks when \( \theta_i = 10^\circ \). They are more noticeable in the green BRDF, and the magnitudes are smaller than those of the other two peaks. Geometrical shadowing plays a role in the backscattering phenomenon. The surface elements that are oriented perpendicular to the incoming light are the prime contributors to the backscattering radiance. They are fully illuminated and reflect most light back along the incident direction. Viewed from a different angle, these surface elements are likely to be shadowed or occluded when they are located in concavities.

Because the shot fabric in this study is made of silk, which is a shiny material, one expects specular reflection to occur in both the red and the green BRDF’s. In Fig. 4 we have marked the places with vertical black lines where the specular reflection is expected to occur, namely, \( \theta_r = -\theta_i \). Specular reflection takes place at the material–air interface, and the specularly scattered beam retains the spectral composition of the irradiance. The profile spotlight used in this study has a higher luminous flux around red than around green. Around specular angles, namely, \( \theta_r = -\theta_i \), red BRDF’s are found to be higher than green (see Fig. 5).

After careful examination, we found that 14 of the 24 specular peaks in Fig. 4 occur at viewing angles that differ slightly by \(-5^\circ\) from the mirror angles of the incident direction. One explanation for this is that adjacent peaks appearing at viewing angles near each other seem to merge together. We mentioned above that three peaks are found in the BRDF, one around area \( \theta_r = -\theta_i \), one around area \( \theta_r = \theta_i \), and one around area \( \theta_r = -70^\circ \). When the incident angle \( \theta_i \) is 10°, the peaks around \( \theta_r = -\theta_i = -10^\circ \) and \( \theta_r = \theta_i = 10^\circ \) seem to blend together. When the incident angle \( \theta_i \) is 70°, the peaks around \( \theta_r = -\theta_i = -70^\circ \) and \( \theta_r = \theta_i = -70^\circ \) are hardly distinguishable. Some of the off-specular peaks occur because the specular maximum is combined with another peak nearby; the resultant crest is found at viewing angles other than the specular angle \( \theta_r = -\theta_i \).

A study of the literature shows that off-specular peak phenomena are found frequently in reflectance data. In their analytical model Middleton and Mun-gall used the specular reflection from small mirror-like facets on nonmetallic surfaces to explain the peaks. Torrance and Sparrow attributed the phenomenon mainly to the shadowing effect of adjacent facets. Foreshortening certainly contributes to the shift of specular peaks even when shadowing effects are absent, though. According to Minnaert the shift occurs because the sides of the surface facets are turned toward the viewing direction, which makes the surface seem like a slanting mirror. In our study the specular peaks appear at viewing angles slightly different from the mirror angle of the incident light, because the threads of the shot fabric are undulated. On one scale the surface structure of shot fabric is similar to interwoven sine waves. When light shines on the surface at an angle, the sine waves cast shadows on adjacent cavities. Furthermore, on a smaller scale, each sine wave consists of a multitude of silk fibers, which in turn can shadow the adjacent filaments.
Another distinct feature is that the BRDFs are high around $\theta_r = -70^\circ$ at different incident angles in all strips and are more pronounced in green. The magnitudes of these peaks are comparable with those of the specular maxima. These peaks are possibly related to the surface structure of the shot fabric, which is similar to interwoven sine waves. To check this assumption, we can simplify the scattering process and perform a ray-tracing calculation of the radiance from a single sine-shaped thread (see Appendix A for details of derivation). Assume that the light is incident on a sine thread in the $Y$–$Z$ plane and scattered in all directions [see Fig. 7(a)]. We trace a large number of light rays and collect the statistical information on the scattered radiance. The latter is found to depend on the angle of the incident light and the amplitude-to-wavelength ratio of the sine thread. The deviation is possibly due to the nonsinusoidal shape of the interwoven threads. Strip orientations are also given.

Fig. 7. (a) Light is incident on a sine thread and is scattered into space. Straight gray lines indicate the incident rays, and straight dark lines denote the reflected rays. The reflected radiance has a bimodal structure and is bounded by two angular limits. (b) We have plotted the incident angles against the two sets of viewing angles at which the scattered flux is at its maximum. The black filled circles are the experimental measurements, the open circles are the model results when the amplitude-to-wavelength ratio of the sine thread is set to 0.08, and the gray filled circles are the model results when the amplitude-to-wavelength ratio of the sine thread is set to 0.11. The ratios are different from the geometrical measurements, which are 0.23 for the green threads, 0.35 for the red ones. The deviation is possibly due to the nonsinusoidal shape of the interwoven threads. Strip orientations are also given.
In Fig. 7(b) we have plotted the two sets of viewing angles at which the scattered radiance is at its maximum against the incident angles. The black filled circles are the experimental measurements, which include the off-specular peaks and the maxima around $\theta_r = -70^\circ$. The open circles are what our ray-tracing calculation generates when the amplitude-to-wavelength ratio of the sine thread is set to 0.08, and the gray filled circles are the calculation results when the amplitude-to-wavelength ratio of the sine thread is set to 0.11. In strips 1, 2, and 6 the black filled circles (measurements) overlap the open ones. In strips 4, 3, and 5 the black filled circles correspond to the gray ones. Strip 1 is parallel to the green threads; strip 4 is parallel to the red threads. The green threads have a smaller amplitude-to-wavelength ratio than the red threads. In Subsection 2.C we mentioned that the amplitude-to-wavelength ratio of the green thread is 0.23 and that of the red thread is 0.35. This shows that this ray-tracing calculation predicts the right order of amplitudes of the sine threads. The fact that measured ratios are different from the ones used in the calculation may possibly be due to the nonsinusoidal shape of the profile of interwoven threads.

We can see that each set of circles forms two parallel slant lines and that the distances between the two slant lines are different in the six plots. In the ray-tracing calculation this distance can be fine tuned by means of changing the amplitude-to-wavelength ratio of the sine thread. Note that the distances between the slant lines in strips 2 and 6 are the same length; this is because the two strips are at symmetric angular directions from strip 1. Similarly, the distances between the slant lines in strips 3 and 5 are found to be equal. The model predictions for two amplitudes correspond to the experimental measurements. This indicates that the BRDF peaks around $-70^\circ$ are related to the surface structure of the shot fabric.

The cylindrical setup made it relatively easy to acquire a complete set of BRDF data; these were obtained by means of tilting the cylinder about its center. However, the sample points of the BRDF measurements are irregularly distributed discrete data points in the scattering space. We interpolated the experimental measurements to derive a continuous representation of surface scattering. The interpolation scheme is explained as follows. One data point consists of coordinates in a four-dimensional angular space, namely, $\theta_i, \phi_i, \theta_r, \phi_r$, and a measured BRDF value. The number of data points is quadrupled after the Helmholtz reciprocity principle and geometrical surface symmetries are applied to the actual BRDF measurements. For a random point without a BRDF value in the four-dimensional angular space a distance-weighted sum of nearby data points normalized by the sum of weights is used as the interpolated BRDF value for this point. The interpolated BRDF $v_j$ can be expressed as

$$v_j = \frac{\sum_{i=1}^{n} v_i (d_j/\alpha d_i + r_i)}{\sum_{i=1}^{n} (d_j/\alpha d_i + r_i)},$$

where $v_i$ is measured BRDF value, $r_i$ is distance from this point to the nearest $n$ data points, $d_j$ is the shortest distance of $r_i$, $\alpha$ is a damping factor whose numerical value is set to 1.0 in this calculation, and the weight is a ratio of $d_j$ to the sum of $\alpha d_i$ and $r_i$.

Figure 8 shows contour plots of BRDF’s in the hemispherical scattering space obtained from interpolation of the experimental measurements. The plots show the BRDF as a function of the scattering direction when the position of the source is held fixed. The directions of incident and scattered rays are represented as unit vectors on a unit hemisphere, and their projection on the X–Y plane defines the coordinate of the BRDF’s. The magnitudes of the BRDF’s are shown as gray-scale contours; i.e., the higher the magnitude, the whiter the contour. The direction of the incident ray is depicted by a white point in a white circle. The contour plots are made in both red and green, when the incident angles are $\theta_i = 30^\circ$ and $70^\circ$, $\phi_i = 0^\circ$, $30^\circ$, $60^\circ$, and $90^\circ$. The orientations of the red threads are drawn as black lines, and the green threads are drawn as gray lines.

We can see that the contour structures are different in the red and the green BRDF’s. In the leftmost and the rightmost columns, where the light is incident on the fabric in the plane perpendicular to the orientation of one of the color threads, the BRDF contours are found to have symmetric structures. The characteristics of the BRDF’s in the plane of incidence can be found on straight bands passing through the center of the contour plots. The three-peak feature appears in some plots. The specular peaks are evident in the red contours, high around the symmetric position of the incident ray on the opposite side of the center. The backscattering phenomenon is found either as a contour maximum next to the incident ray or as a contour plateau surrounding it. The peaks around $\theta_r = -70^\circ$ in BRDF of the strips are apparent in the lower two rows of contours. There is a gradual decline in contours from the incident ray toward the edge of the plots.

4. Conclusions

This study demonstrates that cylindrical sample geometry is a convenient arrangement for making BRDF measurements in the full scattering space. It enables one to collect a stream of data points in one scanned image; data can be acquired more easily and efficiently with this method than with the method of pasting a sample on a flat surface. A series of images allows one to obtain a large number of BRDF samples with relatively little effort. For BRDF studies of nonisotropic samples, strips of different orien-
tations can be cut from the material and wrapped around a cylinder.

Wrapping strips from different orientations of an anisotropic material around a cylinder makes it relatively easy to acquire a large set of BRDF data. However, the samplings of the measurements are irregularly distributed discrete data points in the scattering space. We have interpolated the experimental measurement to derive a continuous representation of surface scattering over the whole hemisphere.

This BRDF investigation reveals that shot fabric has unusual reflectance properties. There are three
peaks in the BRDF in the plane of incidence; one is attributed to backscattering phenomenon, and the other two peaks are attributed to the interwoven surface structure of the shot fabric. A simplified ray-tracing program confirmed that there are two peaks in the radiance reflected from interwoven sine waves. The color-changing property of the shot fabric is largely explained by the occlusion effect. The cangiante effect is especially distinct when the light source is behind the observer. Portrait painters have used this peculiar scattering property of shot fabric when depicting draped garments. The results presented here have direct relevance for the further understanding of the reflectance properties of anisotropic materials.

Appendix A: Trace Light Rays Scattered from a Sine Thread

Assume a sine thread on the $Y-Z$ plane, the profile of the thread being characterized by

$$ p(y, a) = \{0, y, a \sin y\}. $$

The tangent vector on the sine wave is

$$ t(y, a) = \left\{0, \frac{1}{\left(1 + a^2 \cos^2 y\right)^{1/2}}, \frac{a \cos y}{\left(1 + a^2 \cos^2 y\right)^{1/2}} \right\}. \quad (A2) $$

Two unit vectors that are orthogonal to the tangent vector are given by

$$ e_1 = \{1, 0, 0\}, \quad (A3) $$

$$ e_2 = \left\{0, -\frac{a \cos y}{\left(1 + a^2 \cos^2 y\right)^{1/2}}, \frac{1}{\left(1 + a^2 \cos^2 y\right)^{1/2}} \right\}. \quad (A4) $$

The incident light is denoted by

$$ i(\theta) = \{0, \sin \theta, \cos \theta\}. \quad (A5) $$

The direction of the randomly scattered beam can be given by

$$ r(y, a, \theta) = -[t(y, a)i(\theta)]t(y, a) + \left\{1 - [t(y, a)i(\theta)]^2\right\}^{1/2} \times (e_1 \cos \phi + e_2 \sin \phi), \quad (A6) $$

where $\phi$ is a random angle between 0 and $\pi$. In Fig. 9 we show a simulation result of 3000 reflected rays when the light is incident at $25^\circ$. The distribution of the reflected rays lies between two angular limits and forms a curved band with thick edges. It shows that the reflected radiance has a bimodal structure.

References