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CS 647 - Image Synthesis
Assignment 2: Radiometry:

1. The left half of the hemisphere's projected area is still $\frac{\pi}{2}$. The right half is just the projection of the tilted hemisphere:

$$E = \frac{\pi L}{2} + \frac{\pi L}{2}\cos\theta \tag{1}$$

$$E = \frac{\pi L}{2}(1 + \cos\theta) \tag{2}$$

2. If you project one edge of the square onto the unit sphere, then find the projected area, you can multiply by 4 to get your answer.

   The area of the projected angle is θ, the angle at the origin of the triangle made by the edge and the origin. Multiply θ by the cosine of the angle, ϕ, made between the triangle and the $z = 0$ plane, and you get the area of the projected triangle.

$$E = 4(A_{triangle}) \tag{3}$$

$$E = 4(\theta\cos\phi) \tag{4}$$

$$E = 4(\frac{\pi}{4}\cos\frac{\pi}{4}) \tag{5}$$

$$E = \pi\frac{\sqrt{2}}{2} \tag{6}$$

3. This derivation follows easy from the cosine-cubed law. The only difference is to cosine-weight the infinitesimal area dA.

$$\Phi = EdA = Id\omega \tag{7}$$

$$EdA = \frac{\Phi}{4\pi}d\omega \tag{8}$$

$$EdA = \frac{\Phi}{4\pi}\frac{\cos\theta}{r^2}(dA\cos\theta) \tag{9}$$

$$E = \frac{\Phi}{4\pi} \frac{\cos^2 \theta}{r^2} \tag{10}$$

Plugging in that the radius r is $\frac{f}{\cos \theta}$, we get:

$$E = \frac{\Phi}{4\pi} \frac{\cos^4 \theta}{f^2} \tag{11}$$