Problem 1. [22 points]
Do problems 1.4 and 1.15 from Sipser.

Problem 2. [12 points]
Do problem 1.23 from Sipser.

Problem 3. [9 points]
True or false (prove your answer):

(a) All subsets of a regular language are regular.
(b) All supersets of a regular language are non-regular.
(c) All finite languages are regular.

Problem 4. [10 points]
Are these languages regular? Prove your answer. \(x^R\) is \(x\) written backwards.

(a) \(\{xwx^R \mid x, w \in (0 + 1)^+\}\)
(b) \(\{xx^Rw \mid x, w \in (0 + 1)^+\}\)

Problem 5. [10 points]
Do problem 1.24 from Sipser.

Problem 6. [20 points]
Do problem 1.31 from Sipser.

Problem 7. [20 points]
Let \(L\) be any language, and define

\[L_{\frac{1}{2}} = \{x \mid \exists y \text{ such that } |x| = |y| \text{ and } xy \in L\}.
\]

\(L_{\frac{1}{2}}\) is the set of all *first halves* of strings in \(L\). Prove that if \(L\) is regular, so is \(L_{\frac{1}{2}}\).