Problem 1.  [15 points]

Show that there is no program for solving the following problem: Given a program $Q$ and an input $y$, during the execution of $Q$ on $y$ do two variables $A$ and $B$ ever take on the same value?

Problem 2.  [15 points]

Consider the deterministic Turing machine $M$ given by:

$$M = (\{q_0, q_1, q_2\}, \{a, b\}, \{a, b, B\}, \delta, q_0, B, \{q_2\})$$

which has exactly four transitions defined:

1. $\delta(q_0, a) = (q_0, B, R)$
2. $\delta(q_0, b) = (q_1, B, R)$
3. $\delta(q_1, b) = (q_1, B, R)$
4. $\delta(q_1, B) = (q_2, B, R)$

(a) specify the execution trace of $m$ on the two input strings $abb$ and $bb$. give the complete sequence of id’s for both inputs.

(b) provide a regular expression for $L(m)$.

(c) suppose we added the following transition to the above machine:

$$\delta(q_1, a) = (q_0, b, r).$$

provide a regular expression for the language of the resulting turing machine.

Problem 3.  [20 points]

Design a Turing machine that takes as input a number $N$ and adds 1 to it in binary. To be precise, the tape initially contains a $ followed by $N$ in binary. The tape head is initially scanning the $ in state $q_0$. Your TM should halt with $N+1$, in binary, on its tape, scanning the leftmost symbol of $N+1$, in state $q_f$. You may destroy the $ in creating $N+1$, if necessary. For instance, $q_0$\$10011 \not\overset{\star}{\rightarrow} q_f$\$10100, and $q_0$\$11111 \not\overset{\star}{\rightarrow} q_f$\$100000.

(a) Give the transitions of your Turing machine, and explain the purpose of each state.

(b) Show the sequence of ID’s of your TM when given input $\$111$.
Problem 4.  [20 points]

Consider the nondeterministic Turing machine

\[ M = (\{q_0, q_1, q_2, q_f\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, B, \{q_f\}) \]

Informally but clearly describe the language \( L(M) \) if \( \delta \) consists of the following sets of rules:

1. \( \delta(q_0, 0) = \{(q_0, 1, R), (q_1, 1, R)\} \)
2. \( \delta(q_1, 1) = \{(q_2, 0, L)\} \)
3. \( \delta(q_2, 1) = \{(q_0, 1, R)\} \)
4. \( \delta(q_1, B) = \{(q_f, B, R)\} \)