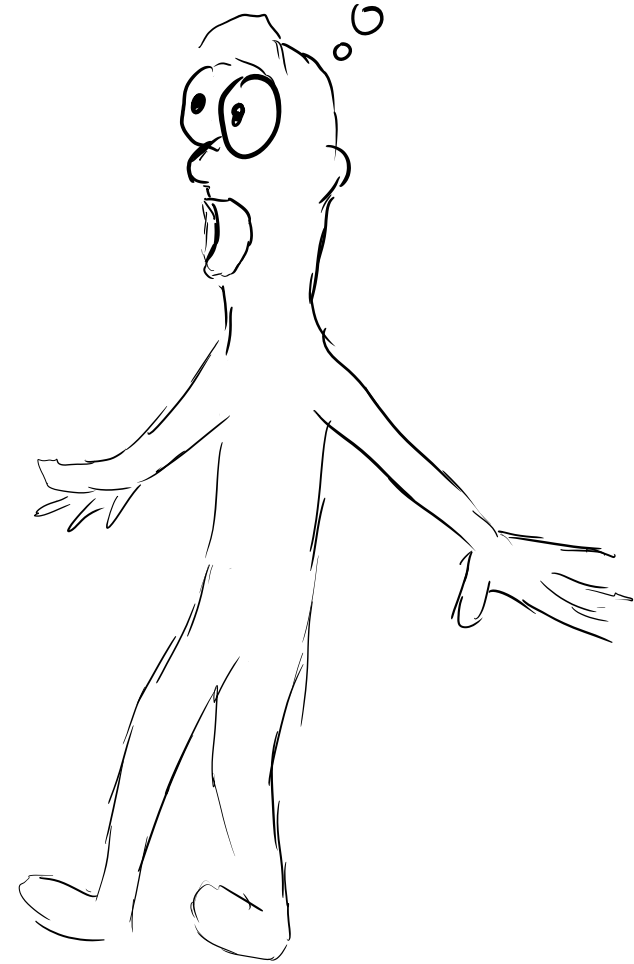


Why are they  
staring at me?



Whoah! That guy's  
top set of hands  
are backwards!



$$\emptyset \subseteq Q$$

$$G = \{ \{1\} \} \quad P(G) = \{ \{ \}, \{ \{1\} \} \}$$

$$G = \{ \blacksquare \}$$

$$P(G) = \{ \{ \}, \{ \blacksquare \} \}$$

$$H = \{1, 2, 3\}$$

$$\{1, 1\} \rightarrow \{1\}$$

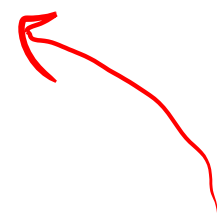
$$\{1, 2\}$$

$$\{1, 3\}$$

$$\{2\}$$

$$\{2, 3\}$$

$$\{3\}$$



$$\{2, 1\}$$

Easy

Hard

concept

Boolean  
arithmetic

↑  
↓  
↑

Pacing

conceptual

Number  
arithmetic

+ -  
x ÷

Time

5yr - 8yr

Boolean  
Algebra

$x$	$y$	$3(x+y)$	$= 3x + 3y$
0	0	$\underbrace{3(0+0)}_0$	$= \underbrace{3 \cdot 0 + 3 \cdot 0}_0$
0	1	$\underbrace{3(0+1)}_3$	$= \underbrace{3 \cdot 0 + 3 \cdot 1}_3$
0	2		
⋮			
⋮			
⋮			

✓

✓

$\neg\neg P \equiv P$  double negation  
 is equivalent to

$- - x = x$

$P$	$\neg(\neg P)$
$\top$	$\top$
$\perp$	$\top$

$\neg\neg P$                        $P$   
 $P$                                        $\neg\neg P$

$R \rightarrow (P \wedge \neg Q)$   
 $\equiv$

$\neg\neg\neg\neg R \rightarrow \neg\neg\neg\neg (P \wedge \neg Q)$

$$X \cdot 0 = 0$$

$$X \cdot 1 = X$$

$$X + 0 = X$$

$$\underline{P} \rightarrow Q \equiv \neg P \vee Q \quad \text{Defn of Implication}$$

DeMorgan's Law

$$\neg(P \wedge Q) \equiv (\neg P \vee \neg Q)$$

$$\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$$

Distributive laws

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$$

$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$$

$$3(x+y) = 3x+3y$$



# Properties of Binary Operators

2 operands

+

x

^

-

x ^ y

x + y

-x

-x

-x

x ? y : z

Commutative

$$x + y = y + x$$

Operands can  
swap positions

Associative

$$(x + y) + z = x + (y + z)$$

Paren can change  
w/o change meaning

$$x - y - z$$

$$(1 - 2) - 5 = -6$$

$$1 - (2 - 5) = 4$$

$$1 - 2 = -1$$

$$2 - 1 = 1$$

$\neg$   
 $\wedge$   
 $\vee$   
 $\oplus$   
 $\rightarrow$   
 $\leftrightarrow$

Associative

nonsense

✓

✓

✓

✗

✗

Commutative

nonsense

✓

✓

✓

✗

✓

$$A \leftrightarrow B \equiv B \leftrightarrow A$$

$$(A \leftrightarrow B) \leftrightarrow C \neq A \leftrightarrow (B \leftrightarrow C)$$

$$(A \wedge B) \leftrightarrow C$$

$$X \wedge (Y \vee Z)$$

$$x \cdot y \cdot z$$

$$x + y + z$$

$$(x \cdot y) + z$$

$$x \wedge (y \wedge z)$$

$$(x \wedge y) \wedge z$$

$$P \rightarrow Q \equiv \neg P \vee Q$$

$$Q \rightarrow P \equiv \neg Q \vee P$$

$$(P \rightarrow Q) \rightarrow R \equiv \neg(P \rightarrow Q) \vee R$$

$$P \rightarrow (Q \rightarrow R) \equiv \neg P \vee (Q \rightarrow R)$$