
GO

$$((P \to Q) \land (Q \to R)) \to (P \to R)$$

$$\neg ((\neg P \lor Q) \land (\neg Q \lor R)) \lor (\neg P \lor R)$$

$$\neg (((\neg P \lor Q) \land \neg Q) \lor ((\neg P \lor Q) \land R)) \lor (\neg P \lor R)$$

$$(\neg P \land \neg Q) \lor (Q \land \neg Q)$$

$$\bot$$

$$\neg ((\neg P \land \neg Q) \lor ((\neg P \lor Q) \land R)) \lor (\neg P \lor R)$$

$$(\neg (\neg P \land \neg Q) \land \neg ((\neg P \lor Q) \land R)) \lor \neg P \lor R$$

$$((P \lor Q) \land (\neg (\neg P \lor Q) \lor \neg R)) \lor \neg P \lor R$$

no sorting algorithm based on comparisons is faster than n log(n)

$$\nexists a \cdot (S(a) \land C(a)) \to \neg F(a)$$

$S(a)$: a is a sorting algorithm

Predicate

function that return true or false

incomplete proposition

Predicate

Proposition template

$P(x, Q(x))$

almost always wrong

B_1 : my algorithm is better than yours

$B_2(a)$: my algorithm is better than $\underline{\quad\quad}_a$

$B_3(m, a)$: m is better than a

$B_4(m, c, a)$: $\underline{m}$ is $\underline{c}$ than $\underline{a}$

$B_4($~~sorting~~, ~~more interesting~~, ~~printing~~$)$

S: sorting

m: more interesting

p: printing

not Boolean

$B_4(s, m, p)$

$$P(x) \leftrightarrow Q(x)$$

$$P(x) \leftrightarrow Q(y) \quad \text{- diff}$$

$$P(y) \leftrightarrow Q(y)$$

```
int f(int x) { return 2*x; }
```

```
def f(x):
    return 2*x
```

$$f(x) = x^2 + 2$$

$$f(y) = y^2 + 2$$

testing code

works for Everything

opposites?

breaks for Something

break for Nothing

for all

$\forall x \in S.\ P(x) \leftrightarrow Q(x)$

$(P(x_1) \leftrightarrow Q(x_1)) \wedge (P(x_2) \leftrightarrow Q(x_2)) \wedge (P(x_3) \leftrightarrow Q(x_3)) \wedge \ldots$

there exists

$\exists x \in S.\ \neg P(x)$

such that

$(\neg P(x_1)) \vee (\neg P(x_2)) \vee (\neg P(x_3)) \vee \ldots$

does not exist

$\nexists x \in S.\ Q(x) \equiv \neg(\exists x \in S.\ Q(x))$

Quantifiers

$\forall \quad \exists \quad \nexists$

F : set of all types of food

domain : fruit

$P(x)$: I like $\underline{\qquad}_{x}$

a : apples

$P(a) \equiv$ I like apples

$\forall y \in F .\ P(y) \equiv$ I like every type of food

$\exists y \in F .\ P(y) \equiv$ There is some type of food I like

E $\quad \nexists y \in F .\ P(y) \equiv$ there does not exist some type of food I like
I don't like any food

A $\quad \forall y \in F .\ \neg P(y) \equiv$ same

$\neg \exists y \in F .\ P(x)$

De Morgan

$\neg (P \vee Q \vee R \vee \ldots) \neg \exists$

$\forall \neg$

$(\neg P \wedge \neg Q \wedge \neg R \wedge \ldots)$

$\neg (\forall y \in F .\ \neg P(x))$

$\exists y \in F .\ \neg\neg P(y)$

$\exists y \in F .\ P(y)$