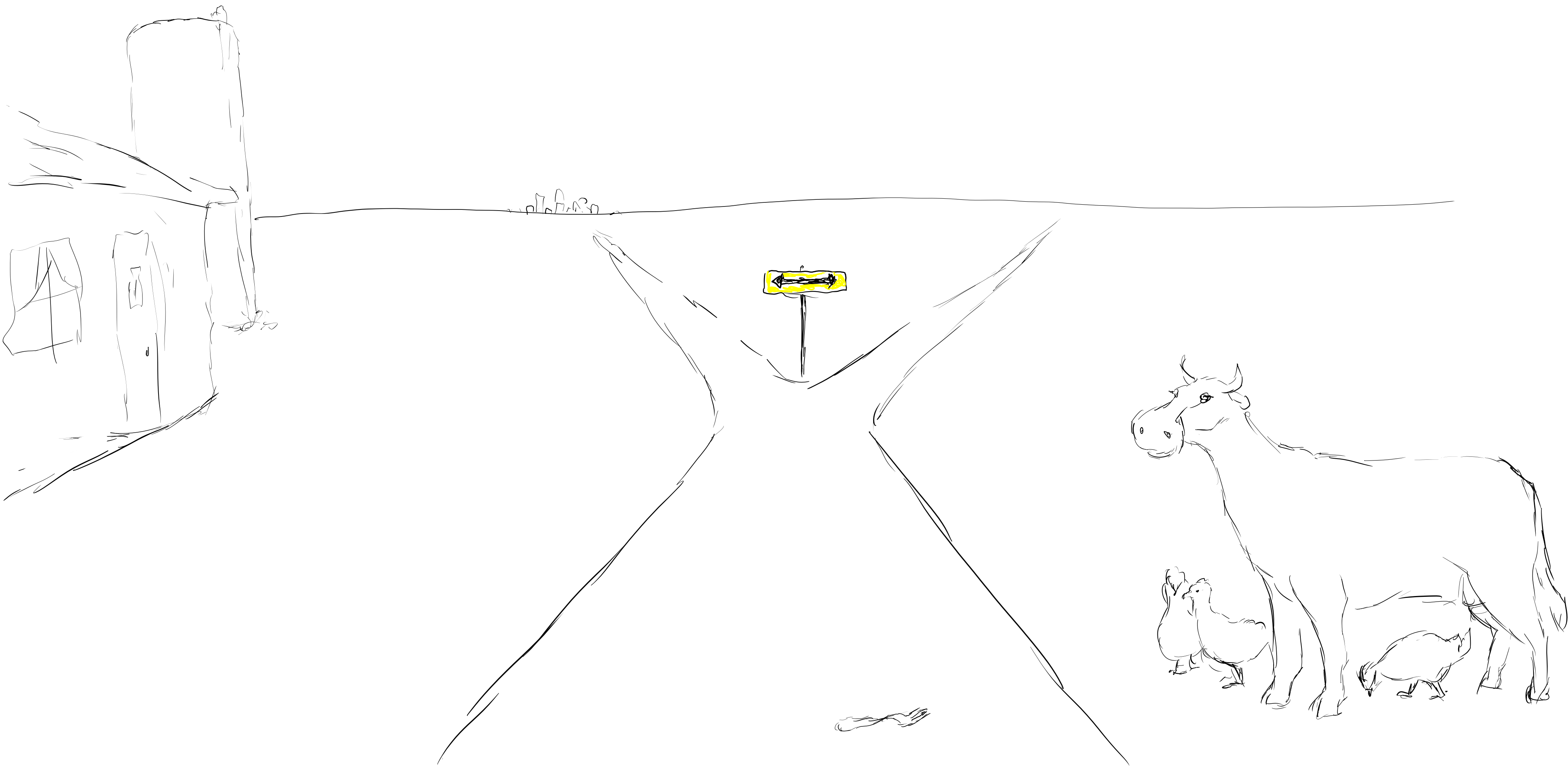


$$\forall x \in \mathbb{N} \, . \, P(x) \models P(2102) \models \exists x \in \mathbb{N} \, . \, P(x)$$

$$P(3)$$

$$\vdots$$

$$\left(\begin{array}{c} |S| = 0 \\ S = \{\} \\ S = \emptyset \end{array} \right) \wedge \left(\forall x \in S \, . \, P(x) \right) \models \top$$

$$\left(S \neq \{\} \right) \wedge \left(\forall x \in S \, . \, P(x) \right) \models \exists x \in S \, . \, P(x)$$

Vacuously true

Trivially true

$P(y)$

$\forall x \quad P(x)$

$\forall x \in \{2, 3\} \quad P(x)$ $\qquad P(x) : \left|x - \frac{5}{2}\right| = \frac{1}{2}$

if $x = 2$, $\left|2 - \frac{5}{2}\right| = \left|\frac{-1}{2}\right| = \frac{1}{2}$

if $x = 3$, $\left|3 - \frac{5}{2}\right| = \left|\frac{1}{2}\right| = \frac{1}{2}$

done!

$\forall x \in \{2, 3\}$

$(x=2) \vee (x=3)$

Disjunctive Tautology

$A \vee B \vee C \vee D \equiv T$

Case A
....
$\therefore X$

assum A is true

Case B
....
$\therefore X$

Case C
....
$\therefore X$

assum C is true

Case D
....
$\therefore X$

X in general

Prove

$P \to Q \equiv \neg P \vee Q$

$\boxed{P \vee \neg P \equiv T}$

Case $\boxed{P}$ is T:

$P \to Q \equiv T \to Q \equiv Q$

$\neg P \vee Q \equiv \neg T \vee Q \equiv \bot \vee Q \equiv Q$

$\therefore \boxed{P \to Q \equiv \neg P \vee Q}$

Case $\boxed{\neg P}$ is T:

$P \to Q \equiv \neg\neg P \to Q \equiv \neg T \to Q$

$\equiv \bot \to Q \equiv T$

$\neg P \vee Q \equiv T \vee Q \equiv T$

$\therefore \boxed{P \to Q \equiv \neg P \vee Q}$

$P \to Q \equiv \neg P \vee Q$

in this case

Because

English read aloud

Tautology

Proof. Either $\underbrace{P}_{x}$ or $\underbrace{\neg P}_{y}$.

Case 1: $\underbrace{P}_{x}$

Because $P = T$ in this case,
$P \to Q$ is $T \to Q$ which simplifies to Q;
and $\neg P \vee Q$ is $\bot \vee Q$ which simplifies to Q.
Because both are equiv to Q, they are equiv to each other.

Case 2: $\underbrace{\neg P}_{y}$

in this case, $\neg P \vee Q \equiv T$ by simplification
and $P \to Q$ is also T because the antecedent is false. Thus, $\neg P \vee Q \equiv P \to Q$

Because $\underbrace{P \to Q \equiv \neg P \vee Q}_{z}$ is true in both cases,
it is true in general. $\square$

$f(x)$

→ if $x \% 2 = 0$, return $\overbrace{x/2}$

→ else return $(\underbrace{\underbrace{3x}_{\text{odd}}+1}_{\text{even}})/2$

$\forall x \in \mathbb{N}.\ f(x) \in \mathbb{N}$

Proof.

→ Either x is even or x is odd.

Case x is even:

in this case we use the if (not else) and return an even non-neg integer divided by 2, which is a non-neg integer.

Case x is odd:

in this case, we use the else branch.

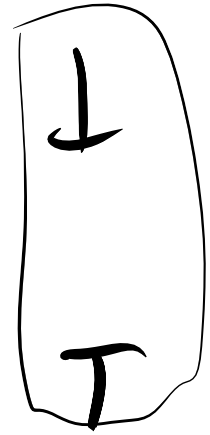
$3 \cdot x$ is the prod of 2 odd num, and is thus odd.

1 + an odd num is even

an even num / 2 is an integer.

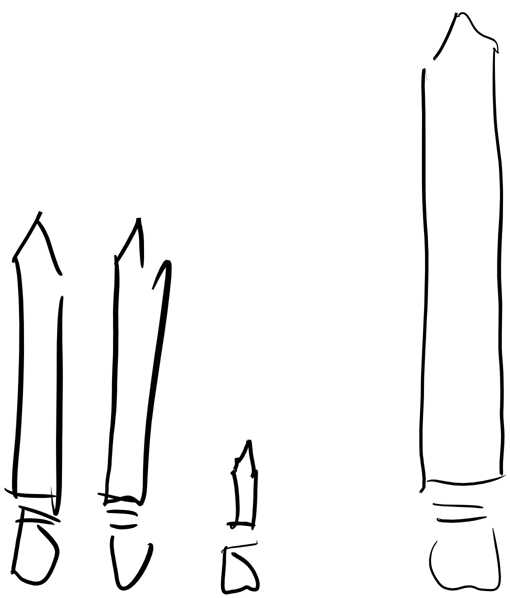
Because $f(x)$ is $\in \mathbb{N}$ in both cases, it is $\in \mathbb{N}$ in general.

You are here



you are 20' tall

you are [illegible]



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