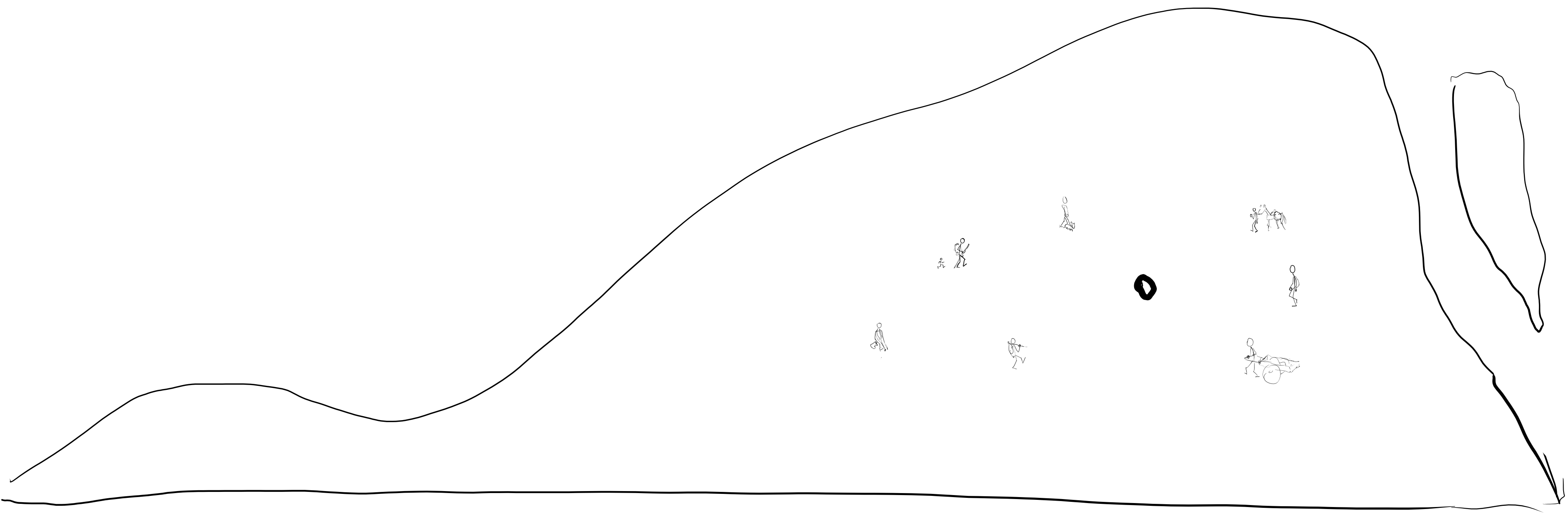

$$\neg \forall \quad \neg(\quad \wedge \quad \wedge \quad \wedge \wedge)$$

$$(\neg \quad \vee \neg \quad \vee \neg \quad)$$

$$\neg \left(\forall x \in \mathbb{N}^{+} . \exists y \in \mathbb{N}^{+} . xy = 1 \right)$$

$$\exists x \in \mathbb{N}^{+} . \forall y \in \mathbb{N}^{+} . xy = 1$$

$$\neg \exists$$

$$\forall y \in \mathbb{N}^{+} . \neg (xy = 1)$$

Proof by induction

$\forall n \geq 0 .\ P(n) \to P(n+1)$

$\vdash P(0)$ - base case

$\forall n > 0 .\ \underbrace{P(n-1) \to P(n)}$ - inductive step

assume $\boxed{P(n-1)}$ inductive hypothesis

$\to$ Prove $P(n)$

$\therefore\ \forall x \in \mathbb{N} .\ P(x)$

non-thm $\forall x \in \mathbb{N} .\ x = 0$

Base $x = 0 = 0$ ✓

ind assm $(n-1) = 0$

$n = 1 \neq 0$

$$\sum_{x \in S} f(x)$$

$$\sum_{x=0}^{n} x = 0 + 1 + 2 + 3 + \dots + (n-1) + n$$

$$x \in \{ i \mid i \in \mathbb{Z} \wedge i \geq 0 \wedge i \leq n \}$$

$$\sum_{x=2}^{1} f(x) = \sum_{x \in \{\}} f(x) = 0$$

$f(x)=1$

thm: $\forall n \in \mathbb{Z}^+ . \sum_{i=1}^{n} 1 = n$

Base case $\boxed{n=1}$

sum is $1 = n = 1$

Base case $n=0$

sum zero value $=0 = n=0$

Inductive step

assum $\sum_{i=1}^{n-1} 1 = n-1$.

$1+1+1+1 \ldots +1$

then $\underbrace{1 + \sum_{i=1}^{n-1} 1}_{} = \underbrace{1+n-1}_{n}$

$= \boxed{\sum_{i=1}^{n} 1}$

$\sum_{i=1}^{n-1} 1 = n-1 \longrightarrow \sum_{i=1}^{n} 1 = n$

By Principle of induction, the thm holds $\forall n \in \mathbb{Z}^+ \square$

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$$

0	0	$\frac{0(1)}{2}$
$0+1$	1	$\frac{1(2)}{2}$
$0+1+2$	3	$\frac{2(3)}{2}$
$0+1+2+3$	6	$\frac{3(4)}{2}$

Base case: $n=0$

$$\sum_{i=0}^{0} i = 0 = \frac{0(0+1)}{2} = 0$$

inductive step: assum $\sum_{i=0}^{n-1} i = \frac{(n-1)(n-1+1)}{2}$

$$n + \sum_{i=0}^{n-1} i = n + \frac{(n-1)(n)}{2} = n + \frac{n^2-n}{2} = \frac{n^2+n}{2} = \frac{n(n+1)}{2}$$

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$$