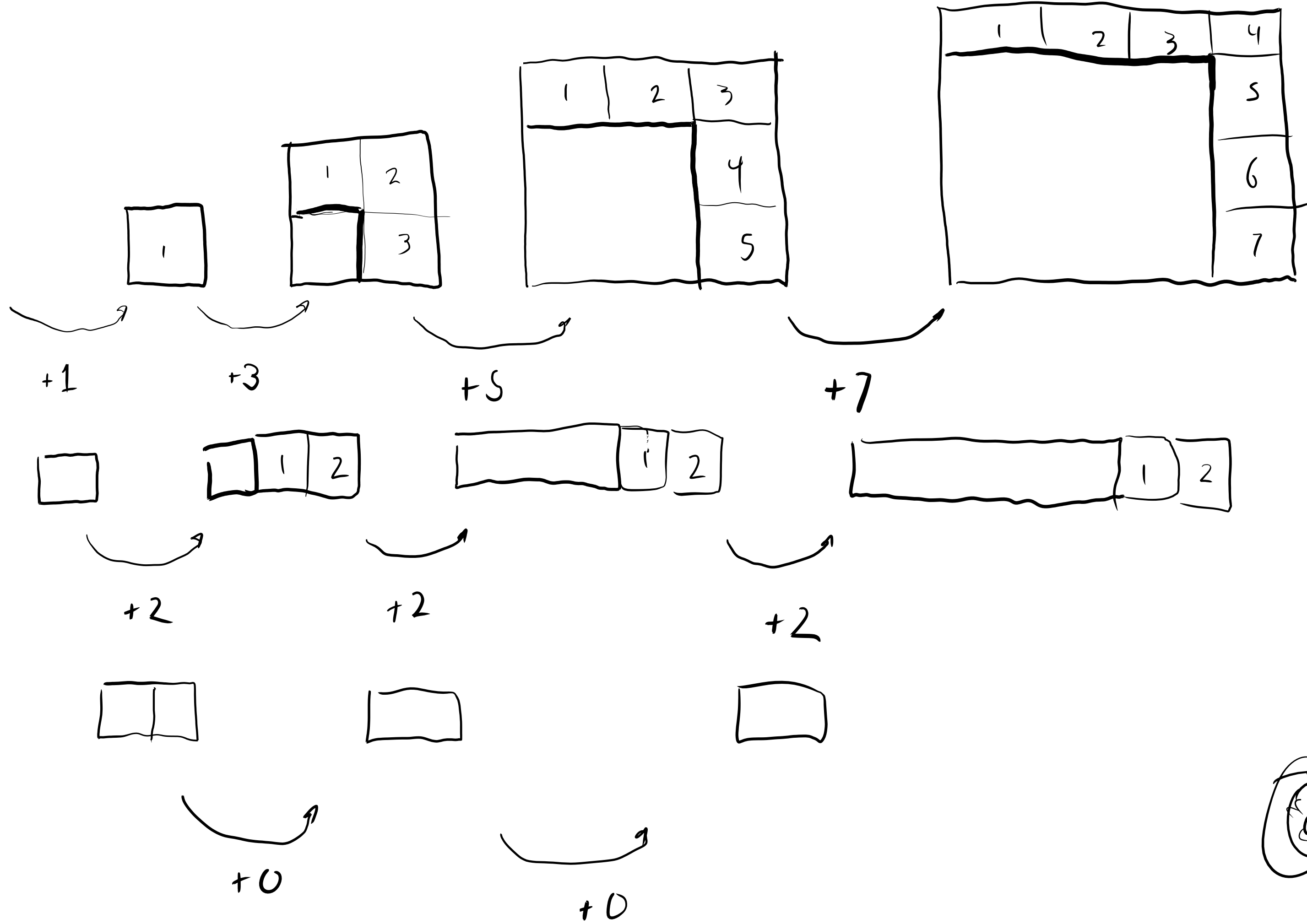
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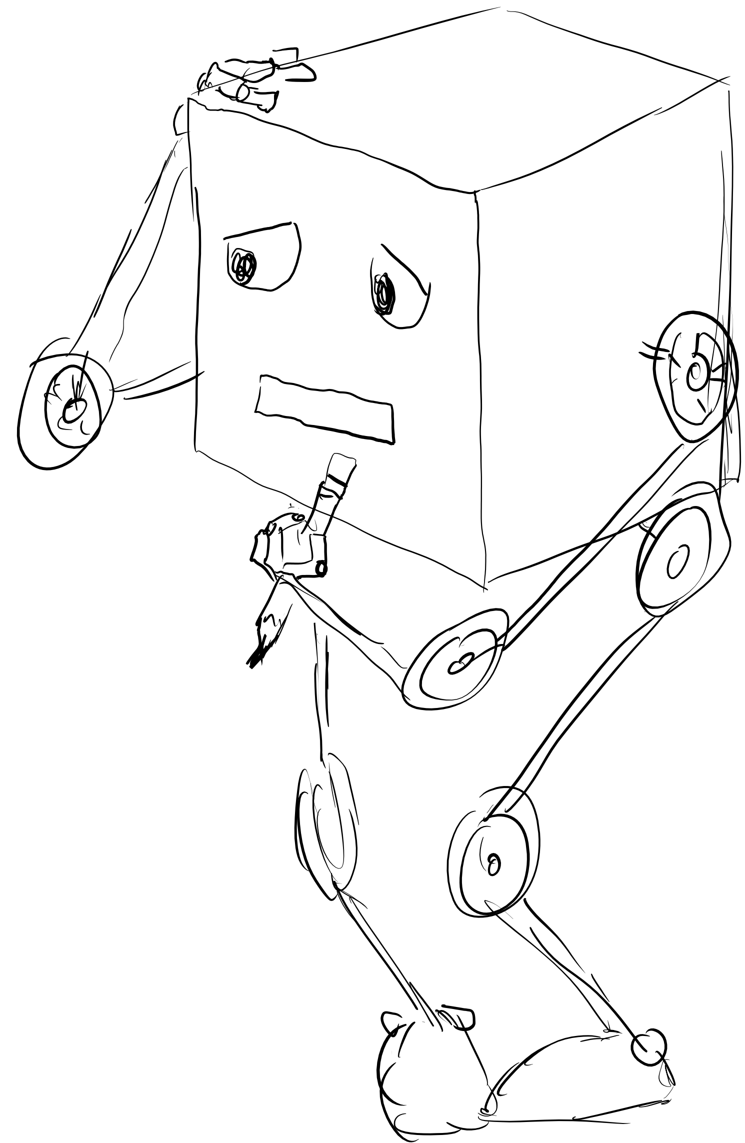

$$\forall n \in \mathbb{N} . \sum_{i=1}^{n} 2i-1 = n^2$$

0 $\quad 0 = 0$

1 $\quad 1 = 1$

2 $\quad 1+3 = 4$

3 $\quad 1+3+5 = 9$

Proof.

We proceed by induction.

Base cases:

case $n=0$: $\sum_{i=1}^{n} 2i-1$ is empty, which means it is zero, as is 0^2.

case $n=1$: $\sum_{i=1}^{n} 2i-1 = 2(1)-1 = 1$, which the same as $n^2 = 1^2 = 1$.

Inductive step:

assume $\sum_{i=1}^{n-1} 2i-1 = (n-1)^2$. Adding $2n-1$ to both sides gives $(2n-1) + \sum_{i=1}^{n-1} 2i-1 = (2n-1) + (n-1)^2$.

The left-hand side is just $\sum_{i=1}^{n} 2i-1$.

The right hand side is $2n-1 + n^2-2n+1$, which simplifies to just n^2. Hence, $\sum_{i=1}^{n} 2i-1 = n^2$

By principle of induction, it follows that $\forall n \in \mathbb{N} . \sum_{i=1}^{n} 2i-1 = n^2$ ∎

→ induction 1: Simple ind

smallest

⋮

non-empty sum

2. Base $n=0$ sum of nothing $= 0^2$

$n=1$ $\quad 2 \cdot 1 - 1 = 1 = 1^2$

3. inductive

assume $\sum_{i=1}^{n-1} 2i-1 = (n-1)^2$

$(2n-1) + \sum_{i=1}^{n-1} 2i-1 = (n-1)^2 + (2n-1)$

$(n^2 - \cancel{2n} + \cancel{1}) + \cancel{2n} - \cancel{1}$

$\sum_{i=1}^{n} 2i-1 = n^2$

4. By induction, . . .

$$\sum_{i=1}^{n} i$$

-3	0
-2	0
-1	0
0	0
1	1
2	3
3	6
	$\vdots$

$+1$

$+2$

$+3$

$$n + \sum_{i=1}^{n-1} i$$

$$= \sum_{i=1}^{n} i$$

non-empty

Steps

ind

Base $\overbrace{y=0}$; loop run 0 times
and then pow returns.

induction step
assume pow(x, $\overbrace{y-1}$) terminates
then pow(x,y) will run
the loop once, and then run it
as many more times as pow(x,y-1) runs it,
which is finite by our assumption.
Hence, pow(x,y) terminates

By prin. of ind, pow(x,y) term. $\forall y \geq 0$

pow(x,y)
$Z = 1$
while $y > 0$
$Z *= x$
$y -= 1$
return Z

$\forall x \in \mathbb{R}.\ \forall y \in \mathbb{N}.\ \ pow(x,y) = x^y$

ind
Base $y=0$ $pow(x,y) = 1$
$x^0 = 1$ $\forall x$ ✓

$y=1$ $pow(x,y) = x$
$x^1 = x$ $\forall x$ ✓

ind. step
assume $pow(x,y-1) = x^{y-1}$
then pow(x,y) runs the loop once,
makes $z = x$ and $y = y-1$, and then
runs loop as pow(x,y-1) does, which
multiplies z by x^{y-1}. $x \cdot x^{y-1} = x^y$
so $pow(x,y) = x^y$ ▯

pow(x,y)
if $y <= 0$, return 1
else return $x * pow(x, y-1)$

Base
$y=0$, $pow(x,y) = 1$
$x^y = x^0 = 1$

ind
assume $pow(x,y-1) = x^{y-1}$
then pow(x,y) returns
$x\, x^{y-1} = x^y$ ▯