logarithm

$\log_{\boxed{10}}(x) = y$ means

- $10^y = x$ (inverse of exponentiation)
- you can write x in $\lceil y \rceil$ digits
- you can divide x by 10 y times before you get ≤ 1

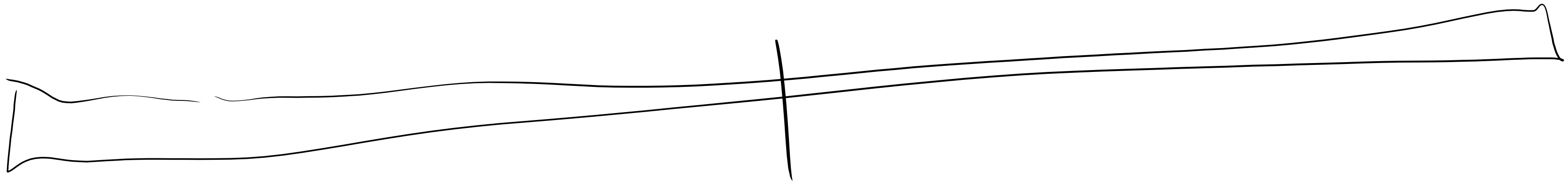
5-digit numbers

9 10 10 10 · 10 $= 9 \cdot 10^4$

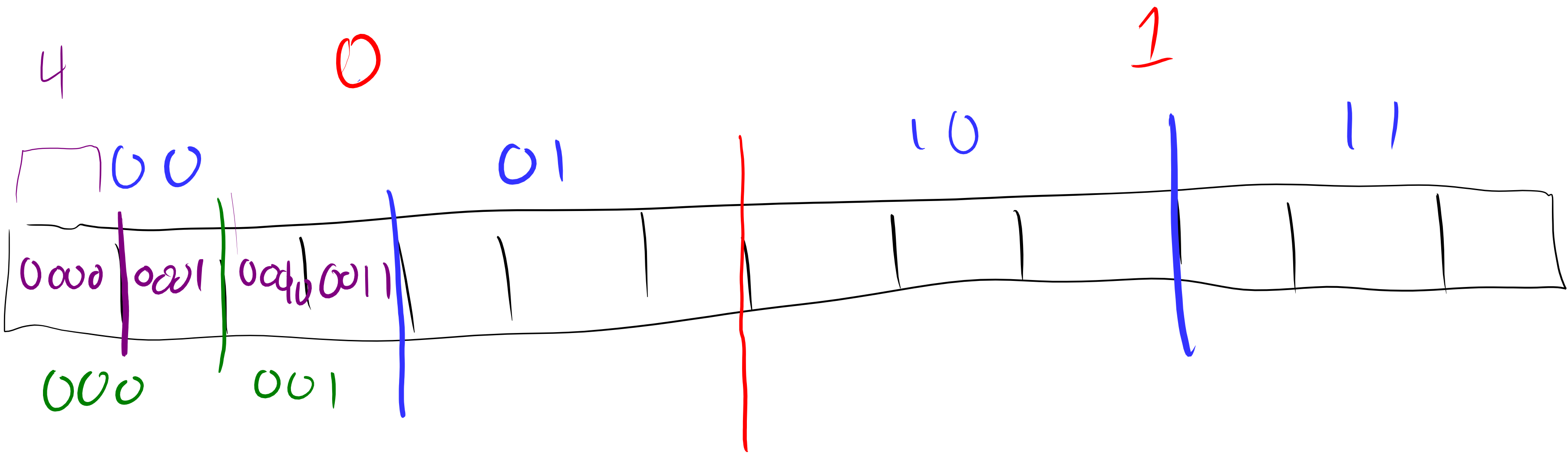
$\leq$ 5-digit

10 10 10 10 10

10^5



$\{0,1\}^*$

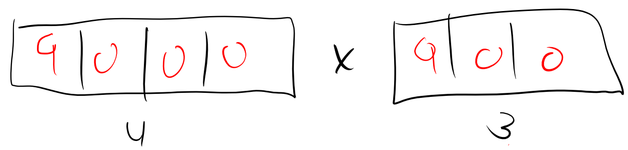


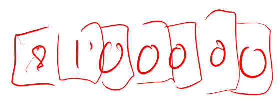
$$4 = \lceil \log_2(13) \rceil$$

log Identities

$$\log_b(b^x) = x$$

$$b^{\log_b(x)} = x$$





$$\log_{\cancel{x}}(a \cdot b) = \log_{\cancel{x}}(a) + \log_{\cancel{x}}(b)$$

$$\log_b(x^y) = y \cdot \log_b(x)$$

$$\log_b\left(\frac{x}{y}\right) = \log_b(x \cdot y^{-1}) = \log_b(x) + \log_b(y^{-1})$$

$$= \log_b(x) - \log_b(y)$$

base-100 digits

00

01

02

⋮

99

base-10

1	2	4	5	0	3	1	2

$= x$ $\log_{10}(x) \sim 8$

base-100

12	45	03	12

$= x$ $\log_{100}(x) \sim 4$

$$\log_{10^2}(x) = \frac{1}{2} \log_{10}(x)$$

$$\log_{a^b}(x) = \frac{1}{b} \log_a(x)$$

$\log_a(x)$

$\log_b$

$\log_{b^y}(x)$

$\boxed{a = b^y}$

$\log_b(a) = y$

$\frac{1}{y}\log_b(x)$

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

$$\log_1(2) = y \quad \stackrel{\Delta}{=} \quad 1^y = 2 \qquad \text{impossible}$$

$$\log_x(1) = 0 \quad \stackrel{\Delta}{=} \quad x^0 = 1$$

base > 1

$$\log_b(x) > \log_b(y)$$

$$\equiv$$

$$x > y$$